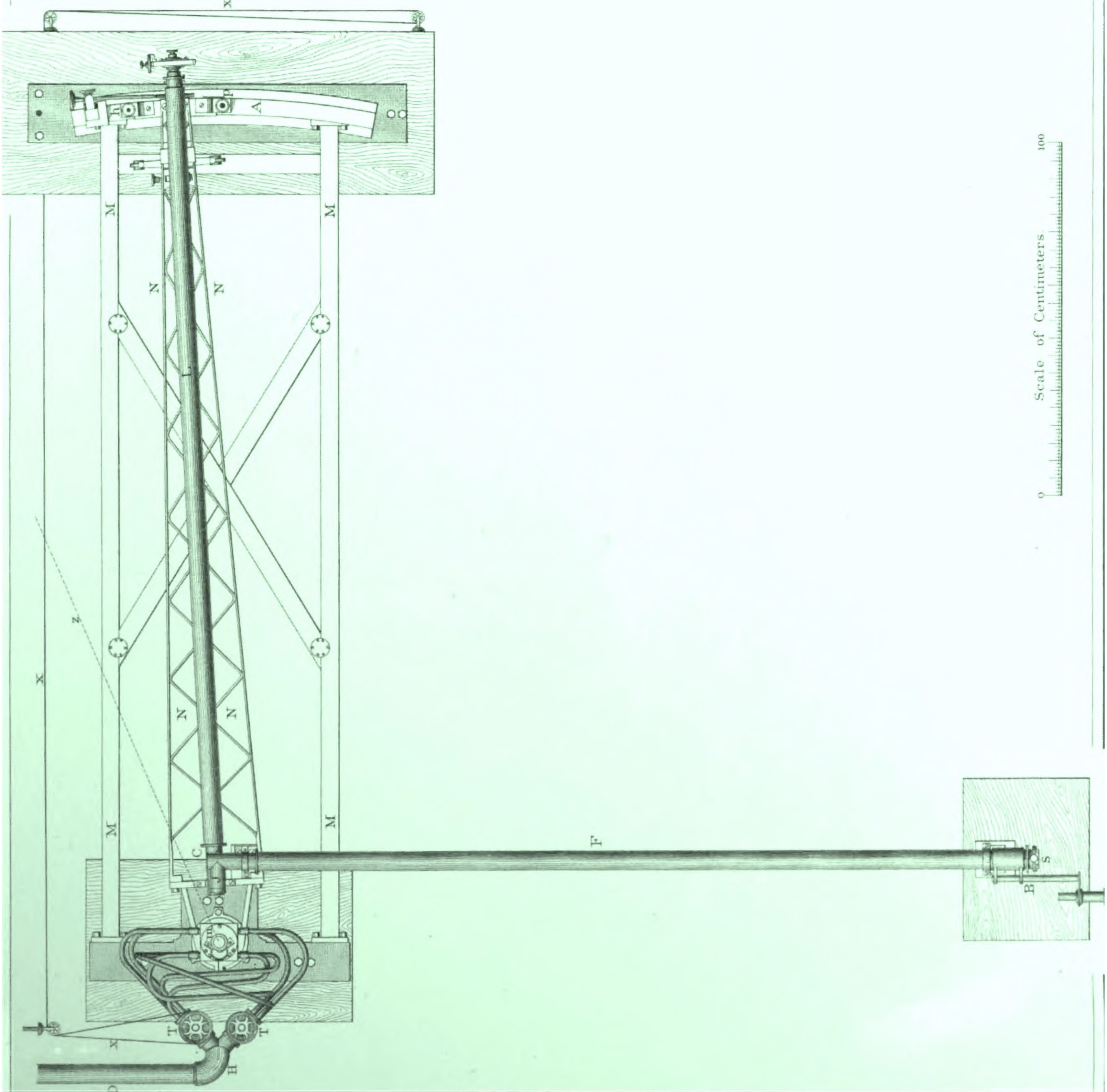
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FORMULÆ AND TABLES

FOR

EXPRESSING CORRECTIONS TO THE GEOCENTRIC PLACE OF A PLANET IN TERMS OF SYMBOLIC CORRECTIONS TO THE ELEMENTS OF THE ORBITS OF THE EARTH AND PLANET.

BY

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CORRECTIONS TO THE GEOCENTRIC PLACE OF A PLANET EXPRESSED IN TERMS OF SYMBOLIC CORRECTIONS TO THE ELEMENTS OF THE ORBITS OF THE EARTH AND PLANET.

The object of the following investigation is to deduce the formulæ necessary for expressing the corrections to the geocentric right ascension and declination of a planet in terms of corrections to the elements. It is founded upon the principle that the geocentric position depends equally upon the position of the earth and that of the planet. In order to utilize such observations to the fullest extent, it is therefore necessary to include the elements of both orbits in the equations of condition. In the special case now in hand, it will be advisable to construct and tabulate the quantities by which the required coefficients of symbolic corrections to the elements may be found, choosing those forms and methods which are best adapted to this purpose. The following method is founded on the same general basis as that of Oppolzer in Vol. II of his *Lehrbuch zur Bahnbestimmung der Kometen und Planeten*, but owing to his formulæ being constructed for a somewhat different purpose, their reconstruction is necessary.

§ I.

RELATIONS OF THE RECTANGULAR AND POLAR GEOCENTRIC AND HELIOCENTRIC CO-ORDINATES.

Let us put

x, y, z, the geocentric rectangular co-ordinates of the planet referred to the equator and equinox;

α, δ, ρ, its geocentric polar co-ordinates, referred to the same plane and line;

a, d, r, its heliocentric polar co-ordinates;

N, the right ascension of the planet's ascending node on the equator;

J, the inclination of its orbit to the plane of the equator;

u, the angular distance of the planet from its ascending node on the equator;

w, the distance from the node on equator to the perihelion;

ε, the obliquity of the ecliptic;

L, the sun's true longitude;

R, the sun's radius vector.

The quantities α, δ and ρ will then be expressed in terms of the heliocentric quantities by the formulæ

$$\begin{aligned} x &= r \cos N \cos u - r \cos J \sin N \sin u + R \cos L \\ y &= r \sin N \cos u + r \cos J \cos N \sin u + R \cos \epsilon \sin L \\ z &= r \sin J \sin u + R \sin \epsilon \sin L \end{aligned} \tag{1}$$

$$\begin{aligned} \rho \cos \delta \cos \alpha &= x \\ \rho \cos \delta \sin \alpha &= y \\ \rho \sin \delta &= z \end{aligned} \tag{2}$$

whence

$$\tan \alpha = \frac{y}{x}$$

$$\tan \delta = \frac{z}{\sqrt{x^2 + y^2}}$$

§ 2.

DIFFERENTIALS OF α AND δ WITH RESPECT TO HELIOCENTRIC CO-ORDINATES.

By differentiating the last two equations and reducing we obtain

$$d\alpha = \frac{1}{\rho \cos \delta} (\cos \alpha \, dy - \sin \alpha \, dx) \tag{3}$$

$$d\delta = -\frac{\sin \delta}{\rho} (\cos \alpha \, dx + \sin \alpha \, dy) + \frac{\cos \delta}{\rho} dz \tag{4}$$

In these equations, we have to substitute the differential coefficients of x, y and z from (1), which are as follows:

$$\begin{aligned} \frac{dx}{du} &= -r (\cos N \sin u + \cos J \sin N \cos u) \\ \frac{dy}{du} &= -r (\sin N \sin u - \cos J \cos N \cos u) \\ \frac{dz}{du} &= +r \sin J \cos u \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{dx}{dN} &= -r (\sin N \cos u + \cos J \cos N \sin u) \\ \frac{dy}{dN} &= +r (\cos N \cos u - \cos J \sin N \sin u) \\ \frac{dz}{dN} &= 0 \end{aligned} \tag{6}$$

$$\frac{dx}{dJ} = + r \sin J \sin N \sin u$$

$$\frac{dy}{dJ} = - r \sin J \cos N \sin u \qquad (7)$$

$$\frac{dz}{dJ} = + r \cos J \sin u$$

$$\frac{dx}{dr} = + \cos N \cos u - \cos J \sin N \sin u$$

$$\frac{dy}{dr} = + \sin N \cos u + \cos J \cos N \sin u \qquad (8)$$

$$\frac{dz}{dr} = + \sin J \sin u$$

$$\frac{dx}{dL} = - R \sin L$$

$$\frac{dy}{dL} = + R \cos \varepsilon \cos L \qquad (9)$$

$$\frac{dz}{dL} = + R \sin \varepsilon \cos L$$

$$\frac{dx}{dR} = + \cos L$$

$$\frac{dy}{dR} = + \cos \varepsilon \sin L \qquad (10)$$

$$\frac{dz}{dR} = + \sin \varepsilon \sin L$$

$$\frac{dx}{d\varepsilon} = 0$$

$$\frac{dy}{d\varepsilon} = - R \sin \varepsilon \sin L \qquad (11)$$

$$\frac{dz}{d\varepsilon} = + R \cos \varepsilon \sin L$$

To adopt the first two equations of (5), (6) and (8) to logarithmic computation we may compute m, n, φ and ψ from the equations:

$$m \sin \varphi = \cos J \sin N$$
$$m \cos \varphi = \cos N$$
$$n \sin \psi = \sin N$$
$$n \cos \psi = \cos J \cos N$$

We shall then have, instead of (5), (6) and (8)

$$\begin{aligned} \frac{dx}{du} &= -\,m\,r \sin(\varphi + u) \\ \frac{dy}{du} &= +\,n\,r \cos(\psi + u) \\ \frac{dz}{du} &= +\,r \sin \mathrm{J} \cos u \end{aligned} \qquad (5)'$$

$$\begin{aligned} \frac{dx}{d\mathrm{N}} &= -\,n\,r \sin(\psi + u) \\ \frac{dy}{d\mathrm{N}} &= +\,m\,r \cos(\varphi + u) \\ \frac{dz}{d\mathrm{N}} &= 0 \end{aligned} \qquad (6)'$$

$$\begin{aligned} \frac{dx}{dr} &= m \cos(\varphi + u) \\ \frac{dy}{dr} &= n \sin(\psi + u) \\ \frac{dz}{dr} &= \sin \mathrm{J} \sin u \end{aligned} \qquad (8)'$$

Supposing all these substitutions made in (3) and (4), we shall have the derivatives of α and δ with respect to u, N, J, r, L, R and ε. But we require the derivatives with respect to the elements. We must therefore express the derivatives of u, r, L and R with respect to the elements.

§ 3.

DIFFERENTIALS OF HELIOCENTRIC CO-ORDINATES WITH RESPECT TO ELEMENTS.

To form the derivatives of u, r, L and R in terms of the elements, we put

$f =$ the true anomaly of the planet;
$g =$ its mean anomaly;
f'', g'', the same quantities with respect to the sun;
w, the distance from the node on the equator to the perihelion.

Then

$$u = w + f = w + \varphi(e, g)$$
$$r = a f(e, g)$$

By differentiation

$$du = dw + \frac{df}{de}\,de + \frac{df}{dg}\,dg$$

$$dr = \frac{dr}{de}\,de + \frac{dr}{dg}\,dg + \frac{r}{a}\,da$$

We then have, from (5)′, (6)′ and (8)′

$$
\begin{aligned}
\frac{dx}{dg} &= \frac{df}{dg}\frac{dx}{du} + \frac{dr}{dg}\frac{dx}{dr} \\
&= -\, m\, r \sin(\varphi + u)\frac{df}{dg} + m \cos(\varphi + u)\frac{dr}{dg} \\
\frac{dx}{de} &= -\, m\, r \sin(\varphi + u)\frac{df}{de} + m \cos(\varphi + u)\frac{dr}{de} \\
\frac{dx}{dw} &= -\, m\, r \sin(\varphi + u) \\
\frac{dx}{da} &= \frac{mr}{a}\cos(\varphi + u)
\end{aligned}
\qquad (12)
$$

$$
\begin{aligned}
\frac{dy}{dg} &= n\, r \cos(\psi + u)\frac{df}{dg} + n \sin(\psi + u)\frac{dr}{dg} \\
\frac{dy}{de} &= n\, r \cos(\psi + u)\frac{df}{de} + n \sin(\psi + u)\frac{dr}{de} \\
\frac{dy}{dw} &= n\, r \cos(\psi + u) \\
\frac{dy}{da} &= \frac{n\, r}{a}\sin(\psi + u)
\end{aligned}
\qquad (13)
$$

$$
\begin{aligned}
\frac{dz}{dg} &= r \sin \mathrm{J} \cos u \frac{df}{dg} + \sin \mathrm{J} \sin u \frac{dr}{dg} \\
\frac{dz}{de} &= r \sin \mathrm{J} \cos u \frac{df}{de} + \sin \mathrm{J} \sin u \frac{dr}{de} \\
\frac{dz}{dw} &= r \sin \mathrm{J} \cos u \\
\frac{dz}{da} &= \frac{r}{a} \sin \mathrm{J} \sin u
\end{aligned}
\qquad (14)
$$

We next require the corresponding expressions for the solar elements. If we put

π'', the longitude of the solar perigee;
e'', the eccentricity
g'', the mean anomaly } of the sun;
l'', the mean longitude

we have, to quantities of the first order with respect to the eccentricity,

$$
\begin{aligned}
\mathrm{L} &= l'' + 2\, e'' \sin g'' \\
&= l'' + 2\, e'' \sin(l'' - \pi'') \\
\mathrm{R} &= 1 - e'' \cos(l'' - \pi'')
\end{aligned}
$$

As the corrections still required to the solar elements must be very small, we may, in the values of the differential coefficients, drop all terms which contain the

eccentricity as a factor. Then, taking l'', e'' and π'' as the solar elements, we shall have

$$\frac{dL}{dl''} = 1$$

$$\frac{dL}{de''} = 2 \sin g'' \tag{15}$$

$$\frac{dL}{e''d\pi''} = - 2 \cos g''$$

$$\frac{dR}{dl''} = 0$$

$$\frac{dR}{de''} = - \cos g'' \tag{16}$$

$$\frac{dR}{e''d\pi''} = - \sin g''$$

From the equations (9) and (10) we obtain the following by applying (15) and (16). Since the eccentricity is neglected, we may suppose

$$R = 1 \text{ and } L = l''$$

$$\frac{dx}{dl''} = - \sin l'' \tag{17}$$

$$\begin{aligned}\frac{dx}{de''} &= - 2 \sin l'' \sin g'' - \cos l'' \cos g'' \\ &= \frac{1}{2} \cos (2 l'' - \pi'') - \frac{3}{2} \cos \pi''\end{aligned} \tag{18}$$

$$\begin{aligned}\frac{dx}{e''d\pi''} &= 2 \sin l'' \cos g'' - \cos l'' \sin g'' \\ &= \frac{1}{2} \sin (2 l'' - \pi'') + \frac{3}{2} \sin \pi''\end{aligned} \tag{19}$$

$$\frac{dy}{dl''} = \cos \varepsilon \cos l'' \tag{20}$$

$$\frac{dy}{de''} = \cos \varepsilon \left\{ \frac{1}{2} \sin (2 l'' - \pi'') - \frac{3}{2} \sin \pi'' \right\} \tag{21}$$

$$\frac{dy}{e''d\pi''} = - \cos \varepsilon \left\{ \frac{1}{2} \cos (2 l'' - \pi'') + \frac{3}{2} \cos \pi'' \right\} \tag{22}$$

$$\frac{dz}{dl''} = \sin \varepsilon \cos L \tag{23}$$

$$\frac{dz}{de''} = \sin \varepsilon \left\{ \frac{1}{2} \sin (2\, l'' - \pi'') - \frac{3}{2} \sin \pi'' \right\} \tag{24}$$

$$\frac{dz}{e''\, d\pi''} = -\sin \varepsilon \left\{ \frac{1}{2} \cos (2\, l'' - \pi'') + \frac{3}{2} \cos \pi'' \right\} \tag{25}$$

§ 4.

DIFFERENTIALS OF GEOCENTRIC CO-ORDINATES WITH RESPECT TO THE ELEMENTS.

Our next step is to substitute the expressions (6), (7), (12), (13), (14), and (17) to (25) in (3) and (4). The analytic substitution is, however, troublesome and unnecessary, and the most convenient course in practical application is to tabulate the derivatives of x, y, and z with respect to the elements, as functions of g in the case of the elements of Mercury and Venus, and of the time of the year in the case of the elements of the sun.

We shall therefore suppose $\frac{dx}{dN}$, $\frac{dx}{dJ}$, $\frac{dx}{dg}$, etc., to be known quantities whose values are to be employed in (3) and (4). Beginning with g, we shall have from (3) and (4),

$$\frac{d\alpha}{dg} = \frac{1}{\rho \cos \delta} \left(\cos \alpha \frac{dy}{dg} - \sin \alpha \frac{dx}{dg} \right)$$

$$\frac{d\delta}{dg} = \frac{\cos \delta}{\rho} \frac{dz}{dg} - \frac{\sin \delta}{\rho} \left(\sin \alpha \frac{dy}{dg} + \cos \alpha \frac{dx}{dg} \right)$$

If we now compute k and K from the equations

$$\left.\begin{aligned} k \sin K &= \frac{dx}{dg} \\ k \cos K &= \frac{dy}{dg} \end{aligned}\right\} \tag{26}$$

we shall have

$$\left.\begin{aligned} \frac{d\alpha}{dg} &= \frac{k}{\rho \cos \delta} \cos (\alpha + K) \\ \frac{d\delta}{dg} &= \frac{\cos \delta}{\rho} \frac{dz}{dg} - \frac{k \sin \delta}{\rho} \sin (\alpha + K) \end{aligned}\right\} \tag{27}$$

The corresponding derivatives for the other elements are found from the equations

$$\left.\begin{aligned}
k' \sin K' &= \frac{dx}{de} \\
k' \cos K' &= \frac{dy}{de} \\
k'' \sin K'' &= \frac{dx}{du} = \frac{dx}{dw} \\
k'' \cos K'' &= \frac{dy}{du} = \frac{dy}{dw} \\
k''' \sin K''' &= \frac{dx}{dN} \\
k''' \cos K''' &= \frac{dy}{dN} \\
k^{iv} \sin K^{iv} &= \frac{dx}{dJ} \\
k^{iv} \cos K^{iv} &= \frac{dy}{dJ}
\end{aligned}\right\} \qquad (28)$$

From these last two equations, combined with (7), we have

$$\begin{aligned}
k^{iv} &= - r \sin J \sin u \\
K^{iv} &= - N
\end{aligned}$$

We then have

$$\left.\begin{aligned}
\frac{d\alpha}{de} &= \frac{k'}{\rho \cos \delta} \cos (\alpha + K') \\
\frac{d\delta}{de} &= \frac{\cos \delta}{\rho} \frac{dz}{de} - \frac{k' \sin \delta}{\rho} \sin (\alpha + K') \\
\frac{d\alpha}{dw} &= \frac{k''}{\rho \cos \delta} \cos (\alpha + K'') \\
\frac{d\delta}{dw} &= \frac{\cos \delta}{\rho} \frac{dz}{du} - \frac{k'' \sin \delta}{\rho} \sin (\alpha + K'')
\end{aligned}\right\} \qquad (29)$$

$$\left(\frac{d\alpha}{dN}\right) = \frac{k'''}{\rho \cos \delta} \cos (\alpha + K''')$$

$$\left(\frac{d\delta}{dN}\right) = - \frac{k''' \sin \delta}{\rho} \sin (\alpha + K''')$$

$$\frac{d\alpha}{dJ} = \frac{k^{iv}}{\rho \cos \delta} \cos (\alpha - N)$$

$$\frac{d\delta}{dJ} = \frac{\cos \delta}{\rho} \frac{dz}{dJ} - \frac{k^{iv} \sin \delta}{\rho} \sin (\alpha - N)$$

The derivatives with respect to N are enclosed in parentheses to distinguish them from another set of derivatives, to be next developed.

In the final formation of the equations of conditions it will be better to substitute for the variables

$$N,\ J,\ u,\ r, \qquad (a)$$

the quantities

$$N,\ J,\ v,\ r, \qquad (b)$$

v being the true longitude of the planet in its orbit from an arbitrary fixed departure point. Then instead of the elements

$$N,\ J,\ w,\ e,\ g,\ a, \qquad (a)'$$

we shall have

$$N,\ J,\ \pi,\ e,\ l,\ a, \qquad (b)'$$

π being the longitude of the perihelion, and l the mean longitude, each counted from the departure point.

To compare the differential coefficients in the two cases we shall inclose the first set of derivatives between parentheses, so that, using the system (a), we have

$$d\alpha = \left(\frac{d\alpha}{dN}\right) dN + \left(\frac{d\alpha}{dJ}\right) dJ + \left(\frac{d\alpha}{du}\right) du + \left(\frac{d\alpha}{dr}\right) dr \qquad (30)$$

with a similar equation for δ.

After the introduction of the elements $(a)'$ this form will become

$$\begin{aligned} d\alpha = & \left(\frac{d\alpha}{dN}\right) dN + \left(\frac{d\alpha}{dJ}\right) dJ + \left(\frac{d\alpha}{du}\right) dw + \frac{d\alpha}{da} da \\ & + \left\{ \left(\frac{d\alpha}{du}\right) \frac{df}{de} + \left(\frac{d\alpha}{dr}\right) \frac{dr}{de} \right\} de \\ & + \left\{ \left(\frac{d\alpha}{du}\right) \frac{df}{dg} + \left(\frac{d\alpha}{dr}\right) \frac{dr}{dg} \right\} dg \end{aligned}$$

so that we may write

$$\left(\frac{d\alpha}{de}\right) = \left(\frac{d\alpha}{du}\right) \frac{df}{de} + \left(\frac{d\alpha}{dr}\right) \frac{dr}{de}$$

$$\left(\frac{d\alpha}{dg}\right) = \left(\frac{d\alpha}{du}\right) \frac{df}{dg} + \left(\frac{d\alpha}{dr}\right) \frac{dr}{dg}$$

The forms (b) are derived from (a) by the substitution

$$\delta u = \delta v - \cos \mathrm{J}\ \delta \mathrm{N}$$

$\delta \mathrm{N}$, $\delta \mathrm{J}$ and δr remaining unaltered. Substituting this value of δu in the expression for $d\alpha$ we have

$$d\alpha = \left\{ \left(\frac{d\alpha}{d\mathrm{N}}\right) - \cos \mathrm{J} \left(\frac{d\alpha}{du}\right) \right\} \delta \mathrm{N} + \left(\frac{d\alpha}{d\mathrm{J}}\right) \delta \mathrm{J}$$
$$+ \left(\frac{d\alpha}{du}\right) \delta v + \left(\frac{d\alpha}{dr}\right) \delta r$$

Hence, omitting the parentheses when the co-ordinates are considered as functions of N, J, v and r, we have

$$\begin{aligned} \frac{d\alpha}{d\mathrm{N}} &= \left(\frac{d\alpha}{d\mathrm{N}}\right) - \cos \mathrm{J} \left(\frac{d\alpha}{du}\right) \\ \frac{d\alpha}{d\mathrm{J}} &= \left(\frac{d\alpha}{d\mathrm{J}}\right) \\ \frac{d\alpha}{dv} &= \left(\frac{d\alpha}{du}\right) \\ \frac{d\alpha}{dr} &= \left(\frac{d\alpha}{dr}\right) \end{aligned} \tag{31}$$

The only derivative which requires to be replaced by another is therefore that with respect to N.

We now have, by writing x for α in the preceding equations and substituting from (5) and (6)

$$\begin{aligned} \frac{1}{\sin \mathrm{J}} \cdot \frac{dx}{d\mathrm{N}} &= - r \sin \mathrm{J} \sin \mathrm{N} \cos u \\ \frac{1}{\sin \mathrm{J}} \cdot \frac{dy}{d\mathrm{N}} &= + r \sin \mathrm{J} \cos \mathrm{N} \cos u \\ \frac{1}{\sin \mathrm{J}} \cdot \frac{dz}{d\mathrm{N}} &= - r \cos \mathrm{J} \cos u \end{aligned} \tag{32}$$

If we put

$$k^{v} = r \sin \mathrm{J} \cos u$$

we shall have

$$\frac{1}{\sin J}\frac{dx}{dN} = -\,k^{v}\sin N$$

$$\frac{1}{\sin J}\frac{dy}{dN} = \quad k^{v}\cos N$$

$$\frac{1}{\sin J}\frac{d\alpha}{dN} = \frac{k^{v}}{\rho\cos\delta}(\cos\alpha\cos N + \sin\alpha\sin N) = \frac{k^{v}}{\rho\cos\delta}\cos(\alpha - N) \qquad (33)$$

$$\frac{1}{\sin J}\frac{d\delta}{dN} = \frac{\cos\delta}{\rho}\frac{1}{\sin J}\frac{dz}{dN} - \frac{k^{v}\sin\delta}{\rho}(\sin\alpha\cos N - \cos\alpha\sin N)$$

$$= \frac{\cos\delta}{\rho}\frac{1}{\sin J}\frac{dz}{dN} - \frac{k^{v}\sin\delta}{\rho}\sin(\alpha - N)$$

If the derivatives with respect to r and v are wanted we may put

$$\left.\begin{aligned} k^{vi}\sin K^{vi} &= \frac{dx}{dr} \\ k^{vi}\cos K^{vi} &= \frac{dy}{dr} \end{aligned}\right\} \qquad (34)$$

and shall then have

$$\left.\begin{aligned} \frac{d\alpha}{dr} &= \frac{k^{vi}}{\rho\cos\delta}\cos(\alpha + K^{vi}) \\ \frac{d\delta}{dr} &= \frac{k^{vi}}{\rho\cos\delta}\frac{dz}{dr} - \frac{k^{vi}\sin\delta}{\rho}\sin(\alpha + K^{vi}) \end{aligned}\right\} \qquad (35)$$

We have for v

$$\left.\begin{aligned} \frac{d\alpha}{dv} &= \frac{d\alpha}{du} = \frac{d\alpha}{dw} \\ \frac{d\delta}{dv} &= \frac{d\delta}{du} = \frac{d\delta}{dw} \end{aligned}\right\} \qquad (36)$$

It is to be remarked that the derivatives with respect to e and g may also be obtained from those with respect to r and v by the formulæ

$$\left.\begin{aligned} \frac{d\alpha}{de} &= \frac{df}{de}\frac{d\alpha}{dv} + \frac{dr}{de}\frac{d\alpha}{dr} \\ \frac{d\delta}{de} &= \frac{df}{de}\frac{d\delta}{dv} + \frac{dr}{de}\frac{d\delta}{dr} \\ \frac{d\alpha}{dg} &= \frac{df}{dg}\frac{d\alpha}{dv} + \frac{dr}{dg}\frac{d\alpha}{dr} \\ \frac{d\delta}{dg} &= \frac{df}{dg}\frac{d\delta}{dv} + \frac{dr}{dg}\frac{d\delta}{dr} \end{aligned}\right\} \qquad (37)$$

If the mean longitude and longitude of the perihelion are to be introduced for correction instead of w and g we express the co-ordinates in terms of the system of variables (b'). To do this, we remark that since

$$v = \pi + f$$

we have, in the system (a) to substitute

$$\delta u = \delta\pi + \delta f - \cos J\, \delta N \tag{38}$$

We shall then have, if φ be any co-ordinate

$$d\varphi = \left\{ \left(\frac{d\varphi}{dN}\right) - \cos J \left(\frac{d\varphi}{du}\right) \right\} dN + \left(\frac{d\varphi}{dJ}\right) dJ + \left(\frac{d\varphi}{du}\right)(d\pi + df)$$
$$+ \left(\frac{d\varphi}{dr}\right)\left(\frac{dr}{d\pi}\,d\pi + \frac{dr}{dl}\,dl + \frac{dr}{de}\,de + \frac{dr}{da}\,da\right)$$

where we have

$$df = \frac{df}{dg}(dl - d\pi) + \frac{df}{de}\,de$$

We thus have

$$\begin{aligned}
\frac{d\varphi}{d\pi} &= \left(1 - \frac{df}{dg}\right)\left(\frac{d\varphi}{du}\right) - \left(\frac{d\varphi}{dr}\right)\frac{dr}{dg} = \frac{d\varphi}{dv} - \frac{d\varphi}{dg} \\
\frac{d\varphi}{dl} &= \left(\frac{d\varphi}{du}\right)\frac{df}{dg} + \left(\frac{d\varphi}{dr}\right)\frac{dr}{dg} = \frac{d\varphi}{dg} \\
\frac{d\varphi}{de} &= \left(\frac{d\varphi}{du}\right)\frac{df}{de} + \left(\frac{d\varphi}{dr}\right)\frac{dr}{de}
\end{aligned} \tag{39}$$

Another modification which may be desirable arises from the circumstance that, by introducing the inclination of the planet's orbit to the ecliptic instead of the equator, J, N and u will become functions of ε and of i and θ, the inclination and longitune of the node. The changes in J, N and u produced by changes in the position of the orbit will be determined by the equations

$$dJ = \cos\psi\,\delta i - \sin\psi \sin i\,\delta\theta + \cos N\,\delta\varepsilon$$

$$\sin J\,\delta N = \sin\psi\,\delta i + \cos\psi \sin i\,\delta\theta - \cos J \sin N\delta\varepsilon$$

$$\delta u = -\cot J \sin\psi\,\delta i - \cot J \cos\psi \sin i\,\delta\theta + \operatorname{cosec} J \sin N\,\delta\varepsilon$$

where ψ is the distance from the node on the equator to the node on the ecliptic.

Although by the introduction of i and θ for J and N, the equations of condition will assume a form with smaller coefficients, the change may not compensate the greater complexity of the formulæ.

Proceeding on the same system with the solar elements we compute

$$
\left.
\begin{aligned}
h' \sin H' &= -\sin L \\
h' \cos H' &= \cos \varepsilon \cos L \\
h'' &= -\sin \varepsilon \sin L \\
h''' \sin H''' &= \tfrac{1}{2} \cos (2L - \pi'') - \tfrac{3}{2} \cos \pi'' \\
h''' \cos H''' &= \cos \varepsilon \left\{ \tfrac{1}{2} \sin (2L - \pi'') - \tfrac{3}{2} \sin \pi'' \right\} \\
h^{iv} \sin H^{iv} &= \tfrac{1}{2} \sin (2L - \pi'') + \tfrac{3}{2} \sin \pi'' \\
h^{iv} \cos H^{iv} &= -\cos \varepsilon \left\{ \tfrac{1}{2} \cos (2L - \pi'') + \tfrac{3}{2} \cos \pi'' \right\} \\
h^{v} \sin H^{v} &= \cos L \\
h^{v} \cos H^{v} &= \cos \varepsilon \sin L
\end{aligned}
\right. \qquad (40)
$$

(We have here written L instead of l'', as being probably a little more accurate and equally convenient.)

The quantities h', h''', etc., being functions of the sun's longitude, can be tabulated as functions of the day of the year. We shall then have

$$
\left.
\begin{aligned}
\frac{d\alpha}{dl''} &= \frac{h'}{\rho \cos \delta} \cos (\alpha + H') \\
\frac{d\delta}{dl''} &= \frac{\cos \delta}{\rho} \frac{dz}{dl''} \sin \varepsilon \cos L - \frac{h' \sin \delta}{\rho} \sin (\alpha + H') \\
\frac{d\alpha}{d\varepsilon} &= -\frac{\cos \alpha}{\rho \cos \delta} \sin \varepsilon \sin L = h'' \frac{\cos \alpha}{\rho \cos \delta} \\
\frac{d\delta}{d\varepsilon} &= \cos \varepsilon \sin L \frac{\cos \delta}{\rho} - h'' \frac{\sin \alpha \sin \delta}{\rho} \\
\frac{d\alpha}{de''} &= \frac{h'''}{\rho \cos \delta} \cos (\alpha + H''') \\
\frac{d\delta}{de''} &= \frac{\cos \delta}{\rho} \frac{dz}{de''} - \frac{h''' \sin \delta}{\rho} \sin (\alpha + H''') \\
\frac{d\alpha}{e'' d\pi''} &= \frac{h^{iv}}{\rho \cos \delta} \cos (\alpha \div H^{iv}) \\
\frac{d\delta}{e'' d\pi''} &= \frac{\cos \delta}{\rho} \frac{dz}{e'' d\pi''} - \frac{h^{iv} \sin \delta}{\rho} \sin (\alpha + H^{iv}) \\
\frac{d\alpha}{dR} &= \frac{h^{v}}{\rho \cos \delta} \cos (\alpha + H^{v}) \\
\frac{d\delta}{dR} &= \frac{\cos \delta}{\rho} \sin \varepsilon \sin L - \frac{h^{v} \sin \delta}{\rho} \sin (\alpha + H^{v})
\end{aligned}
\right. \qquad (41)
$$

§ 5.

OTHER EXPRESSIONS FOR THE COMPUTATION OF THE PRECEDING QUANTITIES.

As a check upon the preceding computations of $\frac{dx}{de}$, etc., it will be well to compute some of these quantities by a slightly different method, thus:

Since

$$u = w + f$$

we have

$$\sin u = \cos w \sin f + \sin w \cos f$$
$$\cos u = \cos w \cos f - \sin w \sin f$$

Hence, considering only the heliocentric co-ordinates of the planet

$$x = r \{\cos N \cos w - \cos J \sin N \sin w\} \cos f$$
$$+ r \{-\cos N \sin w - \cos J \sin N \cos w\} \sin f$$

$$y = r \{\sin N \cos w + \cos J \cos N \sin w\} \cos f$$
$$+ \{-\sin N \sin w + \cos J \cos N \cos w\} \sin f$$

$$z = r \sin J \sin w \cos f + r \sin J \cos w \sin f$$

If we put ξ and η for the rectangular co-ordinates in the orbit referred to the line of apsides as the axis of abscissas, namely,

$$\xi = r \cos f$$
$$\eta = r \sin f$$

these equations become

$$\begin{aligned} x &= \alpha\ \ \xi + \beta\ \ \eta \\ y &= \alpha'\ \xi + \beta'\ \eta \\ z &= \alpha''\ \xi + \beta''\ \eta \end{aligned} \qquad (42)$$

where

$$\begin{aligned} \alpha\ &= \cos N \cos w - \cos J \sin N \sin w \\ \beta\ &= -\cos N \sin w - \cos J \sin N \cos w \\ \alpha' &= \sin N \cos w + \cos J \cos N \sin w \\ \beta' &= -\sin N \sin w + \cos J \cos N \cos w \\ \alpha'' &= \sin J \sin w \\ \beta'' &= \sin J \cos w \end{aligned} \qquad (43)$$

We then have by differentiation

$$
\left.
\begin{aligned}
\frac{dx}{de} &= \frac{dx}{d\xi}\frac{d\xi}{de} + \frac{dx}{d\eta}\frac{d\eta}{de} \\
&= \alpha\frac{d\xi}{de} + \beta\frac{d\eta}{de} \\
\frac{dy}{de} &= \alpha'\frac{d\xi}{de} + \beta'\frac{d\eta}{de} \\
\frac{dz}{de} &= \alpha''\frac{d\xi}{de} + \beta''\frac{d\eta}{de}
\end{aligned}
\right\} \qquad (44)
$$

and similar equations with respect to dg.

ξ and η may be expressed in terms of the eccentric anomaly thus:

$$\xi = a(\cos u - e)$$

$$\eta = a\sqrt{1 - e^2}\sin u$$

where u is the eccentric anomaly, so that $u - e\sin u = g$

The complete differential of this last equation is

$$du(1 - e\cos u) - \sin u\, de = dg$$

So, regarding u as a function of e and g, we shall have

$$\frac{du}{de} = \frac{\sin u}{1 - e\cos u} = \sin u\frac{du}{dg}$$

$$\frac{du}{dg} = \frac{1}{1 - e\cos u} = \frac{1}{\sin}\frac{du}{de}$$

Also from the expressions for ξ and η

$$
\begin{aligned}
\frac{d\xi}{de} &= -a\sin u\frac{du}{de} - a = -a\left(1 + \sin\frac{du}{de}\right) \\
&= -a\left(1 + \frac{\sin^2 u}{1 - e\cos u}\right) \\
\frac{d\xi}{dg} &= \frac{d\xi}{du}\frac{du}{dg} = -a\sin u\frac{du}{dg} = -a\frac{\sin u}{1 - e\cos u} \\
\frac{d\eta}{de} &= a\left(\frac{-e}{\sqrt{1 - e^2}}\sin u + \sqrt{1 - e^2}\cos u\frac{du}{de}\right) \\
&= a\sqrt{1 - e^2}\left(-\frac{e}{1 - e^2}\sin u + \frac{\sin u\cos u}{1 - e\cos u}\right) \\
&= \frac{a(\cos u - e)\sin u}{\sqrt{1 - e^2}\,(1 - e\cos u)} \\
\frac{d\eta}{dg} &= a\sqrt{1 - e^2}\frac{\cos u}{1 - e\cos u}
\end{aligned}
\qquad (45)
$$

By differentiating the equations (43) we have

$$
\begin{aligned}
\frac{d\alpha}{dw} &= \beta & \frac{d\beta}{dw} &= -\alpha \\
\frac{d\alpha'}{dw} &= \beta' & \frac{d\beta'}{dw} &= -\alpha' \\
\frac{d\alpha''}{dw} &= \beta'' & \frac{d\beta''}{dw} &= -\alpha'' \\
\\
\frac{d\alpha}{d\mathrm{N}} &= -\alpha' & \frac{d\beta'}{d\mathrm{N}} &= -\beta' \\
\frac{d\alpha'}{d\mathrm{N}} &= \alpha & \frac{d\beta'}{d\mathrm{N}} &= \beta \\
\frac{d\alpha''}{d\mathrm{N}} &= 0 & \frac{d\beta''}{d\mathrm{N}} &= 0 \\
\\
\frac{d\alpha}{d\mathrm{J}} &= \alpha'' \sin \mathrm{N} & \frac{d\beta}{d\mathrm{J}} &= \beta'' \sin \mathrm{N} \\
\frac{d\alpha'}{d\mathrm{J}} &= -\alpha'' \cos \mathrm{N} & \frac{d\beta'}{d\mathrm{J}} &= -\beta'' \cos \mathrm{N} \\
\frac{d\alpha''}{d\mathrm{J}} &= \cos \mathrm{J} \sin w & \frac{d\beta''}{d\mathrm{J}} &= \cos \mathrm{J} \cos w
\end{aligned}
\tag{46}
$$

By differentiating the equations (42) and substituting the values of the differential coefficients in (45) and (46) when necessary we have

$$
\begin{aligned}
\frac{dx}{de} &= \alpha \frac{d\xi}{de} + \beta \frac{d\eta}{de} \\
\frac{dy}{de} &= \alpha' \frac{d\xi}{de} + \beta' \frac{d\eta}{de} \\
\frac{dz}{de} &= \alpha'' \frac{d\xi}{de} + \beta'' \frac{d\eta}{de} \\
\\
\frac{dx}{dg} &= \alpha \frac{d\xi}{dg} + \beta \frac{d\eta}{dg} \\
\frac{dy}{dg} &= \alpha' \frac{d\xi}{dg} + \beta' \frac{d\eta}{dg} \\
\frac{dz}{dg} &= \alpha'' \frac{d\xi}{dg} + \beta'' \frac{d\eta}{dg}
\end{aligned}
\tag{47}
$$

$$\frac{dx}{dw} = \beta\ \xi - \alpha\ \eta$$

$$\frac{dy}{dw} = \beta'\ \xi - \alpha'\ \eta$$

$$\frac{dz}{dw} = \beta''\ \xi - \alpha''\ \eta$$

$$\frac{dx}{d\mathrm{N}} = -\ \alpha'\ \xi - \beta'\ \eta$$

$$\frac{dy}{d\mathrm{N}} = \alpha\ \xi + \beta\ \eta \tag{48}$$

$$\frac{dz}{d\mathrm{N}} = 0$$

$$\frac{dx}{d\mathrm{J}} = (\alpha''\ \xi + \beta''\ \eta)\sin \mathrm{N}$$

$$\frac{dy}{d\mathrm{J}} = -\ (\alpha''\ \xi + \beta''\ \eta)\cos \mathrm{N}$$

$$\frac{dz}{d\mathrm{J}} = (\xi \sin w + \eta \cos w)\cos \mathrm{J}$$

§ 6.

REDUCTION OF THE PRECEDING EXPRESSIONS TO NUMBERS FOR THE PLANETS MERCURY AND VENUS.

To reduce the preceding expressions to numbers, and tabulate those quantities which depend on the mean anomaly of the planet, the following approximate elements are employed:

	Mercury.	Venus.
N	10° 37′	7° 56′
w	65° 34′	122° 12′
J	28° 42′	24° 32′
e	0.2056	0.0068
a	0.3871	0.7233

Also in case of the solar elements we have

$$\pi'' = 280^\circ\ 45'$$

$$\varepsilon = 23^\circ\ 27'$$

$$e'' = 0.0167$$

We then have the values of the several quantities given in the following tables, the results from which are sufficiently accurate for the corrections of the elements of Mercury and Venus for any time between 1700 and 2000.

Explanation of the Tables.—The object of the tables is to facilitate the computation of the differential coefficients of the geocentric right ascension and declination of Mercury and of Venus, with respect to the elements of the orbits of the earth and of the planet.

The first tables, in each case, give the mean anomaly of the planet, which is the argument g for the tables following.

The next table gives the relations of the anomaly and radius vector to the elements. Its results are employed in computing the subsequent tables.

The tables following give the values of k, K, k', K$'$ etc. The argument for these quantities is the mean anomaly of the planet. The formulæ for their use are (27), (29), (33) and (35). For the derivatives with respect to N, the node on the equator, there are two sets of quantities, one corresponding to each of the two sets of co-ordinates and elements, called respectively a, a', b, b', on page 13. It will probably always be found most convenient to use the set b, b', alone, in which case the quantities k''' and K$'''$ will not be wanted.

Following these tables are those which give the values of h', H$'$, h'', H$''$ etc., for which the argument is the day of the year. The formulæ for their use are (41). Being the same for both planets they are not repeated for Venus.

In the formulæ, we have

α, the geocentric R. A. of the planet;
δ, its geocentric declination;
ρ, its distance from the earth;

all which are to be obtained from ephemerides.

TABLES FOR MERCURY.

MEAN ANOMALY OF MERCURY.

Year	°	Year	°
1750	183.7	1820 B	52.4
51	237.4	21	106.1
52 B	295.2	22	159.8
53	348.9	23	213.5
54	42.6	24 B	271.3
55	96.3	25	325.0
56 B	154.1	26	18.7
57	207.8	27	72.4
58	261.5	28 B	130.2
59	315.2	29	183.9
1760 B	13.0	1830	237.6
61	66.7	31	291.3
62	120.4	32 B	349.1
63	174.1	33	42.8
64 B	231.9	34	96.5
65	285.6	35	150.2
66	339.3	36 B	208.0
67	33.0	37	261.7
68 B	90.8	38	315.4
69	144.5	39	9.1
1770	198.2	1840 B	66.9
71	251.9	41	120.6
72 B	309.7	42	174.3
73	3.4	43	228.0
74	57.1	44 B	285.8
75	110.8	45	339.5
76 B	168.6	46	33.2
77	222.3	47	86.9
78	276.0	48 B	144.7
79	329.7	49	198.4
1780 B	27.5	1850	252.1
81	81.2	51	305.8
82	134.9	52 B	363.6
83	188.6	53	57.3
84 B	246.4	54	111.0
85	300.1	55	164.7
86	353.8	56 B	222.5
87	47.5	57	276.2
88 B	105.3	58	329.9
89	159.0	59	23.6
1790	212.7	1860 B	81.5
91	266.4	61	135.2
92 B	324.2	62	188.9
93	17.9	63	242.6
94	71.6	64 B	300.4
95	125.3	65	354.1
96 B	183.1	66	47.8
97	236.8	67	101.5
98	290.5	68 B	159.3
99	344.2	69	213.0
1800	37.9	1870	266.7
01	91.6	71	320.4
02	145.3	72 B	18.2
03	199.0	73	71.9
04 B	256.8	74	125.6
05	310.5	75	179.3
06	4.2	76 B	237.1
07	57.9	77	290.8
08 B	115.7	78	344.5
09	169.4	79	38.2
1810	223.1	1880 B	96.0
11	276.8	81	149.7
12 B	334.6	82	203.4
13	28.3	83	257.1
14	82.0	84 B	314.9
15	135.7	85	8.6
16 B	193.5	86	62.3
17	247.2	87	116.0
18	300.9	88 B	173.8
19	354.6	89	227.5
1820 B	52.4	1890	281.2

	January.	February.	March.	April.	May.	June.
	°	°	°	°	°	°
1	0.0	126.9	241.4	8.3	131.1	257.9
2	4.1	131.0	245.4	12.4	135.2	262.0
3	8.2	135.1	249.5	16.5	139.3	266.1
4	12.3	139.2	253.6	20.6	143.4	270.2
5	16.4	143.2	257.7	24.7	147.5	274.3
6	20.4	147.3	261.8	28.8	151.6	278.4
7	24.5	151.4	265.9	32.9	155.7	282.5
8	28.6	155.5	270.0	36.9	159.7	286.6
9	32.7	159.6	274.1	41.0	163.8	290.7
10	36.8	163.7	278.2	45.1	167.9	294.8
11	40.9	167.8	282.3	49.2	172.0	298.9
12	45.0	171.9	286.4	53.3	176.1	302.9
13	49.1	176.0	290.5	57.4	180.2	307.0
14	53.2	180.1	294.5	61.5	184.3	311.1
15	57.3	184.1	298.6	65.6	188.4	315.2
16	61.4	188.2	302.7	69.7	192.5	319.3
17	65.5	192.3	306.8	73.8	196.6	323.4
18	69.6	196.4	310.9	77.9	200.7	327.5
19	73.6	200.5	315.0	82.0	204.8	331.6
20	77.7	204.6	319.1	86.1	208.8	335.7
21	81.8	208.7	323.2	90.2	212.9	339.7
22	85.9	212.8	327.3	94.3	217.0	343.8
23	90.0	216.9	331.4	98.4	221.1	347.9
24	94.1	221.0	335.5	102.5	225.2	352.0
25	98.2	225.1	339.6	106.6	229.3	356.1
26	102.3	229.2	343.7	110.7	233.4	0.2
27	106.4	233.3	347.8	114.8	237.5	4.3
28	110.5	237.4	351.9	118.9	241.6	8.4
29	114.6		356.0	122.9	245.7	12.5
30	118.7		0.1	127.0	249.8	16.6
31	122.8		4.2		253.8	

	July.	August.	September.	October.	November.	December.
	°	°	°	°	°	°
1	20.7	147.6	274.4	37.2	164.1	286.8
2	24.8	151.7	278.5	41.3	168.2	290.9
3	28.9	155.8	282.6	45.4	172.3	295.0
4	33.0	159.9	286.7	49.5	176.4	299.1
5	37.1	164.0	290.8	53.6	180.5	303.2
6	41.2	168.1	294.9	57.7	184.5	307.3
7	45.3	172.1	299.0	61.7	188.6	311.4
8	49.4	176.2	303.1	65.8	192.7	315.5
9	53.4	180.3	307.2	69.9	196.8	319.6
10	57.5	184.4	311.3	74.0	200.9	323.6
11	61.6	188.5	315.4	78.1	205.0	327.7
12	65.7	192.6	319.4	82.2	209.1	331.8
13	69.8	196.7	323.5	86.3	213.2	335.9
14	73.9	200.8	327.6	90.4	217.3	340.0
15	78.0	204.9	331.7	94.5	221.4	344.1
16	82.1	209.0	335.8	98.6	225.5	348.2
17	86.2	213.0	339.9	102.7	229.5	352.3
18	90.3	217.1	344.0	106.7	233.6	356.4
19	94.3	221.2	348.1	110.8	237.7	0.5
20	98.4	225.3	352.2	114.9	241.8	4.6
21	102.5	229.4	356.3	119.0	245.9	8.7
22	106.6	233.5	0.4	123.1	250.0	12.7
23	110.7	237.6	4.5	127.2	254.1	16.8
24	114.8	241.7	8.6	131.3	258.2	20.9
25	118.9	245.8	12.6	135.4	262.3	25.0
26	123.0	249.9	16.7	139.5	266.4	29.1
27	127.1	253.9	20.8	143.6	270.5	33.2
28	131.2	258.0	24.9	147.7	274.6	37.3
29	135.3	262.1	29.0	151.8	278.7	41.4
30	139.4	266.2	33.1	155.9	282.8	45.5
31	143.5	270.3		160.0		49.6

In January and February of bissextile years enter this table with the date diminished by one day.

MERCURY.

g	f	$w+f=u$	r	$\log r$	$\frac{df}{dg}$	$\log\frac{df}{dg}$	$\frac{dr}{dg}$	$\log\frac{dr}{dg}$	$\frac{df}{de}$	$\log\frac{df}{de}$	$\frac{dr}{de}$	$\log\frac{dr}{de}$
°	° ′	° ′										
0	0 0	65 34	0.308	9.488	1.551	0.191	+.000	. .	+0.000	. .	−.387	−9.588
5	7 45	73 19	0.308	9.489	1.546	0.189	.011	8.040	0.310	9.491	−.384	−9.584
10	15 27	81 1	0.309	9.491	1.532	0.185	.022	8.336	0.610	9.785	−.373	−9.572
15	23 3	88 37	0.312	9.494	1.509	0.179	.032	8.502	0.894	9.951	−.356	−9.552
20	30 31	96 5	0.315	9.498	1.478	0.170	+.041	8.616	+1.154	0.062	−.334	−9.523
25	37 49	103 23	0.319	9.504	1.442	0.159	.050	8.698	1.383	0.141	−.306	−9.485
30	44 56	110 30	0.324	9.510	1.400	0.146	.057	8.759	1.583	0.199	−.274	−9.438
35	51 49	117 23	0.329	9.517	1.355	0.132	.064	8.806	1.745	0.242	−.239	−9.379
40	58 29	124 3	0.335	9.525	1.309	0.117	+.069	8.841	+1.876	0.273	−.202	−9.306
45	64 54	130 28	0.341	9.533	1.261	0.101	.074	8.867	1.971	0.295	−.164	−9.215
50	71 5	136 39	0.348	9.541	1.214	0.084	.077	8.886	2.038	0.309	−.126	−9.099
55	77 3	142 37	0.354	9.549	1.168	0.067	.079	8.899	2.079	0.318	−.087	−8.938
60	82 46	148 20	0.361	9.558	1.123	0.050	+.081	8.907	+2.095	0.321	−.049	−8.688
65	88 17	153 51	0.368	9.566	1.080	0.033	.081	8.910	2.091	0.320	−.012	−8.068
70	93 34	159 9	0.376	9.575	1.040	0.017	.081	8.909	2.067	0.315	+.024	8.382
75	98 41	164 15	0.383	9.583	1.002	0.001	.080	8.905	2.031	0.308	.058	8.766
80	103 36	169 10	0.390	9.591	0.966	9.985	+.079	8.898	+1.978	0.296	+.091	8.960
85	108 21	173 55	0.396	9.598	0.933	9.970	.077	8.888	1.917	0.283	.122	9.086
90	112 56	178 30	0.403	9.605	0.903	9.956	.075	8.875	1.845	0.266	.151	9.179
95	117 23	182 57	0.409	9.612	0.875	9.942	.072	8.859	1.766	0.247	.178	9.251
100	121 41	187 15	0.416	9.619	0.849	9.929	+.069	8.840	+1.681	0.226	+.203	9.308
105	125 52	191 26	0.421	9.625	0.825	9.917	.066	8.819	1.590	0.201	.227	9.356
110	129 57	195 31	0.427	9.631	0.804	9.905	.062	8.795	1.497	0.175	.248	9.395
115	133 55	199 29	0.432	9.636	0.784	9.895	.059	8.768	1.397	0.145	.268	9.429
120	137 47	203 21	0.437	9.641	0.767	9.885	+.055	8.737	+1.297	0.113	+.287	9.457
125	141 35	207 9	0.442	9.645	0.751	9.876	.051	8.704	1.194	0.077	.303	9.482
130	145 18	210 52	0.446	9.649	0.737	9.867	.046	8.666	1.090	0.037	.318	9.503
135	148 57	214 31	0.450	9.653	0.724	9.860	.042	8.623	0.982	9.992	.332	9.521
140	152 33	218 7	0.453	9.657	0.713	9.853	+.038	8.574	+0.875	9.942	+.344	9.536
145	156 5	221 39	0.457	9.659	0.704	9.847	.033	8.518	0.768	9.885	.354	9.548
150	159 35	225 9	0.459	9.662	0.695	9.842	.028	8.453	0.656	9.817	.363	9.560
155	163 3	228 37	0.461	9.664	0.689	9.838	.024	8.375	0.548	9.739	.370	9.568
160	166 28	232 2	0.463	9.666	0.683	9.834	+.019	8.279	+0.436	9.640	+.376	9.576
165	169 52	235 26	0.465	9.667	0.679	9.832	.014	8.155	0.327	9.514	.381	9.581
170	173 16	238 50	0.466	9.668	0.676	9.830	.010	7.980	0.219	9.340	.384	9.585
175	176 38	242 12	0.466	9.669	0.674	9.829	.005	7.679	0.110	9.041	.386	9.587
180	180 0	245 34	0.467	9.669	0.673	9.828	.000	. .	0.000	. .	+.387	9.588
185	183 22	248 56	0.466	9.669	0.674	9.829	−.005	−7.679	−0.110	−9.041	.386	9.587
190	186 45	252 19	0.466	9.668	0.676	9.830	−.010	−7.980	−0.219	−9.340	.384	9.585
195	190 8	255 42	0.465	9.667	0.679	9.832	−.014	−8.155	−0.327	−9.514	.381	9.581
200	193 32	259 6	0.463	9.666	0.683	9.834	−.019	−8.279	−0.436	−9.640	+.376	9.576
205	196 58	262 32	0.461	9.664	0.689	9.838	−.024	−8.375	−0.548	−9.739	.370	9.568
210	200 25	265 59	0.459	9.662	0.695	9.842	−.028	−8.453	−0.656	−9.817	.363	9.560
215	203 55	269 29	0.457	9.659	0.704	9.847	−.033	−8.518	−0.768	−9.885	.354	9.548
220	207 28	273 2	0.453	9.657	0.713	9.853	−.038	−8.574	−0.875	−9.942	+.344	9.536
225	211 3	276 37	0.450	9.653	0.724	9.860	−.042	−8.623	−0.982	−9.992	.332	9.521
230	214 42	280 16	0.446	9.649	0.737	9.867	−.046	−8.666	−1.090	−0.037	.318	9.503
235	218 25	283 59	0.442	9.645	0.751	9.876	−.051	−8.704	−1.194	−0.077	.303	9.482
240	222 13	287 47	0.437	9.641	0.767	9.885	−.055	−8.737	−1.297	−0.113	+.287	9.457
245	226 5	291 39	0.432	9.636	0.784	9.895	−.059	−8.768	−1.397	−0.145	.268	9.429
250	230 4	295 38	0.427	9.631	0.804	9.905	−.062	−8.795	−1.497	−0.175	.248	9.395
255	234 8	299 42	0.421	9.625	0.825	9.917	−.066	−8.819	−1.590	−0.201	.227	9.356
260	238 19	303 53	0.416	9.619	0.849	9.929	−.069	−8.840	−1.681	−0.226	+.203	9.308
265	242 37	308 11	0.409	9.612	0.875	9.942	−.072	−8.859	−1.766	−0.247	.178	9.251
270	247 4	312 38	0.403	9.605	0.903	9.956	−.075	−8.875	−1.845	−0.266	.151	9.179
275	251 39	317 13	0.396	9.598	0.933	9.970	−.077	−8.888	−1.917	−0.283	.122	9.086
280	256 24	321 58	0.390	9.591	0.966	9.985	−.079	−8.898	−1.978	−0.296	+.091	8.959
285	261 19	326 53	0.383	9.583	1.002	0.001	−.080	−8.905	−2.031	−0.308	.058	8.766
290	266 26	332 0	0.376	9.575	1.040	0.017	−.081	−8.909	−2.067	−0.315	+.024	8.382
295	271 44	337 18	0.368	9.566	1.080	0.033	−.081	−8.910	−2.091	−0.320	−.012	−8.068
300	277 14	342 48	0.361	9.558	1.123	0.050	−.081	−8.907	−2.095	−0.321	−.049	−8.688
305	282 57	348 31	0.354	9.549	1.168	0.067	−.079	−8.899	−2.079	−0.318	−.087	−8.938
310	288 55	354 29	0.348	9.541	1.214	0.084	−.077	−8.886	−2.038	−0.309	−.126	−9.099
315	295 6	0 40	0.341	9.533	1.261	0.101	−.074	−8.867	−1.971	−0.295	−.164	−9.215
320	301 31	7 5	0.335	9.525	1.309	0.117	−.069	−8.841	−1.876	−0.273	−.202	−9.306
325	308 11	13 45	0.329	9.517	1.355	0.132	−.064	−8.806	−1.745	−0.242	−.239	−9.379
330	315 4	20 38	0.324	9.510	1.400	0.146	−.057	−8.759	−1.583	−0.199	−.274	−9.438
335	322 11	27 45	0.319	9.504	1.442	0.159	−.050	−8.698	−1.383	−0.141	−.306	−9.485
340	329 29	35 3	0.315	9.498	1.478	0.170	−.041	−8.616	−1.154	−0.062	−.334	−9.523
345	336 57	42 31	0.312	9.494	1.509	0.179	−.032	−8.503	−0.894	−9.951	−.356	−9.552
350	344 33	50 7	0.309	9.491	1.532	0.185	−.022	−8.336	−0.610	−9.785	−.373	−9.572
355	352 15	57 49	0.308	9.489	1.546	0.189	−.011	−8.040	−0.310	−9.491	−.384	−9.584
360	360 0	65 34	0.308	9.488	1.551	0.191	−.000	. .	−0.000	. .	−.387	−9.588

MERCURY—Continued.

MEAN ANOMALY.

g	k	$\log k$	Δ'	K	$\frac{dz}{dg}$	$\log \frac{dz}{dg}$	Δ'
°				°			
0	.467	9.670	3	281.1	+.095	+8.976	127
5	.471	9.673	1	275.3	.071	8.849	188
10	.472	9.674	1	269.6	.046	8.661	345
15	.471	9.673	3	264.0	+.021	+8.316	715
20	.468	9.670	6	258.5	−.004	−7.601	844
25	.462	9.664	7	253.1	.028	8.445	257
30	.454	9.657	9	247.7	.050	8.702	151
35	.445	9.648	10	242.4	.071	8.853	102
40	.434	9.638	12	237.1	−.090	−8.955	74
45	.423	9.626	12	231.8	.107	9.029	57
50	.411	9.614	13	226.6	.122	9.086	44
55	.399	9.601	13	221.3	.135	9.130	33
60	.387	9.588	13	216.0	−.146	−9.163	25
65	.376	9.575	13	210.8	.154	9.188	20
70	.365	9.562	12	205.5	.161	9.208	14
75	.354	9.550	12	200.2	.167	9.222	9
80	.345	9.538	11	194.9	−.170	9.231	6
85	.336	9.527	11	189.6	.173	9.237	3
90	.328	9.516	9	184.3	.174	9.240	1
95	.322	9.507	8	179.0	.174	9.239	3
100	.316	9.499	7	173.8	−.172	9.236	6
105	.311	9.492	6	168.6	.170	9.230	7
110	.307	9.486	4	163.4	.167	9.223	11
115	.303	9.482	4	158.4	.163	9.212	13
120	.301	9.478	3	153.4	−.158	−9.199	15
125	.299	9.475	1	148.5	.153	9.184	17
130	.298	9.474	1	143.7	.147	9.167	20
135	.297	9.473	0	139.0	.140	9.147	22
140	.297	9.473	0	134.4	−.133	−9.125	25
145	.297	9.473	1	130.0	.126	9.100	29
150	.298	9.474	2	125.6	.118	9.071	32
155	.299	9.476	2	121.3	.109	9.039	36
160	.300	9.478	2	117.1	−.101	−9.003	41
165	.302	9.480	3	113.0	.092	8.962	47
170	.304	9.483	2	109.0	.082	8.915	55
175	.306	9.485	4	105.0	.072	8.860	65
180	.308	9.489		101.1	−.062	−8.795	

ECCENTRICITY.

g	k'	$\log k'$	Δ'	K′	$\frac{dz}{de}$	$\log \frac{dz}{de}$	Δ'
°				°			
0	.348	9.542	14	196.7	−.169	−9.228	15
5	.360	9.556	29	203.4	.163	9.213	1
10	.385	9.585	38	208.9	.163	9.212	13
15	.420	9.623	40	212.6	.168	9.225	25
20	.460	9.663	36	214.6	−.178	−9.250	32
25	.500	9.699	35	215.0	.192	9.282	38
30	.542	9.734	29	214.4	.209	9.320	39
35	.579	9.763	23	213.2	.229	9.359	38
40	.611	9.786	19	211.1	−.249	−9.397	34
45	.638	9.805	14	208.8	.270	9.431	30
50	.659	9.819	11	206.2	.289	9.461	25
55	.676	9.830	7	203.5	.306	9.486	21
60	.687	9.837	4	200.5	−.322	−9.507	17
65	.694	9.841	2	197.7	.335	9.524	13
70	.696	9.843	1	194.7	.344	9.537	9
75	.695	9.842	3	192.0	.352	9.546	5
80	.690	9.839	6	189.3	−.355	−9.551	1
85	.682	9.833	6	186.6	.357	9.552	1
90	.670	9.827	9	184.1	.355	9.551	5
95	.657	9.818	12	181.7	.351	9.546	8
100	.640	9.806	12	179.6	−.345	−9.538	10
105	.623	9.794	14	177.6	.337	9.528	13
110	.604	9.780	17	176.0	.328	9.515	14
115	.580	9.763	17	174.5	.317	9.501	17
120	.558	9.746	19	173.3	−.305	−9.484	19
125	.534	9.727	19	172.6	.292	9.465	20
130	.510	9.708	23	172.0	.279	9.445	22
135	.485	9.685	23	172.1	.265	9.423	22
140	.460	9.663	22	172.5	−.252	−9.401	24
145	.437	9.640	23	173.4	.238	9.377	24
150	.414	9.617	21	175.3	.226	9.353	28
155	.394	9.596	21	177.5	.214	9.330	24
160	.376	9.575	16	180.5	−.202	−9.306	22
165	.362	9.559	12	184.0	.192	9.284	21
170	.352	9.547	5	187.8	.183	9.263	18
175	.348	9.542	1	192.3	.176	9.245	17
180	.348	9.541		196.8	−.169	−9.228	

MERCURY—Continued.

MEAN ANOMALY.

g	k	$\log k$	Δ′	K	$\frac{dz}{dg}$	$\log \frac{dz}{dg}$	Δ′
°				°			
180	.308	9.489		101.1	—.062	—8.795	
185	.310	9.492	3	97.3	.052	8.717	78
190	.312	9.495	3	93.5	.042	8.619	98
195	.314	9.497	2	89.7	.031	8.488	131
200	.316	9.500	3	85.9	—.020	—8.296	192
205	.318	9.503	3	82.2	—.009	—7.932	364
210	.321	9.506	3	78.4	+.003	+7.456	476
215	.323	9.509	3	74.7	.014	8.160	704
220	.325	9.511	2	70.9	+.026	+8.418	258
225	.326	9.514	3	67.1	.038	8.580	162
230	.328	9.516	2	63.2	.050	8.699	119
235	.330	9.518	2	59.3	.062	8.793	94
240	.332	9.521	3	55.3	+.074	+8.870	77
245	.333	9.523	2	51.2	.086	8.936	66
250	.335	9.525	2	46.9	.098	8.993	57
255	.337	9.527	2	42.6	.110	9.042	49
260	.338	9.529	2	38.1	+.122	+9.086	44
265	.340	9.532	3	33.5	.134	9.126	40
270	.342	9.534	2	28.7	.145	9.161	35
275	.344	9.537	3	23.8	.156	9.192	31
280	.347	9.540	3	18.6	+.166	+9.219	27
285	.350	9.544	4	13.3	.175	9.244	25
290	.355	9.550	6	7.8	.184	9.265	21
295	.358	9.554	4	2.1	.191	9.282	17
300	.364	9.561	7	356.2	+.198	+9.296	14
305	.370	9.568	7	350.1	.202	9.306	10
310	.376	9.576	8	343.9	.205	9.312	6
315	.385	9.585	9	337.6	.206	9.314	2
320	.394	9.595	10	331.2	+.205	+9.311	3
325	.402	9.605	10	324.7	.201	9.302	9
330	.414	9.617	12	318.3	.194	9.288	14
335	.424	9.628	11	311.9	.184	9.266	22
340	.435	9.638	10	305.5	+.172	+9.235	31
345	.446	9.649	11	299.3	.156	9.194	41
350	.454	9.657	8	293.1	.138	9.140	54
355	.461	9.664	7	287.0	.117	9.069	71
360	.467	9.670	6	281.1	+.095	+8.975	93

ECCENTRICITY.

g	k'	$\log k'$	Δ′	K′	$\frac{de}{de}$	$\log \frac{de}{de}$	Δ′
°				°			
180	.348	9.541		196.8	—.169	—9.228	
185	.353	9.548	7	201.2	.164	9.215	13
190	.363	9.560	12	205.5	.161	9.207	8
195	.378	9.577	17	209.1	.159	9.202	5
200	.396	9.598	21	212.2	—.159	—9.201	1
205	.419	9.622	24	215.0	.161	9.205	4
210	.442	9.646	24	216.8	.164	9.214	9
215	.469	9.671	25	218.4	.168	9.225	11
220	.495	9.694	23	219.3	—.175	—9.243	18
225	.521	9.717	23	219.6	.183	9.262	19
230	.548	9.739	22	219.7	.192	9.283	21
235	.573	9.758	19	219.2	.203	9.306	23
240	.599	9.777	19	218.6	—.214	—9.331	25
245	.622	9.794	17	217.4	.227	9.356	25
250	.642	9.808	14	216.3	.240	9.381	25
255	.660	9.819	11	214.8	.254	9.405	24
260	.676	9.830	11	213.0	—.268	—9.428	23
265	.689	9.838	8	211.0	.282	9.450	22
270	.699	9.845	7	208.9	.295	9.470	20
275	.706	9.849	4	206.6	.308	9.488	18
280	.707	9.850	1	204.0	—.319	—9.503	15
285	.707	9.850	0	201.5	.328	9.516	13
290	.701	9.846	4	198.6	.335	9.524	8
295	.692	9.840	6	195.8	.339	9.530	6
300	.677	9.831	9	192.9	—.340	—9.532	2
305	.659	9.819	12	190.0	.338	9.529	3
310	.638	9.805	14	187.0	.333	9.522	7
315	.612	9.786	19	184.2	.324	9.510	12
320	.582	9.765	21	181.6	—.311	—9.493	17
325	.547	9.738	27	179.5	.295	9.470	23
330	.510	9.708	30	177.7	.277	9.442	28
335	.473	9.674	34	177.3	.256	9.408	34
340	.433	9.637	37	177.9	—.235	—9.371	37
345	.398	9.600	37	180.1	.214	9.331	40
350	.370	9.568	32	184.2	.196	9.291	40
355	.351	9.546	22	189.8	.180	9.256	35
360	.348	9.541	5	196.8	—.169	—9.228	28

MERCURY—Continued.

LONG. IN ORBIT.

g	k''	$\log k''$	Δ'	K''	$\frac{dz}{du}$	$\log \frac{dz}{du}$	Δ'
°				°			
0	.301	9.479	6	281.1	+.061	+8.786	158
5	.305	9.485	4	274.1	.043	8.628	262
10	.309	9.489	5	267.3	.023	8.366	808
15	.312	9.494	4	260.6	+.004	+7.558	
20	.315	9.498	3	254.0	—.016	—8.205	345
25	.317	9.501	3	247.6	.036	8.550	186
30	.319	9.504	3	241.2	.054	8.736	125
35	.321	9.507	1	234.9	.073	8.861	93
40	.322	9.508	3	228.7	—.090	—8.954	72
45	.324	9.511	2	222.6	.106	9.026	58
50	.326	9.513	2	216.5	.121	9.084	47
55	.328	9.515	4	210.6	.135	9.131	38
60	.330	9.519	4	204.5	—.148	—9.169	32
65	.333	9.523	3	198.6	.159	9.201	26
70	.336	9.526	5	192.9	.169	9.227	21
75	.339	9.531	5	187.2	.177	9.248	16
80	.344	9.536	6	181.7	—.184	—9.264	13
85	.348	9.542	7	176.3	.189	9.277	9
90	.354	9.549	6	171.1	.194	9.286	7
95	.359	9.555	7	166.0	.196	9.293	4
100	.365	9.562	9	161.1	—.198	—9.297	1
105	.372	9.571	7	156.4	.198	9.298	2
110	.379	9.578	8	151.8	.198	9.296	4
115	.386	9.586	8	147.4	.196	9.292	7
120	.393	9.594	8	143.2	—.193	—9.285	9
125	.400	9.602	8	139.1	.189	9.276	11
130	.407	9.610	6	135.1	.184	9.265	14
135	.413	9.616	7	131.3	.178	9.251	17
140	.420	9.623	7	127.6	—.171	—9.234	20
145	.426	9.630	6	123.9	.164	9.214	22
150	.432	9.636	5	120.5	.156	9.192	26
155	.438	9.641	5	117.1	.147	9.166	30
160	.443	9.646	5	113.8	—.137	—9.136	33
165	.447	9.651	3	110.5	.127	9.103	39
170	.451	9.654	4	107.3	.116	9.064	45
175	.454	9.658	3	104.2	.105	9.019	52
180	.458	9.661		101.1	-.093	-8.967	

NODE ON EQUATOR.

g	k'''	$\log k'''$	Δ'	K'''	$\left(\frac{dz}{dN}\right)$	$\log\left(\frac{dz}{dN}\right)$
°				°		
0	.277	9.442	5	286.8	0.000	$-\infty$
5	.274	9.437	2	278.2	. .	. .
10	.272	9.435	2	269.6	. .	. .
15	.274	9.437	5	261.0	. .	. .
20	.277	9.442	8	252.4	. .	. .
25	.282	9.450	11	244.2	. .	. .
30	.289	9.461	12	236.3	. .	. .
35	.297	9.473	14	228.8	. .	. .
40	.307	9.487	15	221.8	. .	. .
45	.317	9.502	14	215.2	. .	. .
50	.328	9.516	14	209.0	. .	. .
55	.339	9.530	14	203.2	. .	. .
60	.350	9.544	12	197.8	. .	. .
65	.360	9.556	12	192.7	. .	. .
70	.370	9.568	11	187.9	. .	. .
75	.379	9.579	10	183.3	. .	. .
80	.388	9.589	9	178.9	. .	. .
85	.396	9.598	7	174.7	. .	. .
90	.403	9.605	7	170.7	. .	. .
95	.410	9.612	6	166.8	. .	. .
100	.415	9.618	5	163.0	. .	. .
105	.420	9.623	4	159.3	. .	. .
110	.424	9.627	3	155.7	. .	. .
115	.427	9.630	3	152.2	. .	. .
120	.429	9.633	2	148.6	. .	. .
125	.431	9.635	1	145.2	. .	. .
130	.432	9.636	1	141.7	. .	. .
135	.433	9.637	0	138.3	. .	. .
140	.433	9.637	0	134.9	. .	. .
145	.433	9.637	2	131.4	. .	. .
150	.432	9.635	1	128.0	. .	. .
155	.430	9.634	2	124.5	. .	. .
160	.429	9.632	1	121.0	. .	. .
165	.427	9.631	3	117.5	. .	. .
170	.425	9.628	2	114.0	. .	. .
175	.423	9.626	3	110.4	. .	. .
180	.420	9.623		106.8	. .	. .

MERCURY—Continued.

LONG. IN ORBIT.

g	k''	$\log k''$	Δ'	K''	$\frac{dz}{du}$	$\log \frac{dz}{du}$	Δ'
°				°			
180	.458	9.661		101.1	—.093	—8.967	
			1				61
185	.459	9.662		98.0	.081	8.906	
			2				74
190	.461	9.664		95.0	.068	8.832	
			0				90
195	.462	9.664		92.0	.055	8.742	
			0				118
200	.461	9.664		89.0	—.042	—8.624	
			1				164
205	.461	9.663		85.9	.029	8.460	
			1				271
210	.459	9.662		82.9	.015	8.189	
			2				893
215	.457	9.660		79.8	—.002	—7.296	
			4				
220	.453	9.656		76.7	+.012	+8.061	
			4				335
225	.449	9.652		73.6	.025	8.396	
			4				186
230	.445	9.648		70.4	.038	8.582	
			6				128
235	.439	9.642		67.1	.051	8.710	
			6				97
240	.433	9.636		63.7	+.064	+8.807	
			7				77
245	.426	9.629		60.2	.077	8.884	
			8				64
250	.418	9.621		56.6	.089	8.948	
			9				53
255	.409	9.612		52.8	.100	9.001	
			9				45
260	.401	9.603		48.9	+.111	+9.046	
			11				39
265	.391	9.592		44.8	.122	9.085	
			11				33
270	.381	9.581		40.5	.131	9.118	
			12				27
275	.371	9.569		35.9	.140	9.145	
			12				23
280	.361	9.557		31.1	+.147	+9.168	
			13				19
285	.350	9.544		26.0	.154	9.187	
			12				15
290	.340	9.532		20.6	.159	9.202	
			13				11
295	.330	9.519		14.9	.163	9.213	
			12				7
300	.321	9.507		8.8	+.166	+9.220	
			12				2
305	.313	9.495		2.4	.167	9.222	
			10				2
310	.305	9.485		355.7	.166	9.220	
			9				6
315	.299	9.476		348.6	.164	9.214	
			7				11
320	.294	9.469		341.3	+.160	+9.203	
			5				17
325	.291	9.464		333.8	.153	9.186	
			3				23
330	.289	9.461		326.1	.145	9.163	
			0				31
335	.289	9.461		318.4	.136	9.132	
			1				39
340	.290	9.462		310.7	+.124	+9.093	
			4				50
345	.292	9.465		303.1	.110	9.043	
			4				64
350	.295	9.469		295.6	.095	8.979	
			5				83
355	.298	9.474		288.3	.079	8.896	
			4				110
360	.301	9.479		281.1	+.061	+8.786	

NODE ON EQUATOR.

g	k'''	$\log k'''$	Δ'	K'''	$\left(\frac{dz}{dN}\right)$	$\log\left(\frac{dz}{dN}\right)$
°				°		
180	.420	9.623		106.8	0.000	— ∞
			3			
185	.417	9.620		103.1	. .	. .
			3			
190	.414	9.617		99.4	. .	. .
			3			
195	.412	9.614		95.6	. .	. .
			3			
200	.409	9.611		91.8	. .	. .
			3			
205	.406	9.608		87.9	. .	. .
			2			
210	.403	9.606		83.9	. .	. .
			3			
215	.400	9.603		80.0	. .	. .
			3			
220	.398	9.600		75.9	. .	. .
			3			
225	.396	9.597		71.9	. .	. .
			2			
230	.393	9.595		67.7	. .	. .
			3			
235	.391	9.592		63.5	. .	. .
			2			
240	.389	9.590		59.3	. .	. .
			2			
245	.387	9.588		55.0	. .	. .
			3			
250	.385	9.585		50.7	. .	. .
			2			
255	.383	9.583		46.4	. .	. .
			2			
260	.381	9.581		42.0	. .	. .
			2			
265	.379	9.579		37.5	. .	. .
			3			
270	.377	9.576		33.0	. .	. .
			2			
275	.375	9.574		28.4	. .	. .
			3			
280	.372	9.571		23.8	. .	. .
			3			
285	.370	9.568		19.3	. .	. .
			5			
290	.366	9.563		14.4	. .	. .
			4			
295	.362	9.559		9.5	. .	. .
			6			
300	.358	9.553		4.6	. .	. .
			5			
305	.353	9.548		359.5	. .	. .
			7			
310	.347	9.541		354.2	. .	. .
			8			
315	.341	9.533		348.8	. .	. .
			9			
320	.334	9.524		343.2	. .	. .
			10			
325	.327	9.514		337.3	. .	. .
			10			
330	.319	9.504		331.1	. .	. .
			12			
335	.311	9.492		324.6	. .	. .
			11			
340	.303	9.481		317.8	. .	. .
			11			
345	.295	9.470		310.6	. .	. .
			11			
350	.288	9.459		303.0	. .	. .
			10			
355	.282	9.449		295.0	. .	. .
			7			
360	.277	9.442		286.8	. .	. .

MERCURY—Continued.

	INCLINATION TO EQUATOR.				NODE ON EQUATOR.			
g	k^{iv}	$\log k^{iv}$	$\frac{dz}{dJ}$	$\log \frac{dz}{dJ}$ Δ'	k^{v}	$\log k^{v}$	$\frac{1}{\sin J}\frac{dz}{dN}$	$\log \frac{1}{\sin J}\frac{dz}{dN}$
°								
0	−.134	−9.129	+.246	+9.390	+.061	+8.786	−.112	−9.047
	8	22		23	19	158	34	157
5	.142	9.151	.259	9.413	.042	8.628	.078	8.890
	5	16		15	19	262	36	263
10	.147	9.167	.268	9.428	.023	8.366	.042	8.627
	3	8		8	19	808	37	807
15	.150	9.175	.273	9.436	+.004	+7.558	−.007	−7.820
	0	2		3	20		36	
20	−.150	−9.177	+.275	+9.439	−.016	−8.205	+.029	+8.467
	1	4		4	19	345	36	345
25	.149	9.173	.272	9.435	.035	8.550	.065	8.812
	3	10		10	19	186	34	185
30	.146	9.163	.266	9.425	.054	8.736	.099	8.997
	4	16		16	19	125	34	126
35	.140	9.147	.256	9.409	.073	8.861	.133	9.123
	7	22		23	17	93	31	93
40	−.133	−9.125	+.243	+9.386	−.090	−8.954	+.164	+9.216
	8	30		29	16	73	30	72
45	.125	9.095	.228	9.357	.106	9.027	.194	9.288
	10	36		36	15	57	28	58
50	.115	9.059	.209	9.321	.121	9.084	.222	9.346
	12	45		45	14	47	25	47
55	.103	9.014	.189	9.276	.135	9.131	.247	9.393
	12	54		55	13	38	23	38
60	−.091	−8.960	+.166	+9.221	−.148	−9.169	+.270	+9.431
	13	68		67	11	32	20	31
65	.078	8.892	.143	9.154	.159	9.201	.290	9.462
	14	84		85	10	26	18	26
70	.064	8.808	.117	9.069	.169	9.227	.308	9.488
	14	110		109	8	21	15	21
75	.050	8.698	.091	8.960	.177	9.248	.323	9.509
	15	152		152	7	16	13	17
80	−.035	−8.546	+.064	+8.808	−.184	−9.264	+.336	+9.526
	15	241		241	5	13	10	13
85	.020	8.305	.037	8.567	.189	9.277	.346	9.539
	15	601		601	4	10	7	9
90	−.005	−7.704	+.009	+7.966	.193	9.287	.353	9.548
	15			300	3	6	6	7
95	+.010	+8.004	−.018	−8.266	.196	9.293	.359	9.555
	15	397		397	2	4	3	3
100	+.025	+8.401	−.046	−8.663	−.198	−9.297	+.362	+9.558
	15	203		202	0	1	0	1
105	.040	8.604	.073	8.865	.198	9.298	.362	9.559
	15	135		136	0	2	1	2
110	.055	8.739	.100	9.001	.198	9.296	.361	9.557
	14	101		101	2	4	3	4
115	.069	8.840	.127	9.102	.196	9.292	.358	9.553
	14	80		80	3	7	6	6
120	+.083	+8.920	−.152	−9.182	−.193	−9.285	+.352	+9.547
	14	66		66	4	9	7	9
125	.097	8.986	.177	9.248	.189	9.276	.345	9.538
	13	55		55	5	11	9	12
130	.110	9.041	.201	9.303	.184	9.265	.336	9.526
	12	47		47	6	14	11	14
135	.122	9.088	.224	9.350	.178	9.251	.325	9.512
	12	40		40	7	17	12	16
140	+.134	+9.128	−.246	−9.390	−.171	−9.234	+.313	+9.496
	12	36		35	7	20	14	20
145	.146	9.164	.266	9.425	.164	9.214	.299	9.476
	10	30		31	8	22	15	23
150	.156	9.194	.286	9.456	.156	9.192	.284	9.453
	10	27		26	9	26	17	25
155	.166	9.221	.304	9.482	.147	9.166	.267	9.428
	9	23		24	10	30	17	30
160	+.175	+9.244	−.320	−9.506	−.137	−9.136	+.250	+9.398
	9	20		20	10	34	19	34
165	.184	9.264	.336	9.526	.127	9.102	.231	9.364
	7	18		18	11	38	20	39
170	.191	9.282	.350	9.544	.116	9.064	.211	9.325
	7	15		15	12	45	20	44
175	.198	9.297	.362	9.559	.104	9.019	.191	9.281
	6	13		12	11	52	22	52
180	+.204	+9.310	−.373	−9.571	−.093	−8.967	+.169	+9.229

MERCURY—Continued.

	INCLINATION TO EQUATOR.							NODE ON EQUATOR							
g	k^{iv}		log k^{iv}		$\frac{dz}{dJ}$	log $\frac{dz}{dJ}$	Δ'	k^{v}		log k^{v}		$\frac{1}{\sin J}\frac{dz}{dN}$		log $\frac{1}{\sin J}\frac{dz}{dN}$	
°															
180	+.204		+9.310		−.373	−9.571		−.093		−8.967		+.169		+9.229	
		5		10			11		12		61		22		61
185	.209		9.320		.382	9.582		.081		8.906		.147		9.168	
		4		9			8		13		74		23		74
190	.213		9.329		.389	9.590		.068		8.832		.124		9.094	
		3		6			7		13		90		24		91
195	.216		9.335		.395	9.597		.055		8.742		.100		9.003	
		3		4			4		13		118		23		117
200	+.219		+9.339		−.399	−9.601		−.042		−8.624		+.077		+8.886	
		1		3			3		13		164		24		165
205	.220		9.342		.401	9.604		.029		8.460		.053		8.721	
		0		0			0		14		272		25		271
210	.220		9.342		.402	9.604		.015		8.188		.028		8.450	
		1		1			1		13		892		24		892
215	.219		9.341		.400	9.603		−.002		−7.296		+.004		+7.558	
		2		4			4		13				25		
220	+.217		+9.337		−.397	−9.599		+.011		+8.061		−.021		−8.322	
		2		5			6		14		335		24		336
225	.215		9.332		.392	9.593		.025		8.396		.045		8.658	
		4		8			7		13		186		25		186
230	.211		9.324		.385	9.586		.038		8.582		.070		8.844	
		5		10			11		13		128		24		128
235	.206		9.314		.376	9.575		.051		8.710		.094		8.972	
		6		13			12		13		97		23		97
240	+.200		+9.301		−.365	−9.563		+.064		+8.807		−.117		−9.069	
		7		15			16		13		77		23		77
245	.193		9.286		.353	9.547		.077		8.884		.140		9.146	
		8		19			18		12		64		22		64
250	.185		9.267		.338	9.529		.089		8.948		.162		9.210	
		9		22			22		11		53		21		53
255	.176		9.245		.321	9.507		.100		9.001		.183		9.263	
		10		26			26		11		45		20		45
260	+.166		+9.219		−.303	−9.481		+.111		+9.046		−.203		−9.308	
		11		30			30		11		39		19		38
265	.155		9.189		.282	9.451		.122		9.085		.222		9.346	
		13		36			36		9		33		17		33
270	.142		9.153		.260	9.415		.131		9.118		.239		9.379	
		13		41			42		9		27		16		28
275	.129		9.112		.236	9.373		.140		9.145		.255		9.407	
		14		50			50		7		23		14		23
280	+.115		+9.062		−.211	−9.323		+.147		+9.168		−.269		−9.430	
		15		60			60		7		19		12		19
285	.100		9.002		.183	9.263		.154		9.187		.281		9.449	
		15		74			74		5		15		10		15
290	.085		8.928		.155	9.189		.159		9.202		.291		9.464	
		17		94			93		4		11		7		10
295	.068		8.834		.125	9.096		.163		9.213		.298		9.474	
		17		124			124		3		7		5		7
300	+.051		+8.710		−.094	−8.972		+.166		+9.220		−.303		−9.481	
		17		180			181		1		2		2		3
305	.034		8.530		.062	8.791		.167		9.222		.305		9.484	
		18		324			324		1		2		1		2
310	+.016		+8.206		−.029	−8.467		.166		9.220		.304		9.482	
		18					926		2		6		5		6
315	−.002		−7.279		+.004	+7.541		.164		9.214		.299		9.476	
		18		1019			1018		4		11		8		12
320	−.020		−8.298		+.036	+8.559		+.160		+9.203		−.291		−9.464	
		18		277			277		7		17		11		17
325	.038		8.575		.069	8.836		.153		9.186		.280		9.447	
		17		164			164		8		23		14		23
330	.055		8.739		.100	9.000		.145		9.163		.266		9.424	
		16		114			115		9		31		18		30
335	.071		8.853		.130	9.115		.136		9.132		.248		9.394	
		16		86			85		12		39		22		40
340	−.087		−8.939		+.159	+9.200		+.124		+9.093		−.226		−9.354	
		14		66			67		14		50		24		50
345	.101		9.005		.185	9.267		.110		9.043		.202		9.304	
		13		52			52		15		64		28		64
350	.114		9.057		.208	9.319		.095		8.979		.174		9.240	
		11		41			40		16		83		30		82
355	.125		9.098		.229	9.359		.079		8.896		.144		9.158	
		9		31			31		18		110		32		111
360	−.134		−9.129		+.246	+9.390		+.061		+8.786		−.112		−9.047	

MERCURY—Continued.

RADIUS VECTOR.

g	k^{vi}	log k^{vi}	K^{vi}	$\frac{dz}{dr}$		log $\frac{dz}{dr}$	
°			°				
0	.899	9.954	16.8	+.437		+9.641	
5	.888	9.948	8.3	.460	23	9.663	22
10	.880	9.945	359.6	.474	14	9.676	13
15	.877	9.943	350.9	.480	6	9.681	5
20	.879	9.944	342.5	+.478	2	+9.679	2
25	.884	9.946	334.2	.467	11	9.670	9
30	.893	9.951	326.3	.450	17	9.653	17
35	.904	9.956	318.8	.426	24	9.630	23
40	.918	9.963	311.8	+.398	28	+9.600	30
45	.931	9.969	305.2	.365	33	9.563	37
50	.944	9.975	299.0	.330	35	9.518	45
55	.956	9.981	293.2	.292	38	9.465	53
60	.968	9.986	287.8	+.252	40	+9.402	63
65	.977	9.990	282.7	.212	40	9.326	76
70	.985	9.994	277.9	.171	41	9.233	93
75	.992	9.996	273.3	.130	41	9.115	118
80	.996	9.998	268.9	+.090	40	+8.956	159
85	.999	9.999	264.7	.051	39	8.707	249
90	1.000	0.000	260.7	+.013	38	+8.099	608
95	1.000	0.000	256.8	—.025	38	—8.392	
100	.998	9.999	253.0	—.061	36	—8.783	391
105	.995	9.998	249.3	.095	34	8.979	196
110	.992	9.996	245.7	.128	33	9.109	130
115	.987	9.994	242.1	.160	32	9.204	95
120	.982	9.992	238.6	—.190	30	—9.280	76
125	.976	9.989	235.2	.219	29	9.341	61
130	.969	9.986	231.7	.246	27	6.392	51
135	.962	9.983	228.3	.272	26	9.435	43
140	.955	9.980	224.9	—.296	24	—9.472	37
145	.948	9.977	221.4	.319	23	9.504	32
150	.940	9.973	218.0	.340	21	9.532	28
155	.933	9.970	214.5	.360	20	9.557	25
160	.926	9.966	211.0	—.379	19	—9.578	21
165	.918	9.963	207.5	.395	16	9.597	19
170	.912	9.960	204.0	.411	16	9.614	17
175	.905	9.957	200.4	.425	14	9.628	14
180	.900	9.954	196.8	—.437	12	—9.641	13

RADIUS VECTOR—Continued.

g	k^{vi}	log k^{vi}	K^{vi}	$\frac{dz}{dr}$		log $\frac{dz}{dr}$	
°			°				
180	.900	9.954	196.8	—.437		—9.641	
185	.894	9.951	193.1	.448	11	9.651	10
190	.889	9.949	189.4	.458	10	9.660	9
195	.885	9.947	185.6	.465	7	9.668	8
200	.882	9.945	181.8	—.472	7	—9.674	6
205	.879	9.944	177.9	.476	4	9.678	4
210	.878	9.943	173.9	.479	3	9.680	2
215	.877	9.943	170.0	.480	1	9.681	1
220	.877	9.943	165.9	—.480	0	—9.681	0
225	.879	9.944	161.9	.477	3	9.679	2
230	.881	9.945	157.7	.473	4	9.674	5
235	.885	9.947	153.5	.466	7	9.668	6
240	.889	9.949	149.3	—.457	9	—9.660	8
245	.895	9.952	145.0	.446	11	9.650	10
250	.901	9.955	140.7	.433	13	9.636	14
255	.909	9.958	136.4	.417	16	9.620	16
260	.917	9.962	131.9	—.399	18	—9.601	19
265	.926	9.967	127.5	.377	22	9.577	24
270	.935	9.971	123.0	.353	24	9.548	29
275	.945	9.976	118.4	.326	27	9.513	35
280	.955	9.980	113.8	—.296	30	—9.471	42
285	.965	9.984	109.2	.262	34	9.419	52
290	.974	9.989	104.4	.226	36	9.353	66
295	.983	9.992	99.5	.185	41	9.268	85
300	.990	9.996	94.6	—.142	43	—9.152	116
305	.996	9.998	89.5	.095	47	8.980	172
310	.999	9.999	84.2	—.046	49	—8.665	315
315	1.000	0.000	78.8	+.006	52	+7.746	
320	.998	9.999	73.2	+.059	53	+8.773	1027
325	.993	9.997	67.3	.114	55	9.058	285
330	.986	9.994	61.1	.169	55	9.229	171
335	.974	9.989	54.6	.224	55	9.349	120
340	.961	9.983	47.8	+.276	52	+9.441	92
345	.946	9.976	40.6	.325	49	9.511	70
350	.930	9.968	33.0	.368	43	9.566	55
355	.913	9.961	25.0	.406	38	9.609	43
360	.899	9.954	16.8	+.437	31	+9.641	32

MERCURY—Continued.

SUN'S LONGITUDE.

1850.	h'	$\log h'$	Δ′	H′	$\frac{dz}{dl''}$	$\log \frac{dz}{dl''}$	Δ′
Jan. 0	.998	9.999		81.0°	+.066	+8.823	
10	.991	9.996	3	71.5	.134	9.126	303
20	.980	9.991	5	61.9	.197	9.294	168
30	.966	9.985	6	52.1	.254	9.405	111
Feb. 9	.952	9.979	6	42.0	.303	9.482	77
19	.938	9.972	7	31.6	.343	9.535	53
Mar. 1	.927	9.967	5	21.0	.372	9.571	36
11	.920	9.964	3	10.2	.390	9.592	21
21	.918	9.963	1	359.4	.397	9.599	7
31	.920	9.964	1	348.7	.391	9.593	6
Apr. 10	.928	9.967	3	338.1	.375	9.573	20
20	.939	9.973	6	327.8	.347	9.540	33
30	.952	9.979	6	317.8	.309	9.489	51
May 10	.966	9.985	6	308.2	.262	9.418	71
20	.979	9.991	6	298.8	.207	9.316	102
30	.990	9.995	4	289.8	.147	9.167	149
June 9	.997	9.999	4	280.8	.083	8.916	251
19	1.000	0.000	1	272.0	+.016	+8.196	720
29	.999	9.999	1	263.3	—.052	—8.712	
July 9	.993	9.997	2	254.5	.117	9.069	357
19	.984	9.993	4	245.5	.180	9.255	186
29	.972	9.988	5	236.4	.237	9.375	120
Aug. 8	.959	9.982	6	227.0	.288	9.459	84
18	.945	9 976	6	217.2	.330	9.519	60
28	.933	9.970	6	207.2	.364	9.561	42
Sept. 7	.924	9.966	4	196.9	.386	9.587	26
17	.918	9.963	3	186.3	.398	9.600	13
27	.918	9.963	0	175.6	.398	9.600	0
Oct. 7	.922	9.965	2	164.9	.386	9.587	13
17	.931	9.969	4	154.4	.363	9.560	27
27	.944	9.975	6	144.0	.329	9.517	43
Nov. 6	.958	9.981	6	133.8	.285	9.455	62
16	.972	9.988	7	123.9	.232	9.366	89
26	.985	9.993	5	114.2	.173	9.237	129
Dec. 6	.994	9.997	4	104.7	.108	9.032	205
16	.999	9.999	2	95.2	—.039	—8.594	438
26	1.000	0.000	1	85.9	+.030	+8.481	

OBLIQUITY.

1850.	h''	$\log h''$	Δ′	H″	$\frac{dz}{d\epsilon}$	$\log \frac{dy}{d\epsilon}$	Δ′
Jan. 0	+.392	+9.594		. .	—.889	—9.949	
10	.374	9.573	21	. .	.848	9.928	21
20	.344	9.537	36	. .	.781	9.893	35
30	.304	9.482	55	. .	.689	9.838	55
Feb. 9	.254	9.404	78	. .	.577	9.761	77
19	.196	9.292	112	. .	.446	9.650	111
Mar. 1	.132	9.122	170	. .	.302	9.480	170
11	+.065	+8.814	308	. .	—.149	—9.174	306
21	—.004	—7.570		. .	+.009	+7.932	
31	.072	8.857	1287	. .	.166	9.220	1288
Apr. 10	.138	9.139	282	. .	.318	9.503	283
20	.199	9.299	160	. .	.462	9.664	161
30	.255	9.406	107	. .	.591	9.772	108
May 10	.302	9.481	75	. .	.704	9.848	76
20	.341	9.533	52	. .	.797	9.901	53
30	.371	9.569	36	. .	.866	9.938	37
June 9	.390	9.591	22	. .	.912	9.960	22
19	.398	9.600	9	. .	.932	9.969	9
29	.395	9.596	4	. .	.925	9.966	3
July 9	.381	9.581	15	. .	.892	9.951	15
19	.357	9.552	29	. .	.835	9.922	29
29	.322	9.508	44	. .	.754	9.877	45
Aug. 8	.279	9.446	62	. .	.651	9.814	63
18	.228	9.357	89	. .	.531	9.725	89
28	.170	9.229	128	. .	.395	9.596	129
Sept. 7	.107	9.028	201	. .	.247	9.393	203
17	—.040	—8.604	424	. .	+.093	+8.968	425
27	+.028	+8.444	(160)	. .	—.066	—8.820	
Oct. 7	.095	8.979	535	. .	.220	9.341	521
17	.159	9.201	226	. .	.368	9.566	225
27	.221	9.345	140	. .	.506	9.704	138
Nov. 6	.275	9.440	95	. .	.628	9.798	94
16	.321	9.507	67	. .	.732	9.864	66
26	.358	9.553	46	. .	.813	9.910	46
Dec. 6	.383	9.583	30	. .	.869	9.939	29
16	.396	9.598	15	. .	.898	9.953	14
26	+.397	+9.599	1	. .	—.899	—9.954	1

MERCURY—Continued.

EARTH'S ECCENTRICITY.

1850.	h'''	$\log h'''$	Δ'	H'''	$\frac{dz}{de''}$	$\log \frac{dz}{de''}$	Δ'
Jan. 0	.923	9.965		347.6	+.390	+9.591	
10	.951	9.978	13	357.8	.413	9.616	25
20	1.059	0.025	47	5.5	.458	9.660	44
30	1.209	0.083	58	9.0	.518	9.714	54
Feb. 9	1.371	0.137	54	9.2	.587	9.769	55
19	1.523	0.183	46	7.1	.656	9.817	48
Mar. 1	1.653	0.218	35	3.5	.716	9.855	38
11	1.751	0.243	25	358.9	.760	9.881	26
21	1.815	0.259	16	353.8	.783	9.894	13
31	1.841	0.265	6	348.5	.783	9.894	0
Apr. 10	1.828	0.262	3	343.2	.759	9.880	14
20	1.778	0.250	12	338.2	.716	9.855	25
30	1.692	0.228	22	333.8	.659	9.819	36
May 10	1.574	0.197	31	330.3	.593	9.773	46
20	1.432	0.156	41	328.1	.527	9.722	51
30	1.276	0.106	50	327.7	.468	9.670	52
June 9	1.124	0.051	55	330.0	.422	9.626	44
19	.998	9.999	52	335.7	.395	9.596	30
29	.928	9.967	32	344.6	.388	9.589	7
July 9	.933	9.970	3	354.8	.403	9.606	17
19	1.013	0.005	35	3.1	.439	9.642	36
29	1.142	0.057	52	8.0	.491	9.691	49
Aug. 8	1.292	0.111	54	9.5	.553	9.743	52
18	1.444	0.159	58	8.5	.620	9.792	49
28	1.581	0.199	40	5.8	.683	9.834	42
Sept. 7	1.695	0.229	30	1.8	.735	9.866	32
17	1.779	0.250	21	357.1	.771	9.887	21
27	1.828	0.262	12	352.0	.785	9.895	8
Oct. 7	1.841	0.265	3	346.6	.777	9.890	5
17	1.814	0.259	6	341.4	.746	9.873	17
27	1.749	0.243	16	336.5	.696	9.843	30
Nov. 6	1.646	0.217	26	332.3	.632	9.801	42
16	1.511	0.179	38	329.1	.563	9.750	51
26	1.353	0.131	48	327.6	.496	9.696	54
Dec. 6	1.188	0.075	56	328.6	.440	9.644	52
16	1.039	0.017	58	333.2	.402	9.605	39
26	.941	9.974	43	341.7	+.388	+9.577	28

SUN'S PERIGEE.

1850.	h^{iv}	$\log h^{iv}$	Δ'	H^{iv}	$\frac{dz}{e'' d\pi''}$	$\log \frac{dz}{e'' d\pi''}$	Δ'
Jan. 0	1.995	0.300		260.6	—.142	—9.152	
10	1.970	0.294	6	255.9	.209	9.319	167
20	1.897	0.278	16	251.4	.263	9.420	101
30	1.783	0.251	27	247.3	.298	9.475	55
Feb. 9	1.637	0.214	37	244.1	.310	9.491	16
19	1.470	0.167	47	242.2	.298	9.474	17
Mar. 1	1.299	0.114	53	242.2	.263	9.420	54
11	1.146	0.059	55	245.3	.210	9.322	98
21	1.036	0.015	44	251.2	.145	9.161	161
31	0.997	9.998	17	259.8	.077	8.884	277
Apr. 10	1.040	0.017	19	268.4	—.013	—8.100	784
20	1.151	0.061	44	274.5	+.041	+8.607	
30	1.304	0.115	54	277.5	.074	8.870	263
May 10	1.471	0.168	53	277.9	.088	8.943	73
20	1.633	0.213	45	276.4	.079	8.895	48
30	1.775	0.249	36	273.6	.049	8.686	209
June 9	1.887	0.276	27	270.1	+.001	+7.000	686
19	1.961	0.292	16	266.0	—.059	—8.772	
29	1.993	0.300	8	261.7	.125	9.097	325
July 9	1.983	0.297	3	257.3	.189	9.276	179
19	1.930	0.285	12	253.0	.245	9.389	113
29	1.837	0.264	21	249.0	.286	9.456	67
Aug. 8	1.712	0.233	31	245.5	.308	9.488	32
18	1.562	0.193	40	243.0	.308	9.488	0
28	1.398	0.145	48	241.9	.286	9.456	32
Sept. 7	1.236	0.092	53	242.9	.244	9.388	68
17	1.099	0.041	51	246.9	.187	9.272	116
27	1.013	0.006	35	254.0	.120	9.078	194
Oct. 7	1.003	0.001	5	263.0	—.053	—8.727	351
17	1.073	0.031	30	271.0	+.008	+7.892	
27	1.206	0.081	50	276.0	.055	8.740	848
Nov. 6	1.372	0.137	56	278.0	.082	8.916	176
16	1.547	0.189	52	277.4	.086	8.936	20
26	1.708	0.233	44	275.1	.066	8.819	117
Dec. 6	1.843	0.265	32	271.7	+.023	+8.367	452
16	1.939	0.287	22	267.5	—.036	—8.556	
26	1.988	0.298	11	263.0	—.105	—9.020	464

MERCURY—Continued.

SUN'S RADIUS VECTOR.

1850.	h^v	$\log h^v$	H^v	$\frac{dz}{dr''}$	Diff.	$\log \frac{dz}{dr''}$	Diff.
			°				
Jan. 0	.920	9.964	169.4	−.392	18	−9.594	21
10	.928	9.967	158.4	.374	30	9.573	36
20	.939	9.973	147.7	.344	40	9.537	55
30	.953	9.979	137.2	.304	50	9.482	78
Feb. 9	.967	9.986	127.2	.254	58	9.404	112
19	.981	9.992	117.4	.196	64	9.292	170
Mar. 1	.991	9.996	107.9	.132	67	9.122	308
11	.998	9.999	98.6	−.065	69	−8.814	
21	1.000	0.000	89.5	+.004	68	+7.569	1288
31	.998	9.999	80.4	.072	66	8.857	282
Apr. 10	.990	9.996	71.3	.138	61	9.139	160
20	.980	9.991	62.1	.199	56	9.299	107
30	.967	9.985	52.7	.255	47	9.406	75
May 10	.953	9.979	43.0	.302	40	9.481	52
20	.940	9.973	33.2	.342	29	9.533	36
30	.929	9.968	23.1	.371	19	9.569	22
June 9	.921	9.964	12.8	.390	8	9.591	9
19	.918	9.963	2.4	.398	3	9.600	4
29	.919	9.963	352.0	.395	14	9.596	15
July 9	.924	9.966	341.7	+.381		+9.581	

SUN'S RADIUS VECTOR—Continued.

1850.	h^v	$\log h^v$	H^v	$\frac{dz}{dr''}$	Diff.	$\log \frac{dz}{dr''}$	Diff.
			°				
July 9	.924	9.966	341.7	+.381	24	+9.581	29
19	.934	9.970	331.6	.357	35	9.552	44
29	.946	9.976	321.7	.322	43	9.508	63
Aug. 8	.960	9.982	312.0	.279	51	9.445	88
18	.974	9.988	302.6	.228	58	9.357	128
28	.985	9.994	293.4	.170	63	9.229	201
Sept. 7	.994	9.998	284.3	.107	67	9.028	425
17	.999	0.000	275.3	+.040	68	+8.603	
27	1.000	0.000	266.3	−.028	67	−8.445	535
Oct. 7	.995	9.998	257.3	.095	65	8.980	225
17	.987	9.994	248.0	.160	61	9.205	139
27	.975	9.989	238.5	.221	54	9.344	96
Nov. 6	.961	9.983	228.7	.275	46	9.440	67
16	.947	9.976	218.6	.321	57	9.507	46
26	.934	9.970	208.1	.358	25	9.553	30
Dec. 6	.024	9.966	197.3	.383	13	9.583	15
16	.918	9.963	186.3	.396	1	9.598	1
26	.918	9.963	175.2	.397	12	9.599	13
36	.923	9.965	164.1	−.385		−9.586	

TABLES FOR VENUS.

MEAN ANOMALY OF VENUS.

	°		°
1750	279.9	1820 B	201.6
51	144.7	21	66.3
52 B	11.0	22	291.1
53	235.8	23	155.9
54	100.6	24 B	22.3
55	325.4	25	247.1
56 B	191.7	26	111.8
57	56.5	27	336.6
58	281.3	28 B	203.0
59	146.1	29	67.8
1760 B	12.5	1830	292.5
61	237.2	31	157.3
62	102.0	32 B	23.7
63	326.8	33	248.5
64 B	193.2	34	113.2
65	58.0	35	338.0
66	282.7	36 B	204.4
67	147.5	37	69.2
68 B	13.9	38	294.0
69	238.7	39	158.7
1770	103.4	1840 B	25.1
71	328.2	41	249.9
72 B	194.6	42	114.7
73	59.4	43	339.5
74	284.1	44 B	205.8
75	148.9	45	70.6
76 B	15.3	46	295.4
77	240.1	47	160.2
78	104.9	48 B	26.6
79	329.7	49	251.3
1780 B	196.0	1850	116.1
81	60.8	51	340.9
82	285.6	52 B	207.3
83	150.4	53	72.0
84 B	16.7	54	296.8
85	241.5	55	161.6
86	106.3	56 B	28.0
87	331.1	57	252.8
88 B	197.5	58	117.5
89	62.2	59	342.3
1790	287.0	1860 B	208.7
91	151.8	61	73.5
92 B	18.2	62	298.2
93	242.9	63	163.0
94	107.7	64 B	29.4
95	332.5	65	254.2
96 B	198.9	66	119.0
97	63.7	67	343.7
98	288.4	68 B	210.1
99	153.2	69	74.9
1800	18.0	1870	299.7
01	242.8	71	164.5
02	107.5	72 B	30.8
03	332.3	73	255.6
04 B	198.7	74	120.4
05	63.5	75	345.2
06	288.3	76 B	211.6
07	153.0	77	76.3
08 B	19.4	78	301.1
09	244.2	79	165.9
1810	109.0	1880 B	32.3
11	333.8	81	257.0
12 B	200.1	82	121.8
13	64.9	83	346.6
14	289.7	84 B	213.0
15	154.5	85	77.8
16 B	20.9	86	302.5
17	245.6	87	167.3
18	110.4	88 B	33.7
19	335.2	89	258.5
1820 B	201.6	1890	123.2

	January.	February.	March.	April.	May.	June.
	°	°	°	°	°	°
1	0.0	49.7	94.5	144.2	192.3	241.9
2	1.6	51.3	96.1	145.8	193.9	243.5
3	3.2	52.9	97.7	147.4	195.5	245.1
4	4.8	54.5	99.3	149.0	197.1	246.7
5	6.4	56.1	100.9	150.6	198.7	248.3
6	8.0	57.7	102.5	152.2	200.3	249.9
7	9.6	59.3	104.1	153.8	201.9	251.5
8	11.2	60.9	105.7	155.4	203.5	253.1
9	12.8	62.5	107.3	157.0	205.1	254.7
10	14.4	64.1	108.9	158.6	206.7	256.3
11	16.0	65.7	110.5	160.2	208.3	257.9
12	17.6	67.3	112.1	161.8	209.9	259.5
13	19.2	68.9	113.8	163.4	211.5	261.1
14	20.8	70.5	115.4	165.0	213.1	262.7
15	22.4	72.1	117.0	166.6	214.7	264.4
16	24.0	73.7	118.6	168.2	216.3	266.0
17	25.6	75.3	120.2	169.8	217.9	267.6
18	27.2	76.9	121.8	171.4	219.5	269.2
19	28.8	78.5	123.4	173.0	221.1	270.8
20	30.4	80.1	125.0	174.6	222.7	272.4
21	32.0	81.7	126.6	176.2	224.3	274.0
22	33.6	83.3	128.2	177.8	225.9	275.6
23	35.2	84.9	129.8	179.4	227.5	277.2
24	36.9	86.5	131.4	181.0	229.1	278.8
25	38.5	88.1	133.0	182.6	230.7	280.4
26	40.1	89.7	134.6	184.2	232.3	282.0
27	41.7	91.3	136.2	185.8	233.9	283.6
28	43.3	92.9	137.8	187.4	235.5	285.2
29	44.9		139.4	189.0	237.1	286.8
30	46.5		141.0	190.7	238.7	288.4
31	48.1		142.6		240.3	

	July.	August.	September.	October.	November.	December.
	°	°	°	°	°	°
1	290.0	339.7	29.3	77.4	127.0	175.1
2	291.6	341.3	30.9	79.0	128.6	176.7
3	293.2	342.9	32.5	80.6	130.3	178.3
4	294.8	344.5	34.1	82.2	131.9	179.9
5	296.4	346.1	35.7	83.8	133.5	181.5
6	298.0	347.7	37.3	85.4	135.1	183.1
7	299.6	349.3	38.9	87.0	136.7	184.7
8	301.2	350.9	40.5	88.6	138.3	186.3
9	302.8	352.5	42.1	90.2	139.9	187.9
10	304.4	354.1	43.7	91.8	141.5	189.5
11	306.0	355.7	45.3	93.4	143.1	191.1
12	307.6	357.3	46.9	95.0	144.7	192.7
13	309.2	358.9	48.5	96.6	146.3	194.3
14	310.8	0.5	50.1	98.2	147.9	195.9
15	312.4	2.1	51.7	99.8	149.5	197.5
16	314.0	3.7	53.3	101.4	151.1	199.1
17	315.6	5.3	55.0	103.0	152.7	200.7
18	317.2	6.9	56.6	104.6	154.3	202.3
19	318.8	8.5	58.2	106.2	155.9	204.0
20	320.4	10.1	59.8	107.8	157.5	205.6
21	322.0	11.7	61.4	109.4	159.1	207.2
22	323.6	13.3	63.0	111.0	160.7	208.8
23	325.2	14.9	64.6	112.6	162.3	210.4
24	326.8	16.5	66.2	114.2	163.9	212.0
25	328.4	18.1	67.8	115.8	165.5	213.6
26	330.0	19.7	69.4	117.4	167.1	215.2
27	331.6	21.3	71.0	119.0	168.7	216.8
28	333.2	22.9	72.6	120.6	170.3	218.4
29	334.8	24.5	74.2	122.2	171.9	220.0
30	336.4	26.1	75.8	123.8	173.5	221.6
31	338.0	27.7		125.4		223.2

In January and February of bissextile years enter this table with the date diminished by one day.

VENUS.

g	f	$w+f=u$	r	$\log r$	$\frac{df}{dg}$	$\log \frac{df}{dg}$	$\frac{dr}{dg}$	$\log \frac{dr}{dg}$	$\frac{df}{de}$	Diff.	$\log \frac{df}{de}$	Diff.	$\frac{dr}{de}$	Diff.	$\log \frac{dr}{de}$	Diff.
°	°	°														
0	0.0	122.2	.718	9.856	1.014	0.006	.000	. .	0.000		. .		—.723		—9.859	
5	5.1	127.3	.718	9.856	1.014	0.006	.000	+6.635	+0.177	177	+9.248		.720	3	9.858	1
10	10.1	132.3	.718	9.856	1.014	0.006	+.001	6.934	0.353	176	9.548	300	.712	8	9.853	5
15	15.2	137.4	.719	9.856	1.013	0.006	.001	7.108	0.526	173	9.721	173	.698	14	9.844	9
20	20.3	142.5	.719	9.857	1.013	0.006	+.002	+7.229	+0.695	169	+9.842	121	—.679	19	—9.832	12
25	25.3	147.5	.719	9.857	1.012	0.005	.002	7.320	0.859	164	9.934	92	.654	25	9.815	17
30	30.4	152.6	.719	9.857	1.012	0.005	.002	7.393	1.015	156	0.006	72	.624	30	9.795	20
35	35.5	157.7	.719	9.857	1.011	0.005	.003	7.453	1.163	148	0.066	60	.589	35	9.770	25
40	40.5	162.7	.720	9.857	1.010	0.004	+.003	+7.503	+1.303	140	+0.115	49	—.550	39	—9.740	30
45	45.6	167.8	.720	9.857	1.010	0.004	.004	7.544	1.431	128	0.156	41	.506	44	9.705	35
50	50.6	172.8	.720	9.857	1.009	0.004	.004	7.579	1.549	118	0.190	34	.459	47	9.662	43
55	55.6	177.8	.720	9.858	1.008	0.003	.004	7.608	1.655	116	0.219	29	.408	51	9.611	51
60	60.7	182.9	.721	9.858	1.007	0.003	+.004	+7.632	+1.747	92	+0.242	23	—.354	54	—9.549	62
65	65.7	187.9	.721	9.858	1.006	0.002	.004	7.652	1.826	79	0.262	20	.298	56	9.474	75
70	70.7	192.9	.722	9.858	1.005	0.002	.005	7.668	1.890	64	0.276	14	.239	59	9.378	96
75	75.8	198.0	.722	9.859	1.003	0.002	.005	7.679	1.940	50	0.288	12	.178	61	9.250	128
80	80.8	203.0	.722	9.859	1.002	0.001	+.005	+7.688	+1.975	35	+0.296	8	—.116	62	—9.064	186
85	85.8	208.0	.723	9.859	1.001	0.000	.005	7.693	1.995	20	0.300	4	—.053	63	—8.726	338
90	90.8	213.0	.723	9.859	1.000	0.000	.005	7.695	1.999	4	0.301	1	+.010	63	+7.996	
95	95.8	218.0	.724	9.860	0.999	9.999	.005	7.693	1.990	9	0.299	2	.073	63	8.862	366
100	100.8	223.0	.724	9.860	0.998	9.999	+.005	+7.688	+1.964	26	+0.293	6	+.135	62	+9.131	269
105	105.8	228.0	.725	9.860	0.996	9.998	.005	7.679	1.923	41	0.284	9	.196	61	9.293	162
110	110.7	232.9	.725	9.860	0.995	9.998	.005	7.668	1.868	55	0.272	12	.256	60	9.408	115
115	115.7	237.9	.726	9.861	0.994	9.997	.004	7.652	1.799	69	0.255	17	.314	58	9.496	88
120	120.7	242.9	.726	9.861	0.993	9.997	+.004	+7.628	+1.717	82	+0.235	20	+.369	55	+9.567	71
125	125.6	247.8	.726	9.861	0.992	9.997	.004	7.608	1.622	95	0.210	25	.422	53	9.625	58
130	130.6	252.8	.726	9.861	0.991	9.996	.004	7.579	1.515	107	0.180	30	.471	49	9.673	48
135	135.6	257.8	.727	9.861	0.990	9.996	.003	7.544	1.397	118	0.145	35	.516	45	9.713	40
140	140.5	262.7	.727	9.862	0.990	9.995	+.003	+7.503	+1.269	128	+0.104	41	+.558	42	+9.747	34
145	145.4	267.6	.727	9.862	0.989	9.995	.003	7.453	1.131	138	0.053	51	.596	38	9.775	28
150	150.4	272.6	.728	9.862	0.988	9.995	.003	7.393	0.985	146	9.994	59	.629	33	9.798	23
155	155.3	277.5	.728	9.862	0.988	9.995	.002	7.320	0.832	153	9.920	74	.657	28	9.818	20
160	160.3	282.5	.728	9.862	0.987	9.994	+.002	+7.229	+0.673	159	+9.828	92	+.681	24	+9.833	15
165	165.2	287.4	.728	9.862	0.987	9.994	.001	7.108	0.509	164	9.707	121	.699	18	9.845	12
170	170.1	292.3	.728	9.862	0.987	9.994	+.001	6.934	0.341	168	9.533	174	.713	14	9.853	8
175	175.1	297.3	.728	9.862	0.987	9.994	.000	+6.635	+0.172	169	+9.234	299	.720	7	9.858	5
180	180.0	302.2	.728	9.862	0.986	9.994	.000	. .	0.000	172	. .		+.723	3	+9.859	1

VENUS—Continued.

g	f	$w+f-u$	r	$\log r$	$\frac{df}{dg}$	$\log\frac{df}{dg}$	$\frac{dr}{dg}$	$\log\frac{dr}{dg}$	$\frac{df}{de}$	diff	$\log\frac{df}{de}$	diff	$\frac{dr}{de}$	diff	$\log\frac{dr}{de}$	diff
°	°	°														
180	180.0	302.2	.728	9.862	0.986	9.994	.000	. .	0.000	172	. .		+.723	3	+9.859	1
185	184.9	307.1	.728	9.862	0.986	9.994	.000	—6.635	—0.172	169	—9.234	299	.720	7	9.858	5
190	189.9	312.1	.728	9.862	0.987	9.994	—.001	6.934	0.341	168	9.533	174	.713	14	9.853	8
195	194.8	317.0	.728	9.862	0.987	9.994	.001	7.108	0.509	164	9.707	121	.699	18	9.845	12
200	199.7	321.9	.728	9.862	0.987	9.994	—.002	—7.229	—0.673	159	—9.828	92	+.681	24	+9.833	15
205	204.7	326.9	.728	9.862	0.988	9.995	.002	7.320	0.832	153	9.920	74	.657	28	9.818	20
210	209.6	331.8	.728	9.862	0.988	9.995	.002	7.393	0.985	146	9.994	59	.629	33	9.798	23
215	214.6	336.8	.727	9.862	0.989	9.995	.003	7.453	1.131	138	0.053	51	.596	38	9.775	28
220	219.5	341.7	.727	9.862	0.989	9.995	—.003	—7.503	—1.269	128	—0.104	41	+.558	42	+9.747	34
225	224.4	346.6	.727	9.861	0.990	9.996	.003	7.544	1.397	118	0.145	35	.516	45	9.713	40
230	229.4	351.6	.726	9.861	0.991	9.996	.004	7.579	1.515	107	0.180	30	.471	49	9.673	48
235	234.4	356.6	.726	9.861	0.992	9.997	.004	7.608	1.622	95	0.210	25	.422	53	9.625	58
240	239.3	1.5	.726	9.861	0.993	9.997	—.004	—7.628	—1.717	82	—0.235	20	+.369	55	+9.567	71
245	244.3	6.5	.725	9.861	0.994	9.997	.004	7.652	1.799	69	0.255	17	.314	58	9.496	88
250	249.3	11.5	.725	9.860	0.995	9.998	.005	7.668	1.868	55	0.272	12	.256	60	9.408	115
255	254.2	16.4	.725	9.860	0.996	9.998	.005	7.679	1.923	41	0.284	9	.196	61	9.293	162
260	259.2	21.4	.724	9.860	0.998	9.999	—.005	—7.688	—1.964	26	—0.293	6	+.135	62	+9.131	269
265	264.2	26.4	.724	9.860	0.999	9.999	.005	7.693	1.990	9	0.299	2	.073	63	8.862	866
270	269.2	31.4	.723	9.859	1.000	0.000	.005	7.695	1.999	4	0.301	1	+.010	63	+7.996	
275	274.2	36.4	.723	9.859	1.001	0.000	.005	7.693	1.995	20	0.300	4	—.053	63	—8.726	338
280	279.2	41.4	.722	9.859	1.002	0.001	—.005	—7.688	—1.975	35	—0.296	8	—.116	62	—9.064	186
285	284.2	46.4	.722	9.859	1.003	0.002	.005	7.679	1.940	50	0.288	12	.178	61	9.250	128
290	289.3	51.5	.722	9.858	1.005	0.002	.005	7.668	1.890	64	0.276	14	.239	59	9.378	96
295	294.3	56.5	.721	9.858	1.006	0.003	.004	7.652	1.826	79	0.262	20	.298	56	9.474	75
300	299.3	61.5	.721	9.858	1.007	0.003	—.004	—7.632	—1.747	92	—0.242	23	—.354	54	—9.549	62
305	304.4	66.6	.720	9.858	1.008	0.003	.004	7.608	1.655	106	0.219	29	.408	51	9.611	51
310	309.4	71.6	.720	9.857	1.009	0.004	.004	7.579	1.549	118	0.190	34	.459	47	9.662	43
315	314.4	76.6	.720	9.857	1.010	0.004	.004	7.544	1.431	128	0.156	41	.506	44	9.705	35
320	319.5	81.7	.720	9.857	1.011	0.004	—.003	—7.503	—1.303	140	—0.115	49	—.550	39	—9.740	30
325	324.5	86.7	.719	9.857	1.011	0.005	.003	7.453	1.163	148	0.066	60	.589	35	9.770	25
330	329.6	91.8	.719	9.857	1.012	0.005	.002	7.393	1.015	156	0.006	72	.624	30	9.795	20
335	334.7	96.9	.719	9.857	1.012	0.005	.002	7.320	0.859	164	9.934	92	.654	25	9.815	17
340	339.7	101.9	.719	9.857	1.013	0.006	—.002	—7.229	—0.695	169	—9.842	121	—.679	19	—9.832	12
345	344.8	107.0	.719	9.856	1.013	0.006	.001	7.108	0.526	173	9.721	173	.698	14	9.844	9
350	349.9	112.1	.718	9.856	1.014	0.006	—.001	6.934	0.353	176	9.548	300	.712	8	9.853	5
355	354.9	117.1	.718	9.856	1.014	0.006	.000	—6.635	—0.177	177	—9.248		.720	3	9.858	1
360	360.0	122.2	.718	9.856	1.014	0.006	.000	. .	0.000		. .		—.723		—9.859	

VENUS—Continued.

MEAN ANOMALY.

g	k	log k	K	$\frac{ds}{dg}$		log $\frac{ds}{dg}$	
°			°				
0	.710	9.851	232.3	—.161		—9.207	
					22		55
5	.704	9.848	227.4	.183		9.262	
					20		46
10	.699	9.845	222.5	.203		9.308	
					19		38
15	.693	9.841	217.5	.222		9.346	
					17		33
20	.687	9.837	212.4	—.239		—9.379	
					15		26
25	.682	9.834	207.2	.254		9.405	
					14		22
30	.676	9.830	202.0	.268		9.427	
					11		18
35	.672	9.827	196.6	.279		9.445	
					9		14
40	.667	9.824	191.2	—.288		—9.459	
					6		10
45	.664	9.822	185.8	.294		9.469	
					5		7
50	.662	9.821	180.3	.299		9.476	
					2		3
55	.660	9.820	174.8	.301		9.479	
					0		1
60	.660	9.820	169.3	—.301		—9.478	
					3		3
65	.661	9.820	163.8	.298		9.475	
					4		7
70	.662	9.821	158.3	.294		9.468	
					7		11
75	.665	9.823	152.9	.287		9.457	
					10		14
80	.668	9.825	147.5	—.277		—9.443	
					11		18
85	.672	9.828	142.2	.266		9.425	
					13		22
90	.677	9.831	137.0	.253		9.403	
					15		27
95	.682	9.834	131.8	.238		9.376	
					17		32
100	.688	9.837	126.8	—.221		—9.344	
					19		39
105	.693	9.841	121.8	.202		9.305	
					20		45
110	.698	9.844	116.9	.182		9.260	
					21		55
115	.703	9.847	112.1	.161		9.205	
					23		66
120	.708	9.850	107.4	—.138		9.139	
					24		81
125	.712	9.852	102.7	.114		9.058	
					24		105
130	.715	9.854	98.1	.090		8.953	
					25		142
135	.717	9.855	93.5	.065		8.811	
					26		218
140	.718	9.856	88.9	—.039		—8.593	
					26		465
145	.719	9.857	84.4	—.013		—8.127	
					26		
150	.719	9.857	79.9	+.013		+8.097	
					25		485
155	.718	9.856	75.4	+.038		+8.582	
					26		222
160	.716	9.855	70.8	+.064		+8.804	
					25		144
165	.713	9.853	66.3	.089		8.948	
					24		105
170	.709	9.851	61.6	.113		9.053	
					23		82
175	.705	9.848	57.0	.136		9.135	
					22		65
180	.700	9.845	52.3	+.158		+9.200	

ECCENTRICITY.

g	k'		log k'		K′	$\frac{ds}{de}$		log $\frac{ds}{de}$	
°					°				
0	.677		9.831		137.4	—.254		—9.405	
		3		2			16		26
5	.680		9.833		142.6	.270		9.431	
		18		11			19		30
10	.698		9.844		147.3	.289		9.461	
		32		19			23		33
15	.730		9.863		151.4	.312		9.494	
		42		25			24		32
20	.772		9.888		154.5	—.336		—9.526	
		50		27			26		32
25	.822		9.915		156.6	.362		9.558	
		56		28			26		31
30	.878		9.943		157.7	.388		9.589	
		58		28			26		28
35	.936		9.971		158.0	.414		9.617	
		57		26			25		26
40	.993		9.997		157.7	—.439		—9.643	
		57		24			23		22
45	1.050		0.021		156.8	.462		9.665	
		55		22			21		19
50	1.105		0.043		155.5	.483		9.684	
		51		20			18		16
55	1.156		0.063		153.8	.501		9.700	
		46		17			13		11
60	1.202		0.080		151.8	—.514		—9.711	
		42		15			10		9
65	1.244		0.095		149.6	.524		9.720	
		35		12			6		4
70	1.279		0.107		147.2	.530		9.724	
		29		10			0		0
75	1.308		0.117		144.7	.530		9.724	
		23		7			4		3
80	1.331		0.124		142.1	—.526		—9.721	
		15		5			8		7
85	1.346		0.129		139.6	.518		9.714	
		9		3			12		10
90	1.355		0.132		137.0	.506		9.704	
		1		0			16		14
95	1.356		0.132		134.4	.490		9.690	
		8		2			20		18
100	1.348		0.130		131.9	—.470		—9.672	
		14		5			22		21
105	1.334		0.125		129.4	.448		9.651	
		22		7			24		24
110	1.312		0.118		127.1	.424		9.627	
		30		10			26		27
115	1.282		0.108		124.9	.398		9.600	
		37		13			26		29
120	1.245		0.095		123.0	—.372		—9.571	
		43		15			26		32
125	1.202		0.080		121.2	.346		9.539	
		49		18			24		32
130	1.153		0.062		119.7	.322		9.507	
		54		21			23		32
135	1.099		0.041		118.6	.299		9.475	
		57		23			21		31
140	1.042		0.018		117.9	—.278		—9.444	
		60		26			17		27
145	.982		9.992		117.7	.261		9.417	
		61		28			14		24
150	.921		9.964		118.1	.247		9.393	
		59		29			10		18
155	.862		9.935		119.2	.237		9.375	
		56		29			5		10
160	.806		9.906		121.2	—.232		—9.365	
		49		27			1		2
165	.757		9.879		124.0	.231		9.363	
		40		23			3		7
170	.717		9.856		127.8	.234		9.370	
		27		17			8		14
175	.690		9.839		132.3	.242		9.384	
		13		8			12		21
180	.677		9.831		137.4	—.254		—9.405	

VENUS—Continued.

MEAN ANOMALY.

g	k	log k	K	$\frac{dz}{dg}$	Δ	$\log\frac{dz}{dg}$	Δ
°			°				
180	.700	9.845	52.3	+.158		+9.200	
					22		55
185	.695	9.842	47.5	.180		9.255	
					20		46
190	.690	6.839	42.6	.200		9.301	
					18		38
195	.684	9.835	37.7	.218		9.339	
					17		32
200	.679	9.832	32.7	+.235		+9.371	
					15		27
205	.674	9.828	27.6	.250		9.398	
					13		23
210	.669	9.825	22.4	.263		9.421	
					12		18
215	.664	9.822	17.1	.275		9.439	
					9		14
220	.661	9.820	11.8	+.284		+9.453	
					7		11
225	.658	9.818	6.4	.291		9.464	
					5		7
230	.656	9.817	1.0	.296		9.471	
					2		4
235	.655	9.816	355.5	.298		9.475	
					1		1
240	.656	9.817	350.0	+.299		+9.476	
					2		3
245	.657	9.817	344.5	.297		9.473	
					4		6
250	.659	9.819	339.1	.293		9.467	
					6		9
255	.662	9.821	333.7	.287		9.458	
					9		13
260	.666	9.824	328.3	+.278		+9.445	
					11		18
265	.671	9.827	323.0	.267		9.427	
					12		20
270	.676	9.830	317.8	.255		9.407	
					15		26
275	.682	9.834	312.6	.240		9.381	
					16		31
280	.688	9.838	307.5	+.224		+9.350	
					18		37
285	.694	9.842	302.5	.206		9.313	
					20		44
290	.700	9.845	297.6	.186		9.269	
					21		53
295	.706	9.849	292.8	.165		9.216	
					23		64
300	.712	9.852	288.0	+.142		+9.152	
					24		79
305	.716	9.855	283.3	.118		9.073	
					24		102
310	.720	9.858	278.6	.094		8.971	
					26		138
315	.723	9.860	274.0	.068		8.833	
					26		208
320	.726	9.861	269.4	+.042		+8.625	
					26		424
325	.727	9.861	264.8	+.016		+8.201	
					27		
330	.727	9.862	260.3	—.011		—8.025	
					26		544
335	.727	9.861	255.7	—.037		—8.569	
					26		231
340	.725	9.860	251.1	—.063		—8.800	
					26		148
345	.723	9.859	246.4	.089		8.948	
					25		109
350	.719	9.857	241.8	.114		9.057	
					24		83
355	.715	9.854	237.0	.138		9.140	
					23		67
360	.710	9.851	232.3	—.161		--9.207	

ECCENTRICITY.

g	k′	Δ	log k′	Δ	K′	$\frac{dz}{de}$	Δ	$\log\frac{dz}{de}$	Δ
°					°				
180	.677		9.831		137.4	—.254		—9.405	
		3		1			16		26
185	.680		9.832		142.5	.270		9.431	
		17		11			19		29
190	.697		9.843		147.2	.289		9.460	
		31		19			21		32
195	.728		9.862		151.2	.310		9.492	
		41		24			24		32
200	.769		9.886		154.4	—.334		—9.524	
		48		27			26		32
205	.817		9.913		156.5	.360		9.556	
		55		27			26		30
210	.872		9.940		157.7	.386		9.586	
		57		28			25		28
215	.929		9.968		158.2	.411		9.614	
		57		26			25		26
220	.986		9.994		158.0	—.436		—9.640	
		56		24			24		22
225	1.042		0.018		157.2	.460		9.662	
		54		22			20		20
230	1.096		0.040		155.9	.480		9.682	
		51		20			18		16
235	1.147		0.060		154.3	.498		9.698	
		46		17			15		12
240	1.193		0.077		152.4	—.513		—9.710	
		43		15			10		9
245	1.236		0.092		150.3	.523		9.719	
		36		12			7		5
250	1.272		0.104		147.9	.530		9.724	
		32		11			2		2
255	1.304		0.115		145.5	.532		9.726	
		22		7			3		3
260	1.326		0.122		142.9	—.529		—9.723	
		18		6			7		6
265	1.344		0.128		140.6	.522		9.717	
		9		3			12		9
270	1.353		0.131		137.8	.510		9.708	
		3		1			16		14
275	1.356		0.132		135.2	.494		9.694	
		6		2			18		16
280	1.350		0.130		132.6	—.476		—9.678	
		13		4			22		21
285	1.337		0.126		130.2	.454		9.657	
		21		7			24		24
290	1.316		0.119		127.8	.430		9.633	
		28		9			25		26
295	1.288		0.110		125.6	.405		9.607	
		37		13			27		29
300	1.251		0.097		123.5	—.378		9.578	
		42		15			26		31
305	1.209		0.082		121.7	.352		9.547	
		49		17			25		33
310	1.160		0.065		120.2	.327		9.514	
		53		21			24		32
315	1.107		0.044		119.0	.303		9.482	
		58		23			21		32
320	1.049		0.021		118.2	—.282		—9.450	
		60		26			18		29
325	.989		9.995		117.9	.264		9.421	
		62		28			15		24
330	.927		9.967		118.2	.249		9.397	
		60		29			10		19
335	.867		9.938		119.2	.239		9.378	
		58		30			7		12
340	.809		9.908		121.1	—.232		—9.366	
		50		28			1		2
345	.759		9.880		123.9	.231		9.364	
		40		24			3		6
350	.719		9.856		127.7	.234		9.370	
		28		17			8		14
355	.691		9.839		132.3	.242		9.384	
		14		8			12		21
360	.677		9.831		137.4	—.254		—9.405	

VENUS—Continued.

LONG. IN ORBIT.								(NODE ON EQUATOR.)			
g	k''	$\log k''$	K''	$\frac{dz}{du}$		$\log \frac{dz}{du}$		g	k'''	$\log k'''$	K'''
°			°					°			°
0	.700	9.845	232.3	−.159	21	−9.201	55	0	.672	9.828	227.4
5	.695	9.842	227.4	.180	21	9.256	47	5	.678	9.831	222.2
10	.690	9.839	222.4	.201	19	9.303	39	10	.684	9.835	217.0
15	.684	9.835	217.4	.220	16	9.342	32	15	.689	9.838	212.0
20	.678	9.831	212.3	−.236	16	−9.374	27	20	.695	9.842	207.0
25	.673	9.828	207.1	.252	13	9.401	22	25	.701	9.845	202.1
30	.668	9.825	201.8	.265	11	9.423	18	30	.706	9.849	197.3
35	.664	9.822	196.4	.276	9	9.441	14	35	.710	9.851	192.6
40	.660	9.820	191.0	−.285	7	−9.455	10	40	.714	9.853	187.9
45	.658	9.818	185.5	.292	4	9.465	7	45	.717	9.855	183.2
50	.656	9.817	180.0	.296	3	9.472	3	50	.719	9.857	178.6
55	.655	9.816	174.4	.299	0	9.475	0	55	.720	9.857	174.0
60	.656	9.817	168.9	−.299	3	−9.475	3	60	.720	9.858	169.4
65	.657	9.818	163.4	.296	4	9.472	7	65	.720	9.857	164.9
70	.660	9.819	157.9	.292	8	9.465	11	70	.718	9.856	160.3
75	.663	9.821	152.5	.285	9	9.454	13	75	.716	9.855	155.6
80	.667	9.824	147.1	−.276	11	−9.441	18	80	.713	9.853	151.0
85	.672	9.827	141.8	.265	13	9.423	22	85	.709	9.851	146.3
90	.678	9.831	136.6	.252	15	9.401	27	90	.704	9.848	141.5
95	.683	9.835	131.4	.237	17	9.374	32	95	.699	9.845	136.7
100	.690	9.839	126.4	−.220	19	−9.342	38	100	.694	9.842	131.8
105	.696	9.842	121.4	.201	20	9.304	46	105	.689	9.838	126.8
110	.702	9.846	116.5	.181	21	9.258	54	110	.684	9.835	121.8
115	.708	9.850	111.7	.160	23	9.204	66	115	.679	9.832	116.7
120	.712	9.853	107.0	−.137	23	−9.138	82	120	.674	9.829	111.4
125	.717	9.856	102.4	.114	25	9.056	106	125	.670	9.826	106.2
130	.721	9.858	97.8	.089	25	8.950	144	130	.667	9.824	100.9
135	.724	9.860	93.3	.064	26	8.806	223	135	.664	9.822	95.5
140	.726	9.861	88.7	−.038	26	−8.583	493	140	.662	9.821	90.1
145	.727	9.862	84.2	−.012	26	−8.090		145	.661	9.820	84.7
150	.727	9.862	79.8	+.014	26	+8.137	461	150	.662	9.821	79.2
155	.726	9.861	75.2	+.040	25	+8.598	216	155	.663	9.821	73.8
160	.725	9.860	70.7	+.065	25	+8.814	142	160	.665	9.823	68.4
165	.722	9.859	66.2	.090	25	8.956	104	165	.668	9.825	63.0
170	.719	9.857	61.6	.115	23	9.060	81	170	.672	9.827	57.8
175	.715	9.854	57.0	.138	23	9.141	66	175	.676	9.830	52.5
180	.710	9.851	52.3	+.161		+9.207		180	.682	9.833	47.4

VENUS—Continued.

LONG. IN ORBIT.

g	k''	log k''	K''	$\frac{dz}{du}$	Diff.	log $\frac{dz}{du}$	Diff.
°			°				
180	.710	9.851	52.3	+.161	21	+9.207	54
185	.705	9.848	47.5	.182	20	9.261	45
190	.699	9.845	42.7	.202	19	9.306	38
195	.693	9.841	37.8	.221	17	9.344	32
200	.687	9.837	32.8	+.238	15	+9.376	27
205	.682	9.834	27.7	.253	13	9.403	22
210	.677	9.830	22.6	.266	11	9.425	18
215	.672	9.827	17.4	.277	9	9.443	14
220	.668	9.824	12.1	+.286	7	+9.457	10
225	.665	9.822	6.7	.293	5	9.467	8
230	.662	9.821	1.3	.298	3	9.475	3
235	.661	9.820	355.9	.301	0	9.478	1
240	.660	9.819	350.4	+.301	2	+9.479	3
245	.661	9.820	344.9	.299	4	9.476	6
250	.662	9.821	339.5	.295	7	9.470	10
255	.664	9.822	334.1	.288	8	9.460	13
260	.668	9.824	328.7	+.280	11	+9.447	17
265	.672	9.827	323.4	.269	13	9.430	21
270	.676	9.830	318.2	.256	65	9.409	26
275	.681	9.833	313.0	.241	16	9.383	31
280	.686	9.836	307.9	+.225	19	+9.352	38
285	.691	9.840	302.9	.206	20	9.314	43
290	.697	9.843	298.0	.186	21	9.271	53
295	.702	9.846	293.1	.165	23	9.218	64
300	.706	9.849	288.3	+.142	23	+9.154	79
305	.710	9.852	283.6	.119	25	9.075	101
310	.714	9.854	278.9	.094	25	8.974	136
315	.716	9.855	274.2	.069	26	8.838	204
320	.718	9.856	269.6	+.043	26	+8.634	409
325	.719	9.857	265.0	+.017	27	+8.225	
330	.719	9.857	260.4	—.010	26	—7.978	573
335	.718	9.856	255.8	—.036	26	—8.551	239
340	.716	9.855	251.2	—.062	25	—8.790	150
345	.713	9.853	246.5	.087	25	8.940	109
350	.710	9.851	241.8	.112	24	9.049	84
355	.705	9.848	237.1	.136	23	9.133	68
360	.700	9.845	232.3	—.159		—9.201	

(NODE ON EQUATOR.)

g	k'''	log k'''	K'''
°			°
180	.682	9.833	47.4
185	.687	9.837	42.3
190	.693	9.841	37.3
195	.698	9.844	32.4
200	.704	9.847	27.5
205	.709	9.850	22.8
210	.713	9.853	18.0
215	.717	9.856	13.4
220	.721	9.858	8.8
225	.723	9.859	4.2
230	.725	9.860	359.7
235	.726	9.861	355.2
240	.726	9.861	350.7
245	.725	9.860	346.1
250	.722	9.859	341.6
255	.719	9.857	337.0
260	.716	9.855	332.4
265	.711	9.852	327.7
270	.706	9.849	323.0
275	.700	9.845	318.2
280	.694	9.842	313.3
285	.688	9.838	308.4
290	.682	9.834	303.3
295	.676	9.830	298.1
300	.671	9.827	292.9
305	.666	9.823	287.6
310	.662	9.821	282.2
315	.658	9.818	276.7
320	.656	9.817	271.2
325	.654	9.816	265.6
330	.654	9.815	260.1
335	.655	9.816	254.5
340	.656	9.8[illegible]	[illegible]
345	.659	9[illegible]	[illegible]
350	.663	9.[illegible]	[illegible]
355	.667	9.[illegible]	[illegible]
360	.672	9.[illegible]	[illegible]

VENUS—Continued.

	INCLINATION TO EQUATOR.				NODE ON EQUATOR			
g	k^{iv}	log k^{iv}	$\frac{dz}{dJ}$	log $\frac{dz}{dJ}$	k^{v}	log k^{v}	$\frac{1}{\sin J}\frac{dz}{dN}$	log $\frac{1}{\sin J}\frac{dz}{dN}$
°								
0	—.252	—9.402	+.553	+9.743	—.159	—9.201	+.348	+9.542
	15	27	33	27	22	56	48	55
5	.237	9.375	.520	9.716	.181	9.257	.396	9.597
	17	32	37	32	20	46	44	47
10	.220	9.343	.483	9.684	.201	9.303	.440	9.644
	18	38	41	38	19	39	41	38
15	.202	9.305	.442	9.646	.220	9.342	.481	9.682
	20	46	44	46	17	32	38	33
20	—.182	—9.259	+.398	+9.600	—.237	—9.374	+.519	+9.715
	22	54	47	55	15	27	33	27
25	.160	9.205	.351	9.545	.252	9.401	.552	9.742
	23	67	50	66	13	22	29	22
30	.137	9.138	.301	9.479	.265	9.423	.581	9.764
	24	83	52	83	11	18	24	18
35	.113	9.055	.249	9.396	.276	9.441	.605	9.782
	24	107	55	107	9	14	20	14
40	—.089	—8.948	+.194	+9.289	—.285	—9.455	+.625	+9.796
	26	147	55	147	7	10	15	10
45	.063	8.801	.139	9.142	.292	9.465	.640	9.806
	26	228	57	228	5	7	10	7
50	.037	8.573	.082	8.914	.297	9.472	.650	9.813
	26	523	57	523	2	3	5	3
55	—.011	—8.050	+.025	+8.391	.299	9.475	.655	9.816
	26		58		0	0	0	0
60	+.015	+8.178	—.033	—8.518	—.299	—9.475	+.655	+9.816
	26	437	57	438	3	3	5	3
65	.041	8.615	.090	8.956	.296	9.472	.650	9.813
	26	212	57	212	4	7	10	7
70	.067	8.827	.147	9.168	.292	9.465	.640	9.806
	26	139	56	139	7	10	15	10
75	.093	8.966	.203	9.307	.285	9.455	.625	9.796
	24	102	54	102	9	14	20	14
80	+.117	+9.068	—.257	—9.409	—.276	—9.441	+.605	+9.782
	24	81	52	80	11	18	24	18
85	.141	9.149	.309	9.489	.265	9.423	.581	9.764
	23	64	49	65	13	22	29	22
90	.164	9.213	.358	9.554	.252	9.401	.552	9.742
	21	54	47	54	15	27	33	27
95	.185	9.267	.405	9.608	.237	9.374	.519	9.715
	20	45	44	44	17	32	37	32
100	+.205	+9.312	—.449	—9.652	—.220	—9.342	+.482	+9.683
	18	37	41	38	19	38	41	38
105	.223	9.349	.490	9.690	.201	9.304	.441	9.645
	17	31	36	31	20	45	43	46
110	.240	9.380	.526	9.721	.181	9.259	.398	9.599
	15	27	33	27	21	55	48	54
115	.255	9.407	.559	9.748	.160	9.204	.350	9.545
	13	21	29	21	23	66	49	67
120	+.268	+9.428	—.588	—9.769	—.137	—9.138	+.301	+9.478
	11	18	24	18	23	82	52	82
125	.279	9.446	.612	9.787	.114	9.056	.249	9.396
	9	13	19	13	25	106	54	105
130	.288	9.459	.631	9.800	.089	8.950	.195	9.291
	7	10	15	10	25	144	55	144
135	.295	9.469	.646	9.810	.064	8.806	.140	9.147
	4	7	10	7	26	222	56	223
140	+.299	+9.476	—.656	—9.817	—.038	—8.584	+.084	+8.924
	3	4	5	3	26	491	57	491
145	.302	9.480	.661	9.820	—.012	—8.093	+.027	+8.433
	0	0	0	0	26		57	
150	.302	9.480	.661	9.820	+.014	+8.137	—.030	—8.478
	3	4	5	3	26	461	57	460
155	.299	9.476	.656	9.817	+.040	+8.598	—.087	—8.938
	4	6	9	6	25	216	56	217
160	+.295	+9.470	—.647	—9.811	+.065	+8.814	—.143	—9.155
	7	10	15	10	25	142	55	142
165	.288	9.460	.632	9.801	.090	8.956	.198	9.297
	8	14	19	14	25	104	54	104
170	·280	9.446	.613	9.787	.115	9.060	.252	9.401
	11	17	24	17	23	81	52	81
175	.269	9.429	.589	9.770	.138	9.141	.304	9.482
	13	21	28	21	23	66	49	66
180	+.256	+9.408	—.561	—9.749	+.161	+9.207	—.353	—9.548

VENUS—Continued.

	INCLINATION TO EQUATOR.								NODE ON EQUATOR.							
g	k^{iv}		$\log k^{iv}$		$\frac{dz}{dJ}$		$\log \frac{dz}{dJ}$		k^{v}		$\log k^{v}$		$\frac{1}{\sin J} \frac{dz}{dN}$		$\log \frac{1}{\sin J} \frac{dz}{dN}$	
°																
180	+.256	15	+9.408	26	−.561	33	−9.749	26	+.161	22	+9.207	54	−.353	47	−9.548	54
185	.241	17	9.382	31	.528	36	9.723	31	.183	20	9.261	45	.400	44	9.602	45
190	.224	18	9.351	37	.492	40	9.692	37	.203	18	9.306	38	.444	40	9.647	38
195	.206	20	9.314	44	.452	44	9.655	44	.221	17	9.344	32	.484	37	9.685	32
200	+.186	21	+9.270	52	−.408	46	−9.611	53	+.238	15	+9.376	27	−.521	33	−9.717	27
205	.165	22	9.218	64	.362	49	9.558	63	.253	13	9.403	22	.554	29	9.744	22
210	.143	24	9.154	78	.313	52	9.495	78	.266	12	9.425	18	.583	25	9.766	18
215	.119	24	9.076	99	.261	53	9.417	100	.278	8	9.443	14	.608	20	9.784	14
220	+.095	25	+8.977	134	−.208	55	−9.317	133	+.286	7	+9.457	11	−.628	15	−9.798	10
225	.070	26	8.843	200	.153	57	9.184	200	.293	5	9.468	7	.643	11	9.808	7
230	.044	26	8.643	387	.096	56	8.984	387	.298	3	9.475	3	.654	6	9.815	4
235	+.018	26	+8.256		−.040	58	−8.597		.301	0	9.478	1	.660	0	9.819	1
240	−.008	26	−7.906	627	+.018	57	+8.247	626	+.301	2	+9.479	3	−.660	4	−9.820	4
245	.034	26	8.533	244	.075	56	8.873	245	.299	4	9.476	6	.656	10	9.816	6
250	.060	25	8.777	153	.131	56	9.118	153	.295	7	9.470	10	.646	14	9.810	9
255	.085	25	8.930	111	.187	54	9.271	111	.288	8	9.460	13	.632	19	9.801	13
260	−.110	24	−9.041	85	+.241	52	+9.382	85	+.280	11	+9.447	17	−.613	23	−9.788	18
265	.134	22	9.126	69	.293	50	9.467	68	.269	13	9.430	21	.590	28	9.770	21
270	.156	22	9.195	56	.343	47	9.535	57	.256	15	9.409	26	.562	33	9.749	25
275	.178	20	9.251	47	.390	45	9.592	46	.241	16	9.383	31	.529	36	9.724	31
280	−.198	19	−9.298	39	+.435	41	+9.638	40	+.225	18	+9.352	37	−.493	40	−9.693	37
285	.217	17	9.337	33	.476	38	9.678	33	.207	20	9.315	44	.453	44	9.656	44
290	.234	16	9.370	27	.514	33	9.711	27	.187	22	9.271	53	.409	47	9.612	53
295	.250	13	9.397	23	.547	29	9.738	23	.165	22	9.218	64	.362	49	9.559	64
300	−.263	11	−9.420	18	+.576	25	+9.761	18	+.143	24	−9.154	79	−.313	52	−9.495	79
305	.274	10	9.438	15	.601	21	9.779	15	.119	25	9.075	100	.261	54	9.416	100
310	.284	7	9.453	10	.622	15	9.794	10	.094	25	8.975	136	.207	56	9.316	136
315	.291	5	9.463	8	.637	11	9.804	7	.069	26	8.839	204	.151	57	9.180	205
320	−.296	2	−9.471	3	+.648	5	+9.811	4	+.043	26	+8.635	407	−.094	57	−8.975	406
325	.298	0	9.474	1	.653	1	9.815	1	+.017	27	+8.228		−.037	58	−8.569	
330	.298	2	9.475	3	.654	5	9.816	3	−.010	26	−7.976	576	+.021	57	+8.317	576
335	.296	4	9.472	7	.649	9	9.813	7	−.036	26	−8.552	238	+.078	57	+8.893	238
340	−.292	7	−9.465	10	+.640	15	+9.806	10	−.062	25	−8.790	150	+.135	56	+9.131	150
345	.285	9	9.455	13	.625	19	9.796	14	.087	25	8.940	109	.191	54	9.281	109
350	.276	11	9.442	18	.606	24	9.782	17	.112	24	9.049	85	.245	53	9.390	84
355	.265	13	9.424	22	.582	29	9.765	22	.136	23	9.134	67	.298	50	9.474	63
360	−.252		−9.402		+.553		+9.743		−.159		−9.201		+.348		+9.542	

VENUS—Continued.

RADIUS VECTOR.								RADIUS VECTOR—Continued.							
g	k^{vi}	log k^{vi}	K^{vi}	$\frac{dz}{dr}$	Diff.	log $\frac{dz}{dr}$	Diff.	g	k^{vi}	log k^{vi}	K^{vi}	$\frac{dz}{dr}$	Diff.	log $\frac{dz}{dr}$	Diff.
°			°					°			°				
0	.936	9.971	317.4	+.351		+9.546		180	.936	9.971	137.4	—.351		—9.546	
5	.944	9.975	311.2	.330	21	9.519	27	185	.944	9.975	132.3	.331	20	9.520	26
10	.952	9.978	307.0	.307	23	9.487	32	190	.951	9.978	127.3	.308	23	9.489	29
15	.960	9.982	302.0	.281	26	9.448	39	195	.959	9.982	122.4	.283	25	9.452	37
20	.967	9.986	297.0	+.253	28	+9.403	45	200	.966	9.985	117.5	—.256	27	—9.408	44
25	.975	9.989	292.1	.223	30	9.348	55	205	.974	9.988	112.8	.227	29	9.356	52
30	.982	9.992	287.3	.191	32	9.281	67	210	.981	9.992	108.0	.196	31	9.292	64
35	.987	9.994	282.6	.158	33	9.198	83	215	.986	9.994	103.4	.164	32	9.214	78
40	.992	9.997	277.9	+.123	35	+9.091	107	220	.992	9.996	98.8	—.130	34	—9.115	99
45	.996	9.998	273.2	.088	35	8.944	147	225	.995	9.998	94.2	.096	34	8.981	134
50	.999	9.999	268.6	.052	36	8.715	229	230	.998	9.999	89.7	.060	36	8.782	199
55	1.000	0.000	264.0	+.016	36	+8.192	523	235	.999	0.000	85.2	—.025	35	—8.395	387
60	.999	0.000	259.4	—.021	37	—8.320		240	1.000	0.000	80.7	+.011	36	+8.046	
65	.998	9.999	254.9	.057	36	8.757	437	245	.998	9.999	76.1	.047	36	8.672	626
70	.996	9.998	250.3	.093	36	8.968	211	250	.996	9.998	71.6	.082	35	8.916	244
75	.992	9.996	245.6	.128	35	9.107	139	255	.993	9.997	67.0	.118	36	9.070	154
80	.987	9.994	241.0	—.162	34	—9.210	103	260	.988	9.995	62.4	.152	34	9.181	111
85	.981	9.992	236.3	.195	33	9.289	79	265	.983	9.992	57.7	.185	33	9.267	86
90	.974	9.989	231.5	.226	31	9.354	65	270	.976	9.990	53.0	.216	31	9.335	68
95	.967	9.985	226.7	.255	29	9.407	53	275	.969	9.986	48.2	.246	30	9.392	57
100	.959	9.982	221.8	—.283	28	—9.452	45	280	.961	9.983	43.3	+.275	29	+9.439	47
105	.951	9.978	216.8	.308	25	9.489	37	285	.953	9.979	38.3	.301	26	9.478	39
110	.943	9.975	211.8	.331	23	9.520	31	290	.946	9.976	33.3	.325	24	9.511	33
115	.936	9.971	206.6	.352	21	9.546	26	295	.938	9.972	28.1	.346	21	9.539	28
120	.929	9.968	201.4	—.369	17	—9.567	21	300	.931	9.969	22.9	+.365	19	+9.562	23
125	.923	9.965	196.2	.384	15	9.585	18	305	.924	9.966	17.6	.381	16	9.581	19
130	.918	9.963	190.9	.397	13	9.598	13	310	.919	9.963	12.1	.394	13	9.595	14
135	.914	9.961	185.5	.406	9	9.608	10	315	.915	9.961	6.7	.404	10	9.606	11
140	.911	9.960	180.1	—.412	6	—9.615	7	320	.912	9.960	1.2	+.411	7	+9.614	8
145	.910	9.959	174.6	.415	3	9.618	3	325	.910	9.959	355.6	.414	3	9.617	3
150	.910	9.959	169.2	.415	0	9.618	0	330	.910	9.959	350.1	.415	1	9.618	1
155	.911	9.960	163.8	.411	4	9.614	4	335	.911	9.960	344.5	.412	3	9.615	3
160	.914	9.961	158.4	—.405	6	—9.608	6	340	.914	9.961	339.0	+.406	6	+9.609	6
165	.918	9.963	153.1	.396	9	9.598	10	345	.918	9.963	333.5	.397	9	9.599	10
170	.923	9.965	147.8	.384	12	9.584	14	350	.923	9.965	328.1	.385	12	9.585	14
175	.929	9.968	142.5	.369	15	9.567	17	355	.929	9.968	322.7	.369	16	9.567	18
180	.936	9.971	137.4	—.351	18	—9.546	21	360	.936	9.971	317.4	+.351	18	+9.546	21

INVESTIGATION OF CORRECTIONS

TO THE

GREENWICH PLANETARY OBSERVATIONS

FROM 1762 TO 1830,

PREPARED FOR THE OFFICE OF THE AMERICAN EPHEMERIS

BY

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INTRODUCTORY NOTE.

Previous to the commencement of accurate observations by BESSEL and STRUVE, the observations made at Greenwich by BRADLEY, MASKELYNE, and POND supply almost the only valuable data at the command of astronomers for fixing the positions of the planets between 1750 and 1820.

These observations were, however, made with instruments which, although the best of their time, were very imperfect when measured by our present standard. Investigations of the corrections which their results require have therefore been undertaken from time to time by those interested in the subject. The last of these investigations is embodied in the great work, *Reduction of the Greenwich Observations of the Planets*, which was conducted by Professor AIRY and published in the year 1845.

These investigations were executed with the imperfect data of former times and depended fundamentally upon the star places in the *Tabulæ Regiomontanæ* of BESSEL, which were published more than half a century ago. It is therefore desirable that the modifications required to make their results correspond with the more accurate data of the present time should be investigated. So far as BRADLEY's observations are concerned the work of Dr. AUWERS, now in press, may be expected to supply all that is required by the astronomy of the present and future. In order to utilize the long series of observations made by BRADLEY's successors it is now necessary that the work of MASKELYNE and POND should be similarly reduced. The work of preparing the fundamental formula and tables for this purpose was placed in the hands of Professor TRUMAN HENRY SAFFORD. The object of his work was to furnish data for the reduction of the planetary observations to the system of right ascensions and declinations adopted in the star catalogues of the American Ephemeris. This involved the reduction of the declinations to Boss's system.

SIMON NEWCOMB, *Professor, U. S. N.,*
Superintendent American Ephemeris.

WASHINGTON, *October* 11, 1883.

§ 1.

HISTORY.

With regard to the observations which precede those now discussed it is sufficient to say that FLAMSTEED, Astronomer Royal from 1675 to 1719, and his successor, HALLEY, whose term of office ended in 1742, made observations which were not in general precise enough for any present utility with respect either to the planets or stars. BRADLEY, who succeeded HALLEY, observed with rather indifferent instruments until 1750, when the transit instrument and the brass quadrant were mounted; and these instruments were employed for the greater part of the period I am considering—the quadrant until 1813, and the transit instrument until 1816.

BRADLEY'S observations, as reduced in part by BESSEL in his *Fundamenta Astronomiæ*, have long been used as the basis of all inquiries into the proper motion of the stars and the annual motions of the planets; but this reduction is now superseded by the more accurate and complete study of the same observations which has been made by Professor AUWERS, at the suggestion of OTTO STRUVE, and with the material assistance of the St. Petersburg Academy of Sciences.

The necessary corrections to BRADLEY'S planetary observations themselves cannot well be made till the work of AUWERS is more completely published. Meanwhile it has become possible to employ in the reduction of the Greenwich observations made after the death of BRADLEY, in 1762, star places carried back by the help of the observations of the present century, and thus to diminish the necessary dependence upon the supposed perfection of BRADLEY'S instruments.

BRADLEY'S immediate successor was NATHANIEL BLISS, whose term of office was short (two years only, ending at his decease in 1764), and the larger part of the period now considered was included in MASKELYNE'S directorship.

NEVIL MASKELYNE, born in 1732, was appointed Astronomer Royal in 1765, and held that office until 1811. His meridian observations were confined mainly to a few objects; the sun, the moon, the principal planets, and a few stars; Polaris and the thirty-six stars selected by himself as fundamental, together with some stars in Gemini and a few which pass near the zenith of Greenwich, are nearly all which he often observed.

In the year 1765 his observations began; up to July, 1772, the instruments seem to have been substantially in the same condition as in BRADLEY'S and BLISS'S time. At the latter date achromatic object-glasses were applied, and in case of the transit instrument one of larger aperture than the previous one.

It is to be regretted that for many years of MASKELYNE'S administration there are so few data for rigidly reducing his observations. He appears to have taken pretty good care to keep his transit instrument nearly in adjustment; but it is not always possible to be certain of its exact position, nor to discriminate between errors of col-

limation and of deviation at the pole. And the case is rather worse with the quadrant. In 1802 he published a catalogue of the declinations of his fundamental stars, which was soon found to be seriously in error.

The retention of the quadrant as the principal zenith-distance instrument seems to have been due to its original excellence; but MASKELYNE failed to detect the errors in question until his attention was called to them by observations with another instrument. The quadrant, eight feet in radius, was of course less in danger of errors of division than instruments of smaller radius, other things being equal.

But, during the latter part of the eighteenth century, it began to be seen that more accurate division was possible, and that complete circles were better than quadrants. RAMSDEN constructed a great altazimuth for Palermo, which was employed with eminent success by PIAZZI from 1792; TROUGHTON made the first meridian circles, one of which, ordered for Russia, was retained in England, and did admirable work in Mr. STEPHEN GROOMBRIDGE'S hands, beginning with 1806; and another altazimuth was employed by JOHN POND as early as 1800 in the determination of the declinations of fundamental stars. MASKELYNE'S catalogue of 1802 was compared by POND with observations made with three circles, one of the three circles being that of a large equatorial at Armagh, in a paper presented to the Royal Society in 1806, and it was at once seen that the quadrant had worn at the center so that its arc of 90° was no longer a true quadrant of the circle described by the vernier attached to its telescope. The discrepancy was 8″ or 10″.

MASKELYNE admitted the truth of this correction, and, it seems, soon ordered a mural circle of TROUGHTON; but at his death in 1811 the new instrument was yet unfinished.

POND, originally an amateur astronomer, was appointed MASKELYNE'S successor, and at once began observations to determine more accurately what results the quadrant would give. The series made with the quadrant for this purpose in 1811 and 1812 is more complete than any which MASKELYNE had observed; and in 1812 the mural circle was mounted, so that he could compare the work of the two instruments. Since that time the quadrant has not been used, except at intervals when the circle was undergoing repair, especially in 1824, when preparations were making to set up a second mural by JONES.

The quadrant observations made by MASKELYNE, or, more strictly speaking, those from 1765 to 1810, inclusive, were discussed by OLUFSEN, according to a plan of BESSEL'S. OLUFSEN'S memoir, in Volume 9 of the *Astronomische Nachrichten*, columns 85 to 106, relates, however, only to the corrections of the quadrant observations in general; it is in some respects unsatisfactory, while there is no doubt that it materially advanced the knowledge of the subject. In the first place he overlooked the observations of 1811 and later years, which give better evidence as to the later condition of the instrument than do those few observations after 1802 which he employed; and, while he prepared equations of condition of the form

$$z + \delta = p + q \sin \delta + r \sin^2 \tfrac{1}{2} \delta,$$

he solved them, making $r = 0$, a process about which BESSEL speaks with hesitation in

that part of the introduction to the *Tabulæ Regiomontanæ* where this place is alluded to. To myself it seems clear that if a sine formula is to be used at all it must necessarily be made complete by the addition of the cosine term, which amounts in effect to the same thing as the term containing r as a factor. OLUFSEN seems to have thought that if he retained r his solution would have approached indeterminateness; this difficulty I have avoided by the extremely simple device of multiplying the coefficient of r and dividing r itself by the same factor, ten. The least square equations become then definite enough for solution; but it is plain that the observations are inadequate to this form of reduction.

In looking over OLUFSEN's paper and comparing his periods with the printed journal of observations, I was at once impressed with the apparent suddenness of the change in the quadrant; it almost seemed as if the wear at the center took place in 1787; that before that year the quadrant might be considered as constant in form for the whole period since 1765; and that the form of the pivot after the change might also be considered constant for the remainder of its employment. This was afterwards modified for stars within 30° of the equator. Certain it is that this assumption will represent the observations of fundamental stars after 1787 at least as well as the interpolations between means extending over long periods which OLUFSEN adopted.

Values of OLUFSEN's *q for his periods; summarized.*

(Computed from his table, *Ast. Nach.*, Vol. IX, assuming $r = 0$).

Limits of periods.		Value of q.	Weight.	Limits of periods.		Value of q.	Weight.
		″				″	
1765, May 10	1765, July 6	+ 1.1	10.9	1787, Mar. 6	1787, Aug. 28	+12.0	12.9
July 7	Aug. 30	− 1.6	4.7	Sept. 9	1788, Nov. 10	+12.1	2.8
Sept. 1	1766, June 10	+ 3.7	4.1	1788, Nov. 15	1793, Nov. 1	+11.5	0.9
1767, Oct. 1	1769, Oct. 12	+ 4.2	4.5	1794, Mar. 7	1795, Oct. 24	+11.6	6.0
1770, Jan. 23	1772, Sept. 25	+ 6.7	1.5	1796, July 6	1799, Dec. 31	+11.7	6.6
	Mean . .	+ 1.9	25.7	1800, Apr. 26	1800, Dec. 31	+10.2	17.6
				1801, Jan. 4	1802, Feb. 26	+ 9.8	22.5
1773, Sept. 6	1776, Sept. 23	+ 7.7	0.9	1802, Mar. 12	Dec. 27	+10.4	23.8
1776, Oct. 15	Dec. 30	− 5.8	1.1	1803, Jan. 8	1804, Dec. 13	+ 8.7	4.4
1777, June 11	1777, Mar. 1	+ 2.6	1.6	1804, Dec. 14	1805, Mar. 17	+ 0.7	0.7
Aug. 9	Oct. 4	+ 0.2	2.4	1805, June 24	Sept. 29	+ 5.2	3.7
Nov. 7	1781, Nov. 3	+ 5.2	0.7	1805, Oct. 12	1806, Feb. 27	+11.4	1.7
1782, July 20	1784, Sept. 9	+ 4.2	5.6	1806, May 25	Dec. 12	+ 5.8	2.0
1785, Mar. 11	1786, July 31	+ 0.4	1.8				
	Mean . .	+ 2.3	14.1				

Mr. POND investigated not only the quadrant but the transit instrument as well, and found that it was practically worn out, fit only for differential work on neighboring parallels of declination, and ordered a new one of TROUGHTON, after vainly experimenting with the mural circle in hopes of using it as a transit. To us at the present day, who are used to well-divided portable circles of ten or eleven inches in diameter, with which latitudes can be accurately determined, and who have seen the most perfect instruments for observation constructed with circles of twenty-one to twenty-six inches, the mural seems an utterly absurd instrument. The only reason for such a construction was, I fancy, the supposed necessity of a six-foot circle for the

best work, combined with the difficulty of fixing it to its telescope. GROOMBRIDGE's instrument, already at work in 1806, was a true transit circle, with a telescope of five feet in length, an aperture of three or four inches only, and a circle of four feet. The construction may be seen in the plates of PEARSON's Practical Astronomy, or as imitated by TROUGHTON's successors, in the eighth volume of the Annals of Harvard College Observatory, the east meridian circle of which was designed after the same model. But this instrument was, probably, thought too small for Greenwich, although it needed but a short and easy test, involving a journey from Greenwich to Blackheath only, to show how accurate observations could be made with it. And I doubt if a similar instrument of larger dimensions, like the preposterous altazimuth or vertical circle, built for Dublin, with a circle of eight feet in diameter, would have been available for good transit work.

The new transit instrument set up in 1816, and the mural circle of 1812, served as the principal meridian instruments of Greenwich till 1850, inclusive; a part of the time JONES's mural, mounted in 1825, was employed alongside the other.

Mr. POND's observations were, as to the details, admirably made: he held on rather too long and firmly to MASKELYNE's plan of observing few stars, but in 1828 began a more extensive catalogue. His plan of reduction was less perfect; he failed to calculate his instrumental corrections, deeming it sufficient to adjust the transit from time to time; his plan of using two mural circles side by side caused some waste of time, and he held to BRADLEY's table of refractions long after it had been abandoned as inaccurate by every other good authority.

The re-reduction of his observations on a uniform plan is desirable, but, it is to be feared, too extensive a work to be soon effected.

POND retired in 1835 and died in 1836, and was succeeded by Professor (now Sir GEORGE) AIRY, under whom Greenwich has regained the old pre-eminence which it had lost during the life of BESSEL.

The reduction of the Greenwich Planetary Observations between 1750 and 1830, to which these investigations are supplementary, is that of Sir GEORGE AIRY. His zero-points are those of the *Tabulæ Regiomontanæ;* OLUFSEN's investigation of the quadrant is employed, and in general the instrumental corrections are considered equal to zero, and their effect eliminated, so far as possible, by a proper selection of stars.

LE VERRIER has also reduced the planetary observations, using rather more modern elements of reduction, such as aberration and nutation, and for a few periods, but only a few, noticing the instrumental corrections.

FLEMMING's reduction of the observations of Uranus, and AUWERS's papers on Procyon and Sirius, contain additional values of the instrumental corrections, the latter of which have been utilized; a portion of these corrections are due to BESSEL, and are used in his paper on variable proper motions and quoted by AUWERS from his manuscripts.

BESSEL had suggested in his memoir on the Solar Tables (*Astr. Nachr.*, Vol. 6) that it was necessary to base our knowledge of the earth's mean motion not upon observations of right ascension reduced from two adopted positions of the equinox, as, for instance, he had promisionally done for 1755 and 1824, but from all the observa-

tions of the sun's declination made between those epochs. It became, in consequence, necessary to reduce MASKELYNE's observations of this element.

The plan for doing this was set forth in the introduction to the *Tabulæ Regiomontanæ*, pp. XLVII *seq.;* and it would seem from the later history of the subject that the method here suggested by BESSEL was applied to the solar observations by OLUFSEN. (See *Tables du Soleil, par* HANSEN *et* OLUFSEN, *p.* 4, *ad fin.*)

§ 2.

GENERAL OUTLINE OF THE WORK.

During BLISS's time the observations are quite inferior to those made later. It has been possible to find values of $n + c$, the instrumental correction of the transit at the pole, with some certainty for a part of the time, by opposite culminations of Polaris, and, from the means of those values, to find the probable deviation of the meridian mark during BLISS's time. But for the declinations, considering the manifest errors, probably due to the plumb-line, nothing was attempted.

In reducing MASKELYNE's right ascensions I followed AUWERS's lead, in his papers on the variable proper motions of Sirius and Procyon. In many cases, which are especially designated by the initial A, I have taken his values of n from these papers; and in others designated by B, BESSEL's values quoted by AUWERS from the former's manuscripts. I have assumed that, to reduce these values as deduced from Polaris to such as would be obtained from Capella and Rigel, the corrections found by AUWERS are to be used:

$$+ 0^{s}.0103\ (t - 1787)$$

from 1787 till July, 1803; and later, from December, 1803, till 1810,

$$+ 0^{s}.025.$$

In 1811 and 1812 I used the same value.

These formulæ are given by Professor AUWERS with opposite signs, as he reduces the values of n from the values given by Capella and Rigel to that given by Polaris.

I have also assumed that the n's so obtained would be applicable throughout the regions of the planets and fundamental stars. Other values I have myself computed.

POND's right ascensions are similarly treated. Many values of $n + c$, that is of n, assuming $c = 0$, are available in the paper on Sirius; and I computed many others, employing Polaris or δ Ursæ Minoris, and sometimes Capella and Rigel.

It is of course to be stated here what instruments were employed. The old transit, which BRADLEY used, was relied upon until 1816. Its object-glass, originally unachromatic and of but two inches aperture and eight feet focal length, was exchanged for an achromatic one of two and three-quarter inches in aperture in 1772. It began to show irregular variation about 1787; its pivots were reground in 1803.

The new transit by TROUGHTON, which came into use in 1816, was employed until 1850, when Sir GEORGE AIRY replaced both it and the mural by TROUGHTON by the present transit circle.

In reducing MASKELYNE's declinations I began by following OLUFSEN's footsteps. The paper of this careful computer is in Volume 9 of the *Astronomische Nachrichten.* He first, from BESSEL's declinations of the fundamental stars and of certain stars in Gemini and of γ Draconis, finds the apparent equator-point of the quadrant

$$z + \rho + \delta$$

where z is the apparent south zenith-distance of the star, as obtained from the outer divisions of the quadrant; ρ is the refraction, δ the stars' declination. Here it is to be noted that the difference of my process at this stage from OLUFSEN's consisted, first, in the fact that I employed the Pulcova refractions in the place of BESSEL's; secondly, that I employed Boss's or NEWCOMB's mean declinations for the stars in question, as found in Professor NEWCOMB's catalogue in the first volume of these papers; and third, that in the reduction, from mean to apparent, I employed PETERS's nutation and STRUVE's aberration as given in the *Tabulæ Reductionum* of WOLFERS. The very minute difference between the precessions (for the fractions of the year) between STRUVE and BESSEL disappeared in my calculations; but a trifling inconsistency as to proper motion for this fraction was introduced by the roundabout but labor-saving process employed for the fundamental stars proper.

The mean declinations having been calculated for four epochs, 1765, 1785, 1805, 1825, from Boss's catalogue, they were compared with the *Tabulæ Regiomontanæ;* this gave the "correction of mean declination, Boss–BESSEL." To this, interpolated for separate years, was added the difference of nutation, PETERS–LINDENAU, so far as dependent upon the longitude of the moon's ascending node; and the sum of these two terms gave the correction to the first part of the fundamental stars' declination of the *Tabulæ Regiomontanæ;* and this term was taken from these tables, frequently to every tenth sidereal day for a considerable interval of time, but otherwise for individual dates.

The annual term for 1870 was taken from WOLFERS's *Tabulæ Reductionum*, and its variation for 100 years by direct subtraction of the values for 1760, as given in the *Tabulæ Regiomontanæ*, from those for 1860, as given by ZECH in the latter part of WOLFERS's book, which is a mere continuation of BESSEL's tables, using identically the same elements. The apparent declination of the stars in Gemini and Draco were computed differently. For them, independent tables like those just mentioned of BESSEL and others were computed; for γ Draconis WOLFERS's values for 1870 were utilized to supply the variation for 100 years. The mean declinations were taken from NEWCOMB or Boss, except for ξ Geminorum. This star is not given among BESSEL's 7, nor in NEWCOMB; I added it because, while further south than the others in the same constellation, it was often observed (apparently by mistake) at Greenwich. The distinction between the Greek letters ξ and ζ seems to have been overlooked at times by MASKELYNE's assistants. I also added β Draconis, as it often accompanied γ and is a very well known star.

After reducing the quadrant observations to this point, I found that OLUFSEN had overlooked the very important and continuous series of fundamental star declinations which POND observed with the quadrant; along with others with the then new

mural circle. POND had just been appointed Astronomer Royal at MASKELYNE'S decease. His first paper, so far as I know, was that in the Philosophical Transactions for 1806, which contains his corrections of the quadrant declinations, in which he showed that the instrument was then giving erroneous results. So that his first work on coming into office at Greenwich was to reundertake the same problem with the larger means now at his command.

After reducing these observations, and treating them in the manner OLUFSEN had employed, as suggested by BESSEL, I found that the indeterminateness, of which OLUFSEN complains, did not apply to their observations.

BESSEL suggests, in the *Tabulæ Regiomontanæ*, the formula of correction

$$z + \rho + \delta = p + q \sin \delta + r \sin^2 \frac{1}{2} \delta,$$

but intimates that possibly r may be so small as to merge in the other terms, leaving the decision to the judgment of the computer.

For a series as complete as POND'S of 1811–'12, this is not so; r is easily determined. But in going backwards to MASKELYNE'S own work, I found the same difficulty as OLUFSEN did; or rather, I found r quite wildly discordant, even when I combined several groups. Looking still farther at POND'S observations, I found that either formula

$$z + \rho + \delta = p + q \sin \delta + r \sin^2 \frac{1}{2} \delta,$$

or

$$z + \rho + \delta = p + q \sin \delta,$$

as finally used by OLUFSEN (with the r added as indeterminate), would give no great advantage over the formula

$$z + \rho + \delta = p' + q' \delta,$$

expressing δ in degrees, at least between the declinations 32° of Castor and −31° of Fomalhaut, to represent the variations in this quantity. I then drew curves and found that within the latter region, which includes all the MASKELYNE'S fundamentals but Capella, α Lyræ, and α Cygni, the curve was very nearly a straight line; that in the region north of Castor a sharp bend took place, which, however, seemed to differ in various years as to its magnitude but not as to its general shape and sharpness; and that, on the whole, it was simplest and safest to employ the straight line for the equatorial region up to ± 32°, and consider the northern stars separately.

One thing that perplexed OLUFSEN was at once cleared up. He noticed that the effects of wear at the center seemed to have reached their maximum in 1787 or thereabouts, and then paradoxically enough to have diminished. This I found to be by no means unequivocally shown by the observations; as OLUFSEN had used indifferently to determine his q the combinations of fundamental stars *inter se*, or those of γ Draconis and the stars in Gemini; and as the errors in the former region were always *essentially* different from those in the latter, it became clear that his numbers could not be always safely used. I thought it, on the whole, best to take observations

of fundamental stars from 32° to −31° as the materials for my q', including, where necessary, as it was in one period, the stars in Gemini, but to omit the more northern stars; except that β and γ Draconis were employed to strengthen the resulting values of p'. It must be here noted that the two sets of observations were kept very separate by MASKELYNE. He seems to have considered the quadrant perfect till he was compelled to think otherwise, perhaps by POND's paper; to have observed γ Draconis and the stars in Gemini for collimation, and in quite different periods to have observed his fundamentals for his catalogues. Perhaps in 1787 the great wear in the quadrant which he noted—occasioned by want of oil or other lubricant at the center—caused him some anxiety and made him observe for a time more industriously.

In studying the observations, it is pretty clear that the quadrant was a very troublesome instrument to use, and that it was frequently employed by quite unskilled persons, perhaps for practice.

The plumb-line seems to have given much trouble, and observations deviating 6″ or more have been usually excluded without ceremony, as the errors are more frequently due to this cause than to any other. The plumb-line adjustment of the instrument may have been frequently neglected as too troublesome, as a routine assistant of long standing would be likely to do; or frequently attempted with a disastrous result, the fault of a zealous but inexperienced hand.

It must not be forgotten that a large part of MASKELYNE's own energies was necessarily expended upon the Nautical Almanac, and that the tradition between BRADLEY and himself was broken by the incapacity of BRADLEY's successor; so that the observatory had lost the habit of good work.

After studying the results of my calculations a long time, with several repetitions of the trials and hypotheses, I finally concluded to employ for q' the value

0″.07 for 1767,
0″.14 for 1787,

interpolating between these values from 1767 to 1787, extrapolating back to 1765, and suffering q' after 1787 to remain constant as long as the quadrant was used. I then computed p' by the formulæ

$$p' = z + \rho + \delta - q' \delta$$

for OLUFSEN's periods, varying them in some cases, but keeping the fundamental stars at first separate from those in Gemini.

This now gave the means of finding the $z + \rho + \delta - p'$, which is no longer equal to $q' \delta$, for β and γ Draconis and the other northern stars.

For the latter, α Aurigæ, α Cygni, and α Lyræ, I got tolerably consistent values of this quantity for the three great periods,

1787–1800,
1811–1812,
1824,

when they were observed.

The anomalies which I have noticed in my results thus obtained are due, in part, to the peculiarities of the quadrant; in part to the peculiarities of its use; but also, in

some degree, to the standard declinations employed, which are, in substance, an extrapolation from the present century back into the eighteenth.

Finally, for OLUFSEN'S periods (as modified in some cases) the different values of p' were combined by weights; the unit of weight corresponded to 1–4 observations in the same period; 5–10 received double, and 11 or more triple weight. The stars in Gemini in any period were combined with equal weight to each, as the number of observations was nearly the same; and the total weight of the 8 stars, if so many were observed, taken as 8 or 16, according to the average number of observations, using in doubtful cases the agreement *inter se* of the 8 as a criterion. The probable error to weight unity comes out less for these stars by about one-third than for the others, plainly because the plumb-line was better looked after in these observations, which were quite continuously made for several days at a time, possibly more directly under MASKELYNE'S own control.

In correcting POND'S declinations, observed with the TROUGHTON circle alone, a very simple process is sufficient. The slight flexure found by OLUFSEN in his re-reduction of POND'S observations made with this instrument in 1822, and the fact that the declinations so determined agree very perfectly with BOSS, render it necessary only to correct AIRY'S index-errors for this circle by the reduction from BESSEL'S declinations to Boss's The formulæ necessary for this purpose include only the terms of mean declination and nutation as elsewhere given in this paper; and for aberration as follows:

$$0.009\, i + 0.0094\, h \cos\,(\mathrm{H} + \alpha) \sin\,\delta,$$

when the factor $\cos\,\delta$ has been omitted from it, or rather replaced by 0.96 as the convenient representative of the average $\cos\,\delta$ for the fundamental stars. The numbers h, H, and i are given in the Star Tables, and hold good for any year. It will be noticed that, as AIRY employs it, the sign of the index-error requires the sum of all these terms to be *subtracted* from it.

For the years 1825-1830, where the two circles TROUGHTON and JONES were employed together, an indirect process was employed to obtain the systematic corrections.

§ 3.

EXPLANATION OF THE TABLES, AND GENERAL REMARKS.

Table I contains, for three epochs, the differences of mean right ascension between NEWCOMB and LE VERRIER: N — L.

Table II, the corrections of these mean right ascensions in the cases of Sirius and Procyon, to be added to the results of Table I for these stars. (As this table is found in Vol. I, pp. 297 and 298, of these papers, it has been omitted in printing.)

Table III contains the correction to the *Tabulæ Regiomontanæ*, first part, to reduce to the Tables of LE VERRIER, for every second year.

Table IV contains for every 30 days the annual term in the same difference.

To reduce an apparent right ascension of a star computed by LE VERRIER to NEWCOMB'S standard, we add the value interpolated from Table I, or the sum of the

values of Tables I and II for Sirius and Procyon. To correct an apparent right ascension of a fundamental star from BESSEL'S *Tabulæ Regiomontanæ* to NEWCOMB'S standard, we add the sum of results from Tables I, III, IV, adding that of Table II as before for Sirius or Procyon.

Table V contains the instrumental corrections (n) for the years 1762 to 1830, inclusive. Those marked B. were computed by BESSEL, from Polaris; those marked A., by AUWERS, from the transits of the same star; those marked F. are from fundamental stars, especially Capella and Rigel,* but including many other combinations, such that the difference of declinations shall be at least 30°. The weight depends upon the number of observations multiplied by the square of the difference of tangents of declinations; but is not given for Polaris, where this formula would be inapplicable.

The periods here given do not include the whole time between 1762 and 1830, but chiefly those times when planets were observed.

The study I have made into these observations has confirmed my view derived from BESSEL, that MASKELYNE was a good observer in detail, but very negligent in the handling of his instrument. He seems to have been methodical, without understanding the importance, which BRADLEY well knew, of keeping a close watch on the adjustments.

Table VI gives the general correction to the stars' declination necessary to reduce to Boss—to which, for Sirius and Procyon, the periodic corrections must be added—together with the principal term of the nutation correction. The formula is:

$$\text{Corrections to } \textit{Tabulæ Regiomontanæ} = \Delta' \delta + \Delta n \sin (\mathrm{N} + \Omega),$$

where Ω is the longitude of the moon's ascending node.

The annual terms to be added to these will be

$$0.0094 \, (i \cos \delta + h \cos (\mathrm{H} + \alpha) \sin \delta),$$

which are readily computed, directly or indirectly, by the method of p. 60.

Tables VII, VIII, and IX are star-tables similar to those in the Star-tables of the American Ephemeris, for the declinations of β and γ Draconis and the 7 stars in Gemini, whose places are given in less detail in the *Tabulæ Regiomontanæ*. The general term is computed for every 200 days during the long period in which they were used, and the annual term is given for 1770 with its centennial variation. The tables were computed by Mr. J. O. WIESSNER, who has ably assisted in various other parts of the work.

Table X gives the empirical correction for the quadrant for the fundamental stars south of α Geminorum, inclusive. After 1787 it is considered constant; before that date the formula is declination in degrees $\times \{0''.07 + 0''.0035 \, (t - 1767)\}$. It is here given with changed sign.

Table XI contains the results, degrees and minutes (51° 28′) omitted, for the equator point of the quadrant from fundamental stars for a portion of OLUFSEN'S periods, in some cases modified in the notes, where I have also placed scattered observations made during long and uncertain periods. MASKELYNE'S fundamental stars were observed with the quadrant in a very desultory way, except for those

* Capella and Rigel were mostly computed by AUWERS; the other fundamentals mainly by myself.—T. H. S.

periods, when he was determining their declinations with more system. This table contains the systematic observations and the notes, those which are more scattered. For α Aurigæ, α Lyræ, and α Cygni, the results in this table are uncorrected; the others are corrected by Table X. The column $z + \rho + \delta$ contains the means of observed zenith distance + refraction + computed declination (Boss–Newcomb); n is the number of observations, and Σe the sums of errors.

Table XII contains the same results for the stars in Gemini. To Bessel's 7 I have added ξ Geminorum. These results are corrected.

Table XIII contains similar results, *uncorrected*, for β and γ Draconis. As these stars were largely employed in finding the equator-point, it was necessary to find the constant difference between them and the mean of fundamental stars, which was found to be 4″.8. I did not consider this to be substantially different for the two stars, which differ 1° in declination. This number, which, as will be seen, attains suddenly almost its full value in 1787, is smaller than either my own formulæ or Olufsen's, would give for the declination 52°, my own formulæ giving 7″.28 after 1787; Olufsen's from 6″.8 to 8″.9 for the same dates. The intermediate stars, α Aurigæ, α Cygni, α Lyræ, whose declinations are between 46° and 38°, give, as will be seen by Table XI, on the whole larger proportionate differences from the other fundamentals and the stars in Gemini.

Table XIV contains the results of Tables XI, XII, XIII, consolidated. I have usually given, in any one period, to 1–4 observations, a weight = 1; to 5–10, a weight = 2; to 11 or more, a weight = 3. This rule was modified a little, for simplicity's sake, in the case of the stars in Gemini, as explained at the foot of the table.

Table XV contains, along with Olufsen's results, my own for his periods. Periods 16 and 19 have been modified a little; perhaps some of the others might also have been with advantage.

In applying this table it must be borne in mind that Gyldén's refractions (the Pulcova tables) were used in preparing the p' and q'. To modify it to Bessel's refractions it will probably be sufficient to increase the p''s by the change of refractions due to the zenith distance, 51°.5. But I conceive the better plan is to employ Gyldén's refractions throughout.

Pond's declinations, as reduced by Airy, require the correction of the zero-points only, using Table VI + annual term $0.0094 \{g' \cos\delta + h \cos(H + a) \sin\delta\}$, so far as the observations to the middle of 1825 are concerned, except that, where the quadrant was used, an additional correction must be applied to Airy's zero-point. This correction will be obtained by adding to Airy's computed declinations of the fundamental stars employed in forming the zero-points the correction appropriate to period 35, and then in the computation of the planet's N. P. D., substituting the value

$$51^\circ\ 28'\ 36''.85 + 0''.140\ \delta^\circ$$

for

$$51^\circ\ 28'\ 36''.02 + 8''.65 \sin\delta.$$

The observations with *two* circles, after the middle of 1825, should be corrected substantially according to Professor Boss's reduction for Gh. 1839, as the observations were made with the same instrument and by very nearly the same method.

Precepts for using the Tables in correcting the Reductions of the Greenwich Observations.

The Reduction of the Greenwich Planetary Observations from 1762 (BLISS's observations) to 1830 requires the following corrections to reduce them to the basis here given.

1. *In Right Ascension.*

The values of clock slow (Sir G. AIRY's) are to be corrected by the sum of the results from Tables I, III, and IV, adding Table I[1] (Vol. I, pp. 297–298 of these Papers) for Sirius and Procyon.

They are also to be corrected by the quantity $-n \tan \delta$, where δ is the declination of each star and n is taken from Table V.

For three periods these values of instrumental corrections $n \tan \delta$, are to be replaced by their differences from MASKELYNE's numbers, which were (exceptionally) employed, namely:

$$= -\overset{s.}{0.014} \tan \delta - \overset{s.}{0.16} \sec \delta \quad \text{for 1768, October 8–November 1.}$$
$$= -0.323 \tan \delta \quad \text{for 1789, June 15–16.}$$
$$= -0.493 \tan \delta \quad \text{for 1793, August 3–15.}$$

For 1793, August 2, the n given in the table is practically the same as MASKELYNE's.

For stars not fundamental, that is, not given in Tables I–IV, the apparent right ascensions will be calculated from NEWCOMB's Catalogue in Vol. I, using the Pulcova tables of log A, B, C, D, from 1750 to 1840. Stars not in NEWCOMB were sometimes employed by AIRY, but may be omitted.

The planet's apparent right ascensions will receive the mean correction of the clock-stars used on that date, and are to be further corrected by $n \tan \delta'$, and for the first of the special periods mentioned above by $-0^s.16 \sec \delta'$, where δ' is the planet's declination.

2. *In North-Polar Distance.*

During BLISS's time, 1762, August 18–1765, March 15, no attempt has been made to find a correction.

MASKELYNE's observations, 1765–1810, have been reduced by AIRY from OLUFSEN's reduction of the quadrant observations. From 1765 to 1812, therefore, the only change required in AIRY's reductions is to substitute my formula

$$p' + q' \delta^\circ \text{ where } \delta^\circ \text{ is equal to } \delta \text{ in degrees}$$

for OLUFSEN's

$$p + q \sin \delta.$$

That is, for each period, we add to AIRY's declinations the quantity,

$$p' + q' \delta^\circ - p - q \sin \delta$$

where δ°, δ are the planet's declination; and p', q', p, q are taken from Table XV.

For period 36 OLUFSEN's value of p, q for period 35 are to be substituted in this formula.

The planetary declinations observed with the quadrant later than period 36, namely, in 1824, see pages 140–143 of the Planetary Reductions, need a special treatment, which will be indicated in an Appendix.

The Mural Circle observations up to 1825, March 19, and between 1828, October 1, and 1829, March 27, require to be corrected for the index errors; that is, on any date on which a planet was observed between 1812, June 28, and 1825, March 19, or between 1828, October 1, and 1829, March 27, find the stars employed for index-error on pages 99 to 144 of the Planetary Reductions, and add to the planet's observed declination the mean of the corrections for these stars computed from Table VI by the formula

$$\triangle \delta = \triangle' \delta + \triangle n \sin (\Omega + N)$$

where N is the longitude of the Moon's Ascending Node.

The Mural Circle declinations later than 1825, June 21, with the exceptions in 1828 and 1829, above stated, need small corrections to reduce to the adopted standard, viz, $0''.1 - 0''.025 \delta^\circ$ for declinations sonth of 4° only.

In all cases the refractions to be used are those of the Pulcova tables.

TABLE I.—*Differences of Mean Right Ascension.*

NEWCOMB—LE VERRIER.

Star's name.	1760.	1800.	1840.
	s.	s.	s.
γ Pegasi	+0.008	+0.020	+0.039
α Arietis	—0.028	—0.010	+0.011
α Ceti	—0.024	+0.012	+0.049
α Tauri	—0.016	—0.011	—0.012
α Aurigæ	+0.008	+0.007	—0.024
β Orionis	—0.025	—0.004	+0.016
β Tauri	—0.009	+0.011	+0.019
α Orionis	—0.010	+0.004	+0.015
α Canis Majoris*	+0.055	+0.008	—0.067
α Geminorum	+0.090	+0.166	+0.238
α Canis Minoris*	+0.063	+0.061	+0.038
β Geminorum	—0.014	0.000	+0.012
α Hydræ	—0.041	—0.007	+0.028
α Leonis	—0.008	+0.003	+0.020
β Leonis	0.000	+0.009	+0.025
β Virginis	—0.037	—0.016	+0.008
α Virginis	—0.019	+0.003	+0.027
α Bootis	—0.013	—0.017	+0.016
α^1 Libræ	+0.042	+0.026	+0.014
α^2 Libræ	—0.024	—0.003	+0.020
α Coronæ	—0.035	—0.009	+0.019
α Serpentis	—0.038	+0.003	+0.044
α Scorpii	—0.013	+0.009	+0.028
α Herculis	+0.020	+0.034	+0.046
α Ophiuchi	—0.055	—0.019	+0.022
α Lyræ	+0.011	+0.024	+0.028
γ Aquilæ	—0.003	+0.021	+0.046
α Aquilæ	+0.005	+0.023	+0.036
β Aquilæ	+0.001	+0.021	+0.050
α^1 Capricorni	—0.021	+0.022	+0.064
α^2 Capricorni	—0.031	+0.008	+0.048
α Cygni	—0.014	+0.010	+0.033
α Aquarii	—0.024	+0.009	+0.045
α Piscis Austrini	+0.079	+0.071	+0.065
α Pegasi	—0.006	+0.003	+0.016
α Andromedæ	—0.080	—0.037	+0.013

*The periodic corrections P, found in *Astronomical Papers*, Vol. I, pp. 297–298, are to be taken account of in using the places of Sirius and Procyon.

TABLE III.—*Corrections to the Tabulæ Regiomontanæ in Right Ascension: Part I.* LE VERRIER—BESSEL; *Mean Right Ascension + Lunar Nutation.*

Year.	γ Pegasi.	α Arietis.	α Ceti.	α Tauri.	α Aurigæ.	β Orionis.	β Tauri.	α Orionis.	α Canis Maj.
	s.	s.	s.	s.	s.	s.	s.	s.	s.
1760	+0.062	—0.065	—0.017	—0.048	—0.043	+0.023	+0.107	+0.008	+0.055
1762	+0.063	—0.059	—0.010	—0.039	—0.056	+0.031	+0.111	+0.015	+0.065
1764	+0.075	—0.042	+0.008	—0.020	—0.032	+0.046	+0.126	+0.031	+0.082
1766	+0.089	—0.018	+0.026	+0.001	—0.006	+0.063	+0.144	+0.050	+0.101
1768	+0.099	+0.002	+0.040	+0.018	+0.014	+0.072	+0.156	+0.062	+0.116
1770	+0.099	+0.007	+0.041	+0.023	+0.019	+0.074	+0.155	+0.062	+0.120
1772	+0.087	0.000	+0.031	+0.013	+0.007	+0.062	+0.138	+0.049	+0.114
1774	+0.066	—0.016	+0.013	—0.005	—0.028	+0.044	+0.112	+0.029	+0.102
1776	+0.045	—0.034	—0.004	—0.023	—0.043	+0.028	+0.084	+0.010	+0.090
1778	+0.031	—0.043	—0.013	—0.032	—0.058	+0.020	+0.072	+0.001	+0.085
1780	+0.028	—0.041	—0.009	—0.028	—0.055	+0.024	+0.070	+0.003	+0.091
1782	+0.036	—0.027	+0.004	—0.011	—0.034	+0.037	+0.073	+0.017	+0.106
1784	+0.050	—0.006	+0.023	+0.011	—0.008	+0.054	+0.101	+0.036	+0.125
1786	+0.061	+0.014	+0.037	+0.029	+0.014	+0.066	+0.114	+0.051	+0.141
1788	+0.066	+0.025	+0.043	+0.038	+0.026	+0.070	+0.119	+0.056	+0.150
1790	+0.057	+0.022	+0.036	+0.032	+0.018	+0.061	+0.107	+0.046	+0.148
1792	+0.038	+0.009	+0.020	+0.007	—0.003	+0.045	+0.083	+0.028	+0.138
1794	+0.015	—0.010	+0.001	—0.001	—0.031	+0.027	+0.056	+0.008	+0.124
1796	—0.003	—0.022	—0.010	—0.014	—0.047	+0.016	+0.038	—0.005	+0.117
1798	—0.010	—0.024	—0.011	—0.014	—0.049	+0.016	+0.033	—0.005	+0.120
1800	—0.004	—0.013	0.000	0.000	—0.033	+0.027	+0.041	+0.006	+0.133
1802	+0.008	+0.007	+0.018	+0.021	—0.008	+0.043	+0.060	+0.024	+0.152
1804	+0.021	+0.028	+0.035	+0.042	+0.019	+0.059	+0.077	+0.041	+0.170
1806	+0.027	+0.043	+0.044	+0.054	+0.034	+0.065	+0.084	+0.050	+0.182
1808	+0.022	+0.045	+0.040	+0.054	+0.034	+0.061	+0.077	+0.045	+0.183
1810	+0.005	+0.034	+0.026	+0.040	+0.016	+0.045	+0.056	+0.028	+0.174
1812	—0.018	+0.015	+0.007	+0.021	—0.009	+0.027	+0.028	+0.006	+0.161
1814	—0.038	—0.001	—0.008	+0.006	—0.030	+0.013	+0.005	—0.010	+0.152
1816	—0.048	—0.007	—0.011	+0.002	—0.037	+0.011	—0.005	—0.014	+0.152
1818	—0.046	0.000	—0.004	+0.012	—0.026	+0.018	+0.002	—0.006	+0.163
1820	—0.034	+0.018	+0.014	+0.032	0.000	+0.035	+0.018	+0.011	+0.182
1822	—0.021	+0.041	+0.031	+0.054	+0.027	+0.050	+0.036	+0.029	+0.201
1824	—0.012	+0.058	+0.044	+0.070	+0.047	+0.060	+0.048	+0.040	+0.216
1826	—0.013	+0.062	+0.043	+0.072	+0.051	+0.059	+0.045	+0.040	+0.220
1828	—0.026	+0.054	+0.032	+0.061	+0.038	+0.046	+0.029	+0.026	+0.215
1830	—0.049	+0.037	+0.014	+0.043	+0.016	+0.028	+0.002	+0.005	+0.202

TABLE III.—*Corrections to the Tabulæ Regiomontanæ, etc.*—Continued.

Year.	ε Gemin.	Canis Min.	β Gemin.	α Hydræ.	α Leonis.	β Leonis.	β Virginis.	α Virginis.	α Bootis.
	s.	s.	s.	s.	s.	s.	s.	s.	s.
1760	−0.078	−0.086	−0.129	−0.109	−0.132	−0.116	−0.016	−0.048	−0.049
1762	−0.061	−0.078	−0.113	−0.099	−0.118	−0.101	−0.006	−0.042	−0.037
1764	−0.035	−0.059	−0.089	−0.080	−0.095	−0.078	+0.012	−0.026	−0.018
1766	−0.009	−0.041	−0.064	−0.059	−0.073	−0.057	+0.031	−0.008	−0.001
1768	+0.009	−0.028	−0.046	−0.044	−0.057	−0.044	+0.043	+0.004	+0.011
1770	+0.017	−0.027	−0.044	−0.036	−0.054	−0.043	+0.045	+0.009	+0.010
1772	+0.002	−0.038	−0.054	−0.042	−0.064	−0.053	+0.034	−0.001	−0.001
1774	−0.017	−0.057	−0.074	−0.056	−0.079	−0.068	+0.017	−0.018	−0.016
1776	−0.034	−0.074	−0.090	−0.069	−0.092	−0.080	−0.001	−0.037	−0.029
1778	−0.038	−0.081	−0.094	−0.075	−0.096	−0.082	−0.007	−0.046	−0.032
1780	−0.027	−0.077	−0.083	−0.068	−0.087	−0.072	−0.002	−0.044	−0.024
1782	−0.004	−0.060	−0.061	−0.052	−0.066	−0.052	+0.014	−0.032	−0.006
1784	+0.022	−0.041	−0.036	−0.032	−0.044	−0.030	+0.034	−0.013	+0.011
1786	+0.045	−0.025	−0.015	−0.013	−0.026	−0.013	+0.049	+0.002	+0.025
1788	+0.052	−0.021	−0.008	−0.004	−0.020	−0.009	+0.054	+0.009	+0.026
1790	+0.046	−0.029	−0.014	−0.007	−0.025	−0.015	+0.046	+0.003	+0.019
1792	+0.028	−0.045	−0.032	−0.017	−0.040	−0.029	+0.031	−0.012	+0.004
1794	+0.010	−0.064	−0.050	−0.032	−0.056	−0.044	+0.013	−0.032	−0.011
1796	+0.002	−0.074	−0.057	−0.040	−0.062	−0.049	+0.002	−0.044	−0.018
1798	+0.008	−0.074	−0.042	−0.038	−0.058	−0.043	+0.003	−0.046	−0.013
1800	+0.028	−0.061	−0.032	−0.025	−0.040	−0.027	+0.017	−0.037	+0.001
1802	+0.055	−0.041	−0.006	−0.004	−0.017	−0.003	+0.036	−0.019	+0.019
1804	+0.080	−0.023	+0.016	+0.015	+0.003	+0.015	+0.054	−0.002	+0.034
1806	+0.092	−0.015	+0.028	+0.028	+0.014	+0.025	+0.061	+0.008	+0.040
1808	+0.089	−0.018	+0.025	+0.030	+0.011	+0.020	+0.058	+0.005	+0.034
1810	+0.075	−0.033	+0.010	+0.020	−0.002	+0.006	+0.043	−0.008	+0.019
1812	+0.055	−0.052	−0.009	+0.005	−0.019	−0.010	+0.025	−0.027	+0.004
1814	+0.042	−0.066	−0.022	−0.006	−0.029	−0.019	+0.011	−0.044	−0.006
1816	+0.044	−0.068	−0.021	−0.007	−0.027	−0.016	+0.009	−0.050	−0.007
1818	+0.061	−0.057	−0.004	+0.003	−0.013	−0.001	+0.018	−0.044	−0.005
1820	+0.088	−0.038	+0.021	+0.023	+0.010	+0.021	+0.037	−0.027	+0.023
1822	+0.114	−0.019	+0.046	+0.044	+0.031	+0.040	+0.055	−0.009	+0.037
1824	+0.131	−0.008	+0.062	+0.058	+0.045	+0.052	+0.067	+0.004	+0.047
1826	+0.133	−0.006	+0.064	+0.064	+0.047	+0.053	+0.066	+0.004	+0.043
1828	+0.120	−0.018	+0.051	+0.058	+0.036	+0.041	+0.054	−0.006	+0.030
1830	+0.102	−0.036	+0.033	+0.042	+0.018	+0.025	+0.037	−0.024	+0.014

Table III.—*Corrections to the Tabulæ Regiomontanæ, etc.*—Continued.

Year.	α^1 Libræ.	α^2 Libræ.	α Coronæ.	α Serpentis.	α Scorpii.	α Herculis.	α Ophiuchi.	α Lyræ.	γ Aquilæ.
	s.	s.	s.	s.	s.	s.	s.	s.	s.
1760	+0.090	+0.170	−0.048	+0.115	+0.056	−0.077	−0.139	−0.064	+0.124
1762	+0.093	+0.170	−0.037	+0.118	+0.060	−0.068	−0.126	−0.060	+0.125
1764	+0.106	+0.181	−0.021	+0.129	+0.075	−0.053	−0.107	−0.049	+0.136
1766	+0.122	+0.195	−0.005	+0.142	+0.092	−0.036	−0.085	−0.035	+0.148
1768	+0.134	+0.205	+0.004	+0.147	+0.105	−0.025	−0.068	−0.025	+0.155
1770	+0.132	+0.201	+0.004	+0.143	+0.104	−0.024	−0.063	−0.022	+0.151
1772	+0.120	+0.187	−0.006	+0.127	+0.090	−0.033	−0.066	−0.028	+0.137
1774	+0.097	+0.162	−0.021	+0.104	+0.067	−0.048	−0.077	−0.036	+0.115
1776	+0.073	+0.136	−0.032	+0.085	+0.042	−0.063	−0.086	−0.047	+0.094
1778	+0.060	+0.121	−0.035	+0.073	+0.027	−0.068	−0.088	−0.051	+0.080
1780	+0.058	+0.117	−0.026	+0.072	+0.027	−0.064	−0.081	−0.050	+0.078
1782	+0.067	+0.124	−0.011	+0.081	+0.038	−0.050	−0.062	−0.041	+0.086
1784	+0.084	+0.139	+0.005	+0.093	+0.057	−0.033	−0.041	−0.028	+0.098
1786	+0.097	+0.150	+0.018	+0.102	+0.071	−0.020	−0.021	−0.015	+0.106
1788	+0.100	+0.151	+0.019	+0.101	+0.075	−0.016	−0.013	−0.010	+0.106
1790	+0.089	+0.138	+0.012	+0.088	+0.064	−0.022	−0.014	−0.013	+0.095
1792	+0.069	+0.116	−0.002	+0.067	+0.043	−0.035	−0.023	−0.022	+0.075
1794	+0.045	+0.090	−0.015	+0.045	+0.018	−0.051	−0.035	−0.033	+0.052
1796	+0.028	+0.071	−0.021	+0.031	−0.001	−0.061	−0.039	−0.039	+0.035
1798	+0.023	+0.064	−0.015	+0.026	−0.005	−0.059	−0.035	−0.040	+0.030
1800	+0.029	+0.068	−0.002	+0.033	+0.002	−0.047	−0.019	−0.033	+0.035
1802	+0.044	+0.081	+0.014	+0.045	+0.020	−0.031	+0.001	−0.020	+0.047
1804	+0.059	+0.094	+0.028	+0.056	+0.036	−0.016	+0.022	−0.007	+0.058
1806	+0.066	+0.099	+0.034	+0.057	+0.043	−0.009	+0.034	+0.001	+0.060
1808	+0.060	+0.091	+0.029	+0.049	+0.039	−0.012	+0.036	0.000	+0.052
1810	+0.042	+0.071	+0.015	+0.028	+0.019	−0.025	+0.028	−0.007	+0.033
1812	+0.017	+0.044	0.000	+0.005	−0.006	−0.041	+0.016	−0.018	+0.010
1814	−0.002	+0.023	−0.008	−0.013	−0.027	−0.053	+0.008	−0.028	−0.008
1816	−0.012	+0.011	−0.006	−0.019	−0.038	−0.055	+0.011	−0.029	−0.018
1818	−0.009	+0.012	+0.004	−0.016	−0.033	−0.046	+0.024	−0.024	−0.017
1820	+0.004	+0.023	+0.021	−0.004	−0.018	−0.030	+0.044	−0.013	−0.005
1822	+0.020	+0.037	+0.035	+0.007	0.000	−0.014	+0.064	0.000	+0.006
1824	+0.030	+0.046	+0.043	+0.012	+0.012	−0.005	+0.080	+0.009	+0.013
1826	+0.027	+0.041	+0.042	+0.007	+0.009	−0.004	+0.084	+0.012	+0.007
1828	+0.012	+0.024	+0.030	−0.011	−0.006	−0.015	+0.079	+0.007	−0.009
1830	−0.012	−0.002	+0.016	−0.033	−0.030	−0.030	+0.068	−0.004	−0.031

Table III.—*Corrections to the Tabulæ Regiomontanæ, etc.*—Continued.

Year.	α Aquilæ.	β Aquilæ.	α¹ Capr.	α² Capr.	α Cygni.	α Aquarii.	α Pisc. Aust.	α Pegasi.	α Andro.
	s.	s.	s.	s.	s.	s.	s.	s.	s.
1760	−0.039	+0.101	−0.080	−0.020	−0.114	−0.116	−0.098	−0.021	−0.034
1762	−0.033	+0.102	−0.070	0.013	−0.112	−0.106	−0.087	−0.016	−0.029
1764	−0.017	+0.113	−0.051	+0.005	−0.102	−0.087	−0.069	−0.001	−0.013
1766	−0.001	+0.124	−0.030	+0.023	−0.086	−0.067	−0.053	+0.018	+0.008
1768	+0.011	+0.131	−0.017	+0.035	−0.071	−0.053	−0.047	+0.031	+0.026
1770	+0.012	+0.127	−0.017	+0.032	−0.061	−0.050	−0.055	+0.036	+0.035
1772	+0.003	+0.113	−0.030	+0.017	−0.060	−0.059	−0.074	+0.027	+0.031
1774	−0.014	+0.091	−0.049	−0.005	−0.066	−0.075	−0.098	+0.010	+0.017
1776	−0.030	+0.069	−0.068	−0.025	−0.078	−0.092	−0.116	−0.008	−0.003
1778	−0.039	+0.055	−0.075	−0.035	−0.087	−0.098	−0.122	−0.018	−0.013
1780	−0.036	+0.052	−0.070	−0.032	−0.089	−0.092	−0.115	−0.017	−0.014
1782	−0.023	+0.060	−0.053	−0.017	−0.080	−0.076	−0.098	−0.005	−0.001
1784	−0.006	+0.072	−0.032	+0.001	−0.067	−0.055	−0.081	+0.013	+0.018
1786	+0.007	+0.080	−0.017	+0.015	−0.050	−0.040	−0.071	+0.027	+0.038
1788	+0.012	+0.080	−0.011	+0.018	−0.038	−0.033	−0.075	+0.036	+0.049
1790	+0.006	+0.068	−0.021	+0.006	−0.036	−0.039	−0.092	+0.031	+0.049
1792	−0.009	+0.048	−0.040	−0.015	−0.040	−0.053	−0.116	+0.016	+0.037
1794	−0.027	+0.025	−0.059	−0.037	−0.050	−0.070	−0.138	−0.003	+0.016
1796	−0.039	+0.007	−0.071	−0.051	−0.061	−0.080	−0.147	−0.016	+0.004
1798	0.039	+0.002	−0.069	−0.052	−0.065	−0.079	−0.143	−0.019	−0.001
1800	−0.029	+0.006	−0.055	−0.040	−0.060	−0.065	−0.130	−0.009	+0.007
1802	−0.012	+0.017	−0.035	−0.022	−0.048	−0.045	−0.111	+0.007	+0.025
1804	+0.003	+0.028	−0.016	−0.006	−0.030	−0.026	−0.100	+0.024	+0.046
1806	+0.011	+0.030	−0.008	0.000	−0.018	−0.017	−0.098	+0.033	+0.061
1808	+0.007	+0.022	−0.012	−0.007	−0.012	−0.019	−0.112	+0.033	+0.064
1810	−0.007	+0.002	−0.030	−0.027	−0.015	−0.032	−0.134	+0.020	+0.054
1812	−0.025	−0.022	−0.051	−0.050	−0.025	−0.049	−0.158	+0.001	+0.037
1814	−0.038	−0.040	−0.065	−0.067	−0.035	−0.063	−0.172	−0.015	+0.020
1816	−0.043	−0.050	−0.067	−0.070	−0.042	−0.064	−0.171	−0.022	+0.011
1818	−0.037	−0.049	−0.056	−0.062	−0.039	−0.055	−0.160	−0.016	+0.017
1820	−0.020	−0.038	−0.037	−0.045	−0.030	−0.035	−0.142	0.000	+0.033
1822	−0.004	−0.028	−0.018	−0.028	−0.014	−0.016	−0.128	+0.017	+0.053
1824	+0.008	−0.021	−0.005	−0.017	+0.001	−0.003	−0.123	+0.029	+0.070
1826	+0.007	−0.027	−0.006	−0.021	+0.009	−0.002	−0.132	+0.032	+0.077
1828	−0.004	−0.044	−0.021	−0.038	+0.009	−0.013	−0.153	+0.022	+0.071
1830	−0.021	−0.067	−0.042	−0.061	+0.001	−0.029	−0.178	+0.004	+0.056

TABLE IV.—LE VERRIER *(Annales)*—BESSEL *(Tabulæ Regiomontanæ)*.—*Annual Term in Right Ascension for* 1800.

Star's name.	Jan. 0.	Jan. 30.	Mar. 1.	Mar. 31.	Apr. 30.	May 30.	June 29.	July 29.	Aug. 28.	Sept. 27.	Oct. 27.	Nov. 26.	Dec. 26.
	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.
γ Pegasi	−.002	−.007	−.011	−.012	−.009	−.004	+.002	+.007	+.011	+.012	+.010	+.005	−.001
α Arietis	+.004	−.002	−.002	−.012	−.012	−.009	−.004	+.002	+.008	+.012	+.013	+.010	+.005
α Ceti	+.007	+.001	−.005	−.010	−.012	−.011	−.007	−.001	+.005	+.010	+.012	+.011	+.007
α Tauri	+.011	+.006	−.001	−.007	−.011	−.013	−.011	−.007	−.001	+.006	+.011	+.013	+.011
α Aurigæ	+.016	+.010	+.002	−.007	−.015	−.018	−.017	−.011	−.003	+.006	+.013	+.018	+.017
β Orionis	+.012	+.007	+.001	−.005	−.010	−.013	−.012	−.008	−.003	+.004	+.009	+.012	+.012
β Tauri	+.013	+.009	+.002	−.005	−.011	−.014	−.013	−.009	−.003	+.004	+.010	+.014	+.013
α Orionis	+.012	+.009	+.003	−.003	−.009	−.012	−.012	−.010	−.004	+.002	+.008	+.012	+.013
α Canis Maj.	+.013	+.011	+.006	−.001	−.007	−.011	−.013	−.011	−.007	−.001	+.006	+.011	+.013
α Gemin.	+.015	+.014	+.009	+.002	−.005	−.012	−.015	−.014	−.010	−.004	+.004	+.011	+.014
α Canis Min.	+.012	+.012	+.008	+.002	−.004	−.010	−.012	−.012	−.009	−.003	+.003	+.009	+.012
β Gemin.	+.014	+.013	+.009	+.003	−.005	−.011	−.014	−.014	−.010	−.004	+.003	+.010	+.014
α Hydræ	+.009	+.012	+.011	+.008	+.002	−.004	−.009	−.012	−.012	−.008	−.002	+.004	+.009
α Leonis	+.008	+.012	+.012	+.009	+.004	−.003	−.008	−.011	−.012	−.010	−.005	+.002	+.008
β Leonis	+.004	+.010	+.013	+.012	+.009	+.003	−.003	−.009	−.013	−.013	−.010	−.005	+.002
β Virginis	+.003	+.009	+.012	+.011	+.009	+.004	−.002	−.008	−.012	−.012	−.009	−.004	+.002
α Virginis	−.002	+.004	+.009	+.012	+.011	+.008	+.002	−.004	−.009	−.012	−.012	−.009	−.003
α Bootis	−.005	+.002	+.008	+.012	+.013	+.010	+.006	−.001	−.007	−.011	−.013	−.011	−.006
α¹ Libræ	−.007	.000	+.006	+.011	+.013	+.012	+.007	+.001	−.006	−.011	−.013	−.012	−.008
α² Libræ	−.007	.000	+.006	+.011	+.013	+.012	+.007	+.001	−.006	−.011	−.013	−.012	−.008
α Coronæ	−.009	−.003	+.004	+.010	+.014	+.013	+.010	+.003	−.004	−.010	−.013	−.014	−.010
α Serpentis	−.008	−.002	+.004	+.009	+.012	+.011	+.008	+.003	−.003	−.008	−.012	−.012	−.011
α Scorpii	−.011	−.006	+.001	+.008	+.012	+.014	+.012	+.007	.000	−.007	−.012	−.014	−.012
α Herculis	−.011	−.007	−.001	+.005	+.009	+.012	+.011	+.008	+.002	−.004	−.009	−.012	−.012
α Ophiuchi	−.011	−.008	−.002	+.004	+.009	+.012	+.011	+.008	+.003	−.003	−.008	−.012	−.012
α Lyræ	−.016	−.013	−.007	+.001	+.009	+.014	+.016	+.014	+.008	+.001	−.007	−.014	−.016
γ Aquilæ	−.012	−.012	−.009	−.003	+.004	+.009	+.012	+.012	+.009	+.004	−.002	−.008	−.012
α Aquilæ	−.012	−.012	−.008	−.003	+.004	+.009	+.012	+.012	+.009	+.004	−.002	−.008	−.012
β Aquilæ	−.012	−.012	−.008	−.003	+.004	+.009	+.012	+.012	+.009	+.004	−.002	−.008	−.012
α¹ Capri.	−.012	−.012	−.008	−.003	+.004	+.009	+.012	+.012	+.009	+.004	−.002	−.008	−.012
α² Capri.	−.011	−.012	−.010	−.004	+.002	+.007	+.011	+.012	+.010	+.005	−.001	−.007	−.011
α Cygni	−.016	−.017	−.014	−.008	+.001	+.010	+.015	+.017	+.015	+.009	+.001	−.008	−.015
α Aquarii	−.011	−.012	−.010	−.004	+.002	+.007	+.011	+.012	+.010	+.005	−.001	−.007	−.011
α Piscis Austr.	−.007	−.012	−.013	−.012	−.007	.000	+.006	+.011	+.013	+.012	+.009	+.002	−.006
α Pegasi	−.005	−.010	−.012	−.011	−.007	−.001	+.005	+.010	+.012	+.011	+.007	+.001	−.005
α Androm.	−.003	−.009	−.013	−.014	−.011	−.005	+.002	+.009	+.013	+.014	+.011	+.006	−.001

TABLE V.—*Values of n from Observations with Greenwich Transits*, 1762–1830.

Period.		Value of n.	Authority.	Weight.	Remarks.
		s.			
1762, Dec. 20	1762, Dec. 23	+0.03	(2.0)	. .	
Dec. 25	1763, Jan. 2	—0.11	(3.2)	. .	
1763, Jan. 14	Jan. 16	—0.21	(3.2)	. .	
May 4	. . .	—0.14	(0.1)	. .	
May 22	May 23	—0.42	(0.2)	. .	
May 28	June 2	—0.25	(2.5)	. .	May 28–29, $n = -0^s.22$; May 30–June 2, $n = -0^s.27$. The adjustment May 29 produced little change.
June 11	June 16	—0.07	(1.5)	. .	June 19 gives $n = -0^s.6$.
June 20	June 24	—0.24	(0.3)	. .	
Nov. 15	Nov. 28	+0.22	(5.0)	. .	
Nov. 30	Dec. 11	—0.17	(5.1)	. .	
Dec. 23	1764, Jan. 3	+0.07	(1.3)	. .	
1764, Jan. 11	Feb. 18	—0.07	(5.6)	. .	At other times during 1763 to 1764 it will be sufficient to employ $n = -0^s.12$; as the instrument was probably set to a mark that amount in error, and the observations are at best inaccurate.
1765, May 7	1765, May 14	—0.22	P. (1.1)	. .	
May 17	May 19	+0.30	B. (1.2)	. .	
June 1	June 9	+0.10	B. (1.2)	. .	
June 15	July 2	—0.18	P. (3.6)	. .	July 2 gives $n = -0^s.42$, is included.
July 8	July 13	—0.14	B. (4.3)	. .	
July 13	July 22	—0.17	B. (7.6)	. .	
July 27	Aug. 10	—0.03	B. (3.2)	. .	
Aug. 15	Dec. 28	0.00	A.	. .	By fundamental stars.
1766, Jan. 4	1766, Jan. 12	+0.10	Interpolated.	. .	
Jan. 28	Feb. 7	+0.20	F. 18	. .	
Feb. 19	April 17	+0.27	A. (1.0) and F.	. .	
May 3	June 10	—0.03	F. 23	7.56	
June 25	Dec. 23	No corr.	. . .	. .	
1767, Mar. 7	. . .	No corr.	. . .	. .	
June 9	1767, June 12	+0.01	F. 5	3.57	
Aug. 2	Aug. 4	+0.10	F. 3	3.06	
Aug. 15	Sept. 29	No corr.	. . .	. .	
Dec. 16	Dec. 23	—0.15	F. 6	2.24	
1768, Jan. 4	1768, Jan. 22	—0.13	Interpolated.	. .	
April 3	April 8	—0.10	F. 8	3.28	
May 1	May 22	0.00	B. (0.5)	. .	
June 20	July 2	—0.11	P. (3.3)	. .	
Oct. 8	Nov. 3	—1.26	. . .	. .	$c = -2^s.16$. I have combined three observations of Polaris above, and one below, with four pairs of fundamental stars, and with the fact that the instrument was on the mark after the disturbance.

TABLE V.—*Values of n, etc.*—Continued.

Period.						Value of n.	Authority.	Weight.	Remarks.
						s.			
1768,	Nov.	5	1768,	Nov.	9	—0.01	P. (5.3)	. .	
	Nov.	10		Nov.	13	+0.13	P. (2.2)	. .	
	Nov.	18		Dec.	2	—0.04	P. (6.4)	. .	
1769,	Jan.	1	1769,	Feb.	8	No corr.	. . .	. .	
	Mar.	21		Mar.	23	No corr.	. . .	. .	
	April	24		June	11	+0.04	P. (2.6)	. .	
	Aug.	7		Aug.	13	+0.10	F. 2	1.56	
	Oct.	24		. . .		No corr.	. . .	. .	
1770,	Jan.	17	1770,	Jan.	23	+0.11	P. (2.1)	. .	
	Mar.	16		. . .		No corr.	. . .	. .	
	April	17		. . .		+0.01	P. (0.1)	. .	
	June	9		Sept.	11	No corr.	. . .	. .	
	Nov.	1		Nov.	6	+0.08	F. 5	3.58	
	Dec.	13		. . .		No corr.	. . .	. .	
1771,	Feb.	1		. . .		No corr.	. . .	. .	
	Mar.	22	1771,	Mar.	24	+0.11	F. 6	3.57	AUWERS finds $n = 0$ from April 4–April 9.
	April	15		May	5	+0.22	F. 10	7.02	
	May	22		. . .		+0.17	B. (0.1)	. .	
	July	10		July	15	—0.03	F. 9	8.74	
	Nov.	18		. . .		+0.25	A. (Interp.)	. .	From fundamental stars.
1772,	Feb.	12	1772,	Feb.	15	+0.29	A.	. .	From fundamental stars.
	June	2		. . .		No corr.	. . .	. .	
	Aug.	24		Nov.	15	—0.07	A.	. .	From fundamental stars.
1773,	Jan.	24		. . .		0.00	A.	. .	
	Feb.	26	1773,	April	25	—0.18	B. (3.1)	. .	
	May	28		. . .		—0.46	A.	. .	
	June	26		July	6	—0.26	F. 3	1.41	
	Sept.	27		Sept.	28	+0.06	F. 5	4.98	
	Dec.	30	1774,	Jan.	19	+0.26	F. 9	5.62	
1774,	Mar.	13		Mar.	28	—0.01	P. (1.1)	. .	
	June	16		July	6	+0.21	A. (0.2)	. .	Includes also fundamental stars.
	Aug.	9		. . .		+0.02	B. (1.1)	. .	
	Nov.	3		Dec.	29	+0.15	P. (2.2)	. .	
1775,	Jan.	28		. . .		No corr.	. . .	. .	
	Feb.	10	1775,	June	4	0	A.	. .	
	May	30		. . .		+0.12	B. (0.1)	. .	
	June	26		. . .		—0.10	B. (1.0)	. .	
	Sept.	22		. . .		+0.11	F. 2	1.54	
	Dec.	9		Dec.	11	+0.14	Interpolated.	. .	
1776,	Feb.	29	1776,	Mar.	5	+0.17	B. (1.0)	. .	
	April	5		Oct.	31	+0.22	B. (0.2)	. .	
	Nov.	16		Nov.	21	+0.06	P. (0.3)	. .	
	Dec.	24		Dec.	25	+0.10	Interpolated.	. .	

TABLE V.—*Values of n, etc.*—Continued.

Period.		Value of n.	Authority.	Weight.	Remarks.
		s.			
1777, Jan. 8	1777, Jan. 28	+0. 10	Interpolated.	. .	
Feb. 19	Mar. 21	+0. 15	B. (4. 2)	. .	
Mar. 30	April 19	+0. 17	Interpolated.	. .	
April 24	May 16	+0. 19	B. (0.2) P. (1.0)	. .	
May 25	June 10	+0. 05	A. (1. 5)	. .	
July 11	Aug. 13	+0. 05	A.	. .	
Nov. 1	Nov. 28	+0. 36	A. (4. 5)	. .	
1778, Feb. 23	1778, June 12	+0. 06	A. (0. 3)	. .	
Aug. 18	Aug. 24	0	A.	. .	
Nov. 12	Dec. 19	+0. 16	A. (1. 5)	. .	
1779, Feb. 2	1779, Feb. 27	+0. 29	F. 13	6. 87	
Mar. 12	Dec. 3	+0. 35	A.	. .	
1780, April 17	. . . .	0. 00	A. (1. 2)	. .	
May 23	1780, June 1	+0. 06	B. (1. 2)	. .	
July 6	July 30	—0. 09	F. 18	15. 87	
1781, Oct. 20	1782, May 29	—0. 11	P. (39. 30)	. .	Includes many observations discussed by BESSEL and AUWERS.
1782, June 14	June 24	+0. 17	B. (2. 5)	. .	
Sept. 10	Dec. 7	—0. 13	B. (3. 4)	. .	
			P. (5. 4)		
Dec. 14	. . . .	—0. 06	Interpolated.	. .	
Dec. 23	1783, Feb. 24	—0. 01	B. (2. 2)	. .	
			P. (2. 1)		
1783, Mar. 4	May 1	—0. 08	B. (3. 3) A.	. .	
May 4	May 31	+0. 08	B. (6. 5)	. .	
July 4	Sept. 27	0. 00	B. (1. 1)	. .	
			P. (1. 1)		
Sept. 29	Oct. 27	—0. 28	P. (2. 3)	. .	
Nov. 1	Dec. 11	—0. 07	B. (2. 1)	. .	
			P. (2. 1)		
1784, Jan. 17	1784, Jan. 28	—0. 09	F. 8	4. 86	With some allowance for Polaris in the neighboring periods. The fundamental stars give $n = -0^s.06$.
Feb. 10	Mar. 23	—0. 16	A. (2. 4)	. .	
May 1	May 19	+0. 24	A. (1. 2)	. .	
May 27	Dec. 21	—0. 10	A. (6. 7)	. .	
1785, Jan. 10	1785, Feb. 13	—0. 12	A. 3	. .	
Feb. 17	Mar. 28	+0. 11	A. 6	. .	
April 11	April 17	—0. 03	A. 4	. .	
April 18	April 19	—0. 24	B. (1. 1)	. .	
April 27	May 5	—0. 11	B. (3. 0)	. .	
May 21	June 12	+0. 01	B. (1. 3)	. .	
			P. (0. 1)	. .	
June 21	July 25	—0. 03	B. (1. 2)		
			F. 12	13. 38	
Aug. 28	. . . .	—0. 05	A. 2	. .	

Table V.—*Values of n, etc.*—Continued.

Period.		Value of *n*.	Authority.	Weight.	Remarks.
		s.			
1785, Oct. 1	1785, Oct. 6	—0. 19	A. (1. 1)	. .	
Oct. 26	Nov. 28	+0. 13	A. (1. 2)	. .	
Dec. 28	Dec. 29	0	F. 2	2. 41	
1786, Jan. 13	. . .	+0. 05	A. 1	. .	
Feb. 10	1786, May 25	—0. 26	A. (5. 6)	. .	
May 26	June 25	—0. 06	A. (1. 3)	. .	
July 4	. . .	+0. 11	A. (0. 1)	. .	
July 24	Aug. 13	—0. 02	A. (1. 1)	. .	
Sept. 9	Nov. 7	—0. 06	A. (2. 2)	. .	
			P. (0. 1)		
Dec. 25	. . .	—0. 18	P. (1. 0)	. .	
1787, Jan. 14	. . .	+0. 34	A. (1. 0)	. .	
Jan. 31	1787, Mar. 22	—0. 05	F. 13	6. 69	
April 6	April 12	—0. 17	B. (2. 1)	. .	
May 4	June 20	+0. 06	A. (10. 17)	. .	
July 22	. . .	—0. 20	A. (0. 1)	. .	
Aug. 1	Aug. 8	+0. 39	A. 2	. .	
Aug. 17	Aug. 22	+0. 01	A. (3. 3)	. .	
Sept. 28	Oct. 24	—0. 06	F. 19	20. 01	
Nov. 12	Dec. 21	—0. 22	F. 12	10. 90	
1788, Jan. 14	. . .	—0. 22	Last period.	. .	
Mar. 8	1788, May 5	—0. 06	A. 6	. .	
May 27	June 2	—0. 02	A. (2. 1)	. .	Correction doubtful, but stars of companion are near the parallel.
July 4	July 26	+0. 15	A. 3	. .	
Aug. 2	Sept. 21	—0. 19	B. (1.1) P. (0.1)	. .	
			A. 10		
Oct. 17	Nov. 5	—0. 02	F. 8	8. 13	
Nov. 27	Dec. 3	—0. 31	P. (0. 1)	9. 44	
			F. 6		
1789, Jan. 14	1789, Jan. 18	No corr.	. . .	. .	
Feb. 20	. . .	—0. 08	A. 6	. .	
Mar. 20	April 18	—0. 21	A. 12	. .	
June 13	June 15	+0. 72	A. (2. 1)	. .	
June 18	. . .	—0. 06	A. (1. 0)	. .	
June 20	. . .	—0. 31	A. (1. 1)	. .	
Aug. 5	. . .	—0. 20	A. 3	. .	
Sept. 6	Sept. 11	—0. 34	F. 6	6. 79	
Sept. 12	Sept. 13	+0. 03	F. 2	2. 02	
Oct. 28	Nov. 10	—0. 04	F. 11	7. 86	
Nov. 26	Dec. 3	+0. 06	F. 3; A. 2	2. 31	
1790, Jan. 7	1790, Feb. 1	+0. 24	A. 5	. .	
Feb. 10	Mar. 16	+0. 06	A. 5	. .	
Mar. 17	Mar. 27	—0. 06	A. 6	. .	

Table V.—*Values of n, etc.*—Continued.

Period.		Value of n.	Authority.	Weight.	Remarks.
		s.			
1790, April 18	1790, Sept. 5	−0.06	A. 21	. .	
Oct. 25	Nov. 15	+0.01	F. 12	9.61	
Dec. 18	1791, Jan. 8	−0.01	F. 13	8.22	
1791, Jan. 17	April 25	+0.09	A. 30	. .	
June 18	Oct. 16	+0.17	A. 9	. .	
Nov. 4	1792, Jan. 7	+0.05	F. 16	10.27	
1792, Jan. 8	April 15	+0.23	A. 5	. .	Auwers has one period—1792, Jan. 8–Dec. 24.
June 14	June 27	+0.39	F. 7	3.26	
July 17	July 23	No corr.	. . .	. .	Observations disagree in this period.
Oct. 21	Nov. 16	+0.18	A. 5; F. 4	5.91	
1793, Jan. 13	1793, May 19	+0.07	F. 8	4.87	Corrections not very certain.
Aug. 2	. . .	+0.80	A. 3	. .	
Aug. 3	Aug. 15	+0.32	A. 3	. .	
Aug. 24	Nov. 6	−0.04	B. (2.1) A. (2.2)	. .	
Nov. 9	Nov. 19	+0.11	F. 8	8.52	
Nov. 19	1794, Jan. 7	−0.04	A. 3; A. (1.0)	. .	
1794, Jan. 23	Feb. 2	0.00	F. 8	8.24	
Feb. 4	Mar. 27	−0.09	A. 13	. .	
April 5	May 17	+0.10	A. 7	. .	
May 31	June 2	+0.07	A. (0.2)	. .	
June 14	June 28	+0.18	A. (0.2) A. 2	. .	
July 3	Aug. 12	−0.18	A. 8	. .	
Aug. 13	. . .	+0.24	B. (1.0)	. .	
Aug. 14	Sept. 17	−0.15	A. 6	. .	
Nov. 6	Nov. 28	−0.03	A. 3; F. 22	18.31	
Dec. 18	Dec. 20	−0.22	F. 8	8.30	
1795, Jan. 3	1795, April 10	+0.02	A. (2.1) A. 12; F. 19	17.41	
April 20	May 5	+0.14	A. 3; F. 6	5.34	
July 22	July 23	+0.32	F. 4	5.10	$c = +0.47$
July 25	. . .	−0.15	F. 3	2.03	$c = +0.47$
July 31	Sept. 26	−0.07	F. 15	. .	Between Aug. 4 and Oct. 30 use $n = -0^s.20$, as given by 6 observations of Capella and Rigel (Auwers). The instrument seems to have been frequently adjusted, or some other cause of discrepancy has intervened.
Aug. 6	Sept. 27	−0.35	B. (1.2) A. (1.0)	. .	
Sept. 30	. . .	+0.10	P. (1.0)	. .	
Oct. 15	Oct. 19	0.00	F. 6	5.25	
Oct. 24	Nov. 3	−0.34	B. (2.2)	. .	
Nov. 8	. . .	−0.02	A. (1.1)	. .	From Nov. 25, 1795, till May 29, 1797, use $c = -0^s.48$.

TABLE V.—*Values of n, etc.*—Continued.

Period.						Value of *n*.	Authority.	Weight.	Remarks.
						s.			
1796,	Feb.	23	1796,	Feb.	24	—0. 11	A. (1. 0)	. .	
	May	22		June	18	—0. 30	F. 10	8. 51	
	June	28		Sept.	21	—0. 26	A. 10	. .	
	Oct.	9		Oct.	15	—0. 28	F. 12	14. 32	
	Nov.	29		Dec.	24	—0. 30	F. 12	10. 33	
1797,	Jan.	12	1797,	May	28	—0. 24	A. 9	. .	
	July	10		July	25	+0. 04	A. 6	. .	
	Sept.	21		Oct.	7	—0. 04	F. 4	3. 76	
	Dec.	9	1798,	Jan.	5	0. 00	F. 7	6. 93	
1798,	Feb.	19	1798,	May	25	—0. 05	F. 9; A. 14	8. 66	
	July	30		Aug.	17	—0. 06	A. 7	. .	
	Aug.	31		. . .	.	+0. 05	A. (1. 1)	. .	
	Oct.	18		Dec.	11	+0. 11	F. 10	. .	
	Dec.	30	1799,	Jan.	19	—0. 05	A. 5	. .	
						+0. 03	F. 3	3. 16	
1799,	Feb.	9		Mar.	12	—0. 18	F. 7	3. 82	
	April	3		June	29	—0. 13	A. 9	. .	
	July	28		Aug.	24	—0. 04	A. 4	. .	
	Oct.	1		. . .	.	+0. 04	P. (1. 1)	. .	
	Oct.	27		. . .	.	No corr.	. . .	. .	
	Dec.	12		Dec.	31	+0. 01	F. 5	4. 11	
1800,	Jan.	20	1800,	Feb.	3	+0. 24	F. 9	12. 4	
	Mar.	5		April	1	+0. 20	A. 8	. .	
	April	18		April	26	—0. 10	A. 3	. .	
	July	16		July	17	+0. 18	F. 2	2. 02	
	July	24		Aug.	17	+0. 14	A. 7	. .	
	Adopted for			July	22	+0. 15	. . .	. .	
	Oct.	22		Nov.	22	+0. 04	F. 7	8. 85	
1801,	Jan.	28	1801,	Mar.	25	+0. 02	A. 10	. .	
	Mar.	31		May	6	—0. 05	A. 11	. .	
	Aug.	10		Aug.	23	+0. 09	F. 4	4. 73	
	Nov.	24		Nov.	26	+0. 01	F. 2	1. 71	
	Dec.	31	1802,	Jan.	1	—0. 07	A. (1. 1)	. .	
1802,	Feb.	10		April	7	+0. 15	A. 17	. .	
	April	12		April	30	+0. 06	F. 8	4. 58	
	May	1		Aug.	19	—0. 02	A. 5	. .	
	Sept.	18		. . .	.	—0. 25	B. (1. 1)	. .	
	Oct.	15		Oct.	16	—0. 17	A. (1. 2) & 3	. .	
	Dec.	13		Dec.	28	—0. 12	F. 6	7. 45	
1803,	Feb.	2	1803,	May	16	—0. 09	A. 18 and (2. 2)	. .	
	June	4		. . .	.	—0. 06	P. (0. 3)	. .	
	June	23		. . .	.	+0. 10	P. (1. 1)	. .	

TABLE V.—*Values of n, etc.*—Continued.

Period.						Value of n.	Authority.	Weight.	Remarks.
						s.			
1803,	July	1	1803,	July	6	+0.02	B. (1.1)	. .	
	July	8		Aug.	10	+0.05	F. 5	7.3	
	July	9		Aug.	25	+0.44	P. (5.5)	. .	
							B. (1.1)		
1804,	Jan.	21	1804,	. . .	.	—0.12	A. (1.1)	. .	
	Mar.	26		. . .	.	—0.53	A. (0.1)	. .	
	June	12		July	15	—0.12	P. (6.4)	. .	
							B. (2.2)		
	July	30		Sept.	23	—0.04	A. 2	2.8	Adopted means, $-0^{s}.12$.
	Sept.	20		Oct.	17	—0.14	F. 12	13.12	
	Oct.	27		Oct.	30	—0.17	Interpolated.	. .	
	Nov.	21		Dec.	3	—0.20	A. (3.3)	. .	
	Dec.	20		. . .	.	—0.32	P. (0.1)	. .	
1805	April	7	1805,	April	13	—0.28	B. (2.3)	. .	
	April	29		May	5	—0.22	P. (2.1)	. .	
							B. (1.1)		
	May	21		May	27	+0.22	B. (4.3)	. .	
	Oct.	2		Oct.	11	—0.01	F. 10	15.85	
	Nov.	2		Nov.	3	+0.06	P. (1.1)	. .	
							A. (1.1)		
1806,	Jan.	8	1806,	Jan.	10	—0.02	A. (0.1)	. .	
							P. (1.0)		
	April	29		July	26	—0.03	B. (2.2)	. .	
							P. (1.1)		
	Nov.	17		Nov.	22	—0.02	A. (2.1)	. .	
	Dec.	6		. . .	.	—0.06	Interpolated.	. .	
	Dec.	22	1807,	April	12	—0.10	A. 12	. .	
									The separate determinations are—
1807,	April	24		Oct.	28	—0.05	A. 4	. .	$+0^{s}.06$, A. 4, April 26–Oct. 19;
							P. (1.2)	. .	$-0^{s}.08$, P. (1.2), May 2–June 4;
							F. 5	7.16	$-0^{s}.08$, F. 5, Oct. 26–Nov. 5.
1808,	Jan.	24	1808,	Feb.	7	+0.10	F. 10	14.	
	Feb.	9		April	14	+0.11	A. 20	. .	Use this value also for April 21.
	Mar.	15		Mar.	16	+0.10	B. (1.1)	. .	
	May	4		May	6	—0.02	B. (1.2)	. .	Use for May 8, 9.
	Sept.	15		Oct.	8	—0.34	F. 8	9.67	$c = -0^{s}.40$.
	Nov.	28		Dec.	3	—0.29	F. 5	5.85	
1809,	Jan.	28	1809,	Feb.	5	—0.20	A. (1.1) & 6	. .	
	Mar.	3		May	14	—0.09	A. 9	. .	
	May	19		May	24	+0.02	F. 5	3.02	
	Oct.	13		Oct.	15	—0.13	F. 5	5.61	
	Dec.	21	1810,	Jan.	16	+0.13	F. 4	5.57	
1810,	May	1		May	11	—0.08	F. 3	2.31	
	May	31		June	4	—0.15	F. 4	2.27	An observation of Capella is probably 1^{s} in error, and has been rejected.

Table V.—*Values of n, etc.*—Continued.

Period.		Value of n.	Authority.	Weight.	Remarks.
		s.			
1811, Feb. 26	1811, Mar. 14	0.00	P. (5.1)	. .	
Mar. 14	April 26	—0.03	F. 20	28.	
April 27	May 12	+0.04	P. (2.2)	. .	
May 13	May 28	+0.06	P. (3.6)	. .	
May 29	July 18	+0.04	P. (6.21)	. .	
July 28	Sept. 8	+0.10	P. (0.13)	. .	
1811, Sept. 10	1811, Sept. 15	+0.21	P. (0.4)	. .	
Sept. 17	Nov. 16	+0.04	P. (5.9)	. .	
Nov. 20	Dec. 19	—0.04	Y. (12.4)	. .	
Dec. 27	1812, Feb. 13	—0.11	P. (8.0)	. .	
1812, Mar. 22	May 23	(+0.21)	P. (8.28)	. .	Instrument reversed.
May 27	June 19	(+0.30)	P. (4.15)	. .	The observations in this position will require a different correction from that already applied; it will be safest to use stars near the parallel of the planets only.
Sept. 5	Sept. 27	+0.02	P. (1.14)	. .	Instrument reversed daily, and in process of adjustment; use stars on parallel.
Oct. 4	Oct. 30	—0.07	P. (0.5)	. .	October 8 is anomalous and has been omitted; I think it was made above pole, not below.
1813, Feb. 12	1813, Mar. 14	—0.08	F. 9	6.4	
Mar. 14	May 25	—0.11	Interpolated.	. .	
May 25	July 9	—0.15	P. (1.11)	. .	It will be sufficient, I think, for the planetary observations up to July 27, 1816, to employ $n = -0.07$.
VALUE OF n FROM POND'S OBSERVATIONS WITH NEW TRANSIT—1816, JULY 27–1830.					
		s.			
1816, July 27	1816, Aug. 29	+0.02	(8.8)	. .	The letter P. is omitted in this table; where A. is referred to, the observations in parentheses are those of Polaris computed by myself.
Sept. 5	. . . .	+0.03	(1.0)	. .	
Sept. 10	Sept. 22	+0.01	(6.8)	. .	
Oct. 14	Oct. 18	+0.02	(4.3)	. .	
Dec. 7	Dec. 8	+0.04	(2.1)	. .	
1817, May 23	1817, May 31	+0.14	(0.2)	. .	
June 10	June 16	—0.07	(3.2)	. .	
July 19	. . . .	+0.12	(0.1)	. .	
Aug. 25	Sept. 3	+0.01	A. (0.1)	. .	
Sept. 7	Sept. 21	No corr.	. . .	. .	
1818, April 4	1818, June 4	+0.02	A. (20.37)	. .	
Sept. 6	Nov. 3	—0.04	(5.3)	. .	And 14 pairs of n and s stars.
1819, June 27	1819, April 27	—0.06	A. (3.4)	. .	And 25 observations of Capella and Rigel A.
May 1	May 17	—0.01	A. (4.5)	. .	
May 27	May 28	+0.07	(2.0)	. .	
May 29	June 28	—0.03	(12.10)	. .	

TABLE V.—*Values of n, etc.*—Continued.

Period.		Value of n.	Authority.	Weight.	Remarks.
		s.			
1819, July 29	1819, Sept. 14	—0.07	. . .	. .	35 pairs of n and s stars.
Sept. 16	Sept. 24	0.00	(3.2)	. .	Partly A.
Sept. 26	Oct. 10	—0.16	(2.4)	. .	
Oct. 15	Oct. 26	+0.09	(8.1)	. .	
Nov. 18	Nov. 27	—0.03	(5.3)	. .	
Dec. 1	Dec. 2	—0.17	(2.1)	. .	
Dec. 8	1820, Jan. 3	—0.02	(10.2)	. .	
1820, Jan. 22	Feb. 9	—0.18	A. (3.0)	. .	
Feb. 10	Feb. 23	—0.05	A.	. .	Capella and Rigel 7.
Feb. 28	April 15	+0.08	A. (2.7)	. .	C. and R. 8; also (1, 2.) computed by myself.
April 17	May 11	+0.02	A. (3.7)	. .	
May 14	July 2	+0.02	(19.20)	. .	
July 10	Aug. 24	—0.03	(5.7) & A. (3.2)	. .	From this point on the observations of Polaris are so numerous that I have sometimes computed those only which were consecutive, omitting such as were far distant from corresponding opposite.
Aug. 28	Oct. 3	+0.01	(11.9) & A. (2.1)	. .	
Oct. 5	Nov. 15	—0.02	(6.8)	. .	
Nov. 17	Dec. 31	—0.03	(11.5)	. .	
1821, Jan. 16	April 24	+0.06	A. (11.14)	. .	
April 24	1821, May 18	+0.05	(3.3) & A. (3.5)	. .	
May 22	June 19	—0.06	(6.6)	. .	
June 21	July 22	—0.11	(11.13)	. .	
July 23	July 26	+0.09	(3.1)	. .	
Aug. 3	Oct. 21	—0.05	(14.16)	. .	
Oct. 22	Oct. 31	—0.12	(6.2)	. .	
Nov. 3	Nov. 29	+0.02	(12.8)	. .	
Nov. 30	Dec. 8	—0.05	(5.4)	. .	December 8, $s.p.$, rejected; 10ˢ wrong? Collimation uncertain; use stars near parallel.
Dec. 10	Dec. 13	+0.17	(1.2)	. .	
Dec. 14	1822, Feb. 27	+0.03	(10.5) & A. (9.4)	. .	
1822, Feb. 27	Mar. 28	+0.01	A. (3.2)	. .	
April 7	June 3	+0.07	A. (4.5) & (0.4)	. .	
June 8	June 9	+0.15	(1.2)	. .	
June 12	June 14	. .	(2.3)	. .	For planets, June 13, use the stars near parallel; the value of n seems to be small if the c (0ˢ.07) has the positive sign. The azimuth was adjusted June 11; the level had been previously corrected. The value of $(n + c)$ comes out $+ 0^s.085$.
June 18	July 20	+0.01	(4.10)	. .	
Aug. 4	Sept. 18	+0.05	A. (0.5)	. .	
Oct. 4	Oct. 22	—0.01	(3.2)	. .	
Oct. 24	Oct. 30	+0.03	(6.4)	. .	
Nov. 2	1822, Nov. 6	0.00	(3.3)	. .	

TABLE V.—*Values of n, etc.*—Continued.

Period.		Value of *n*.	Authority.	Weight.	Remarks.
		s.			
1822, Nov. 8	1822, Dec. 9	—0.04	(12.8)	. .	
Dec. 10	Dec. 30	+0.01	(10.9)	. .	
1823, Jan. 8	1823, Jan. 11	+0.01	A. (3.2)	. .	
Jan. 14	Jan. 23	+0.14	(3.0)	. .	
Feb. 8	April 10	+0.05	A. (5.3)	. .	
April 15	June 20	—0.01	(6.13)	. .	
			A. (6.9)		
June 23	June 24	+0.10	(2.0)		
July 2	July 6	—0.05	(1.3)	. .	
July 9	July 15	—0.06	(3.2)	. .	
July 16*	Aug. 15*	—0.14	(7.6)	. .	* The observations of Polaris do not well agree, but the mean value of *n* is nearly confirmed by δ Ursæ Minoris.
			δ (0.6)		
Aug. 14	Aug. 16	+0.15	(1.0)	. .	δ Ursæ Minoris is here included.
			δ (2.1)	. .	
Aug. 20	Sept. 9	—0.04	A. (4.4)	. .	
			A. δ (2.2)	. .	
Sept. 10	Sept. 18	—0.04	(5.5)	. .	
Sept. 20	Oct. 8	—0.09	(8.5)	. .	
Oct. 9	Nov. 9	+0.04	(13.12)	. .	
Nov. 11	1824, Feb. 2	+0.03	(29.19)	. .	A. (4.0) gives + 0.02 for the last few days of this period.
1824, Feb. 14	. . . .	+0.03	A. (1.0)	. .	
Mar. 9	Mar. 13	—0.01	A. (0.2)	. .	
Mar. 14	. . . .	+0.07	A. (0.1)	. .	
Mar. 16	Mar. 22	—0.16	A. (1.1)	. .	
			(1.1)		
Mar. 28	. . . .	—0.07	(1.1)	. .	
Mar. 29	April 4	—0.01	(3.4)	. .	
April 12	April 13	—0.17	A. (1.2)	. .	
April 13	April 15	—0.13	A. (2.1)	. .	
April 17	April 19	—0.04	A. (1.2)	. .	
April 20	April 29	—0.01	A. (1.1)	. .	
April 30	June 4	—0.04	A. (1.2)	. .	
			(1.7)		
June 6	June 9	—0.12	(0.3)	. .	
July 4	July 15	—0.07	(5.4)	. .	
July 17	July 22	—0.06	(2.5)	. .	
Aug. 27	Sept. 4	—0.09	A. (3.3)	. .	
Sept. 4*	Sept. 5	—0.21	(1.1)	. .	* Beginning with Castor.
			δ (1.0)		
Sept. 6	Sept. 18	—0.16	A.(1.1) A. δ (0.1)	. .	
			(5.5)		
Sept. 25	Sept. 26	—0.12	(1.1)	. .	
Sept. 26†	Sept. 28	—0.02	(1.2)	. .	† From Polaris inclusive.
Sept. 30	Oct. 2‡	—0.03	(2.2)	. .	‡ To Venus inclusive.
Oct. 25	Nov. 21	—0.14	(13.8)	. .	

TABLE V.—*Values of n, etc.*—Continued.

Period.		Value of *n*.	Authority.	Weight.	Remarks.
		s.			
1824, Nov. 22	1824, Dec. 3	—0. 10	(5. 3)	. .	
Dec. 7	1825, Jan. 2	—0. 13	A. (2. 2)	. .	
1825, Jan. 15	Feb. 5	—0. 19	A. (2. 1)	. .	
June 21	July 17	—0. 03	(3. 6)	. .	
July 27	Sept. 22	—0. 18	(1. 7)	. .	
Sept. 27	Oct. 31	—0. 06	(6. 8)	. .	
Nov. 17	Nov. 21	—0. 10	(3. 3)	. .	
Dec. 6	Dec. 7	—0. 06	(1. 1)	. .	
Dec. 20	Dec. 24	—0. 01	(2. 1)	. .	
1826, Jan. 26	1826, Feb. 17	+0. 11	A. (4.4) & (2.2)	. .	
Feb. 22	April 8	—0. 03	A. (4. 0)	. .	
April 10	April 11	+0. 22	(1. 0)	. .	This value should only be used with great caution.
April 14	April 30	+0. 05	(1. 10)	. .	
May 1	May 18	+0. 10	(3. 10)	. .	
May 19	May 21	+0. 07	(1. 3)	. .	
June 7	June 12	—0. 01	(3. 4)	. .	
June 12	June 15	. .	(3. 2)	. .	From June 12, Polaris *s. p.* to June 15, Spica inclusive, the value of $(n + c)$ is $-0^s.33$; we shall most safely employ $n = 0$ and $c = -0^s.33$. AIRY's comparison stars are not far from the parallel of Venus.
June 15	June 18	—0. 02	(2. 1)	. .	
June 19	June 20	—0. 05	(1. 2)	. .	
June 22	. . . .	—0. 14	(0. 1)	. .	
June 23	June 29	+0. 03	(6. 6)	. .	
July 1	July 9	+0. 02	(5. 6)	. .	
July 12	Sept. 15	+0. 02	(11. 14)	. .	
			δ (6. 0)		
Oct. 27	Nov. 1	—0. 17	(3. 2)	. .	
Nov. 2	Nov. 21	—0. 05	(9. 3)	. .	
Dec. 21	1827, Jan. 14	—0. 00	(4. 4)	. .	
1827, Feb. 5	Feb. 17	—0. 06	A. (3. 4)	. .	
Mar. 29	April 27	—0. 07	A. (3. 4)	. .	
July 10	July 28	—0. 06	(6. 9)	. .	
Aug. 23	Sept. 1	—0. 04	(1. 3)	. .	
1828, Jan. 25	1828, Feb. 28	—0. 06	A. (3. 5)	. .	
			A. δ (0. 3)		
Mar. 9	May 17	0. 00	A. (4. 2)	. .	
			(4. 2)		
May 18	. . . .	+0. 10	(1. 1)	. .	
May 19	. . . .	+0. 11	(0. 1)	. .	
May 19	June 13	+0. 01	(5. 7)	. .	
June 14	July 15	—0. 02	(3. 15)	. .	
July 16	Aug. 29	—0. 05	(1. 2)	. .	
			δ (14. 0)		

TABLE V.—*Values of n, etc.*—Continued.

Period.		Value of *n*.	Authority.	Weight.	Remarks.
		s.			
1828, Sept. 2	1828, Nov. 7	—0.02	(9.14) δ (7.0)	. .	
Nov. 9	Nov. 19	—0.09	(5.0)	. .	
Nov. 21	Nov. 23	+0.01	(0.2)	. .	
Nov. 24	Dec. 1	—0.01	(3.3)	. .	
1829, Jan. 17	1829, Jan. 19	+0.03	(1.1)	. .	
Feb. 1	Mar. 24	+0.13	A. (5.0)	. .	
Mar. 25	Mar. 27	+0.09	(2.0)	. .	
May 12	June 16	+0.01	(9.8)	. .	
June 20	July 2	0.00	(1.4)	. .	
July 21	Aug. 7	—0.05	A. (0.1)	. .	
			A. δ (2.0)		
			(0.2)		
			δ (4.0)		
Aug. 8	. . .	0.00	δ (1.0)		
Aug. 9	Aug. 19	—0.08	(0.1) δ (2.0)	. .	
Aug. 21	Sept. 4	—0.01	(2.0) δ (2.0)	. .	
Sept. 7	Sept. 16	—0.08	(2.4) δ (1.0)	. .	
Sept. 30	Oct. 15	—0.07	(0.1)	. .	
Oct. 18	Nov. 26	0.00	(6.1)	. .	
Dec. 10	Dec. 20	—0.02	(3.2)	. .	
1830, Feb. 10	1830, April 7	+0.07	A. (2.1) δ (2.0)	. .	
April 24	June 5	0.00	A. (1.2) (4.10)	. .	
June 13	July 4	0.00	(0.4)	. .	
July 8	July 14	+0.04	(1.2)	. .	
July 22	Aug. 2	—0.07	(0.4) δ (2.0)	. .	
Aug. 3	Sept. 11	—0.06	(0.1) δ (4.0)	. .	Three observations of Br. 2749 = No. 480 of the ABO catalogue, are added to complete the period.
Sept. 12	Oct. 13	+0.04	(2.4)	. .	Does not include morning observations.
Oct. 14	Oct. 19	—0.06	(3.4)	. .	
Oct. 20	Nov. 1	+0.02	(6.4)	. .	
Nov. 3	Nov. 8	+0.04	(3.3)	. .	
Nov. 9	Nov. 12	Doubtful.	. .	. .	Between these dates use no correction and stars near parallel, even if not fundamental.
Nov. 15	Dec. 16	+0.03	(9.9)	. .	
Dec. 23	Dec. 31	+0.18	(2.3)	. .	Observations 1831, Jan. 6, 8 are included, but Dec. 29 omitted as probably 10^s in error. The corrections between Dec. 16 and the end of the year are doubtful; it is better to employ zodiacal stars on parallel.

TABLE VI.—*Reduction of the Declinations of the Tabulæ Regiomontanæ to* Boss's *system, as employed in the Catalogue of the American Ephemeris.*

Star's name.	$\Delta'\delta$ 1765.	$\Delta'\delta$ 1785.	$\Delta'\delta$ 1805.	$\Delta'\delta$ 1825.	Δn 1800.	N. 1800.
	″	″	″	″	″	°
γ Pegasi	+0.14	+0.33	+0.52	+0.70	0.182	179.0
α Arietis	+0.45	+0.47	+0.48	+0.46	0.199	143.2
α Ceti	−1.43	−0.61	+0.21	+1.03	0.214	128.5
α Tauri	+1.42	+1.09	+0.75	+0.40	0.237	108.1
α Aurigæ	+0.64	+0.47	+0.29	+0.09	0.242	100.8
β Orionis	−0.56	−0.09	+0.39	+0.88	0.243	100.3
β Tauri	−0.90	−0.33	+0.23	+0.78	0.244	98.6
α Orionis	−0.42	−0.15	+0.13	+0.40	0.246	92.9
α Canis Majoris*	−0.24	+0.30	+0.88	+1.47	0.245	83.2
α Geminorum	+0.62	+0.50	+0.41	+0.33	0.239	74.6
α Canis Minoris*	+0.50	+1.03	+1.60	+2.20	0.238	73.2
β Geminorum	+0.36	+0.39	+0.44	+0.51	0.237	72.4
α Hydræ	+0.58	+0.74	+0.91	+1.06	0.211	49.2
α Leonis	+1.25	+1.03	+0.81	+0.62	0.201	38.6
β Leonis	+1.59	+1.21	+0.83	+0.48	0.183	7.1
β Virginis	+0.33	+0.60	+0.88	+1.17	0.183	6.6
α Virginis	+0.44	+0.73	+1.03	+1.33	0.190	335.5
α Bootis	+1.86	+1.46	+1.04	+0.58	0.202	320.2
α¹ Libræ	+1.66	+1.61	+1.58	+1.56	0.211	311.5
α² Libræ	+2.05	+1.70	+1.34	+1.01	0.211	311.4
α Coronæ	+2.56	+1.95	+1.33	+0.71	0.223	300.4
α Serpentis	+2.12	+1.92	+1.74	+1.57	0.226	298.6
α Scorpii	+1.38	+1.51	+1.64	+1.78	0.235	289.6
α Herculis	+1.73	+1.61	+1.50	+1.40	0.243	280.2
α Ophiuchi	+2.61	+2.03	+1.46	+0.90	0.245	276.4
α Lyræ	+1.00	+1.00	+0.99	+1.00	0.245	264.4
γ Aquilæ	+0.95	+0.94	+0.94	+0.94	0.236	251.6
α Aquilæ	+1.02	+1.02	+1.03	+1.06	0.236	250.8
β Aquilæ	+1.33	+1.38	+1.47	+1.62	0.235	249.8
α¹ Capricorni	+0.14	+0.68	+1.21	+1.76	0.230	245.5
α² Capricorni	+0.60	+1.12	+1.62	+2.14	0.230	245.4
α Cygni	+0.62	+0.72	+0.81	+0.90	0.223	239.4
α Aquarii	+0.71	+0.89	+1.08	+1.25	0.201	219.2
α Piscis Austrini	+0.71	+1.95	+3.19	+4.40	0.189	204.2
α Pegasi	+2.51	+1.90	+1.28	+0.65	0.188	201.6
α Andromedæ	+1.05	+0.77	+0.49	+0.70	0.182	180.6

*The corrections P¹ on pp. 297–298 of Vol. I are to be applied.

TABLE VII.—*Mean Declinations of the 8 stars in Gemini and of β and γ Draconis for every ten years from* 1760 *to* 1810.

Epoch.	Declination.	Corr. to BESSEL.	Annual Var.	Proper Motion.	Epoch.	Declination.	Corr. to BESSEL.	Annual Var.	Proper Motion.
1 GEMINORUM (*h* or H.).					ε GEMINORUM.				
	° ′ ″	″	″	″		° ′ ″	″	″	″
1760	23 15 5.96	−0.24	+0.815	−0.101	1760	25 20 39.24	−0.21	−2.558	−0.012
1770	15 13.84	−0.15	+0.761	−0.101	1770	20 13.40	−0.34	−2.611	−0.012
1780	15 21.19	−0.07	+0.708	−0.101	1780	19 47.02	−0.48	−2.665	−0.013
1790	15 28.00	+0.01	+0.655	−0.101	1790	19 20.10	−0.62	−2.718	−0.013
1800	15 34.29	+0.09	+0.602	−0.102	1800	18 52.65	−0.77	−2.772	−0.013
1810	15 40.04	+0.17	+0.549	−0.102	1810	18 24.66	−0.92	−2.825	−0.013
η GEMINORUM.					ξ GEMINORUM.				
1760	22 33 7.83	−0.04	−0.050	−0.015	1760	13 7 57.71	. .	−2.992	−0.214
1770	33 7.07	−0.13	−0.102	−0.015	1770	7 27.56	. .	−3.040	−0.214
1780	33 5.78	−0.23	−0.155	−0.015	1780	6 56.92	. .	−3.088	−0.214
1790	33 3.96	−0.32	−0.208	−0.016	1790	6 25.80	. .	−3.136	−0.214
1800	33 1.63	−0.41	−0.261	−0.016	1800	5 54.19	. .	−3.185	−0.214
1810	32 58.77	−0.49	−0.314	−0.016	1810	5 22.10	. .	−3.233	−0.214
μ GEMINORUM.					ζ GEMINORUM.				
1760	22 36 45.73	−0.29	−0.859	−0.121	1760	20 53 59.22	+0.51	−4.349	−0.018
1770	36 36.88	−0.39	−0.912	−0.121	1770	53 15.48	+0.32	−4.399	−0.018
1780	36 27.50	−0.50	−0.965	−0.121	1780	52 31.23	+0.13	−4.450	−0.018
1790	36 17.58	−0.60	−1.018	−0.121	1790	51 46.48	−0.06	−4.501	−0.018
1800	36 7.14	−0.71	−1.071	−0.121	1800	51 1.22	−0.25	−4.551	−0.018
1810	35 56.17	−0.82	−1.124	−0.121	1810	50 15.45	−0.45	−4.602	−0.018
ν GEMINORUM.					β DRACONIS.				
1760	20 20 25.46	+0.51	−1.307	−0.020	1760	52 29 18.44	. .	−3.050	0.000
1770	20 12.12	+0.26	−1.359	−0.021	1770	28 48.03	. .	−3.031	0.000
1780	19 58.27	+0.02	−1.411	−0.021	1780	28 17.82	. .	−3.011	0.000
1790	19 43.90	−0.23	−1.463	−0.021	1790	27 47.81	. .	−2.992	0.000
1800	19 29.01	−0.48	−1.515	−0.021	1800	27 17.99	. .	−2.972	0.000
1810	19 13.59	−0.73	−1.567	−0.021	1810	26 48.36	. .	−2.953	0.000
γ GEMINORUM.					γ DRACONIS.				
1760	16 34 52.31	+0.08	−2.130	−0.047	1760	51 31 35.69	−0.75	−0.815	−0.031
1770	34 30.76	+0.02	−2.180	−0.047	1770	31 27.64	−0.95	−0.794	−0.031
1780	34 8.70	−0.03	−2.231	−0.047	1780	31 19.80	−1.15	−0.774	−0.031
1790	33 46.14	−0.08	−2.281	−0.047	1790	31 12.16	−1.35	−0.754	−0.031
1800	33 23.08	−0.14	−2.331	−0.047	1800	31 4.73	−1.54	−0.733	−0.030
1810	32 59.50	−0.21	−2.382	−0.047	1810	30 57.50	−1.74	−0.713	−0.030

Table VIII.—*Lunar Nutation in Declination.*

$\Delta_{\Omega}\delta$ / Sid. Day.	Geminorum. ι	η	μ	ν	γ	ε	ζ	Draconis. β	γ
	″	″	″	″	″	″	″	″	″
1760 0	+0.27	+0.57	+0.84	+1.03	+1.29	+1.45	+2.05	+0.46	−0.32
200	1.95	2.27	2.50	2.68	2.93	3.08	3.63	−1.22	1.99
400	3.56	3.86	4.07	4.24	4.47	4.60	5.09	2.86	3.60
1761 0	3.29	3.60	3.81	3.98	4.21	4.36	4.86	2.58	3.34
200	4.80	5.07	5.27	5.42	5.62	5.74	6.17	4.14	4.84
400	6.13	6.37	6.53	6.66	6.86	6.93	7.26	5.54	6.16
1762 0	5.92	6.17	6.34	6.47	6.65	6.74	7.10	5.31	5.95
200	7.08	7.29	7.43	7.53	7.67	7.74	7.99	6.56	7.11
400	8.01	8.17	8.27	8.34	8.43	8.48	8.62	7.60	8.03
1763 0	7.87	8.04	8.14	8.22	8.32	8.38	8.54	7.44	7.89
200	8.58	8.69	8.76	8.81	8.86	8.88	8.94	8.26	8.60
400	9.00	9.05	9.08	9.09	9.09	9.10	9.04	8.81	9.02
1764 0	8.95	9.01	9.05	9.06	9.08	9.08	9.04	8.74	8.97
200	9.12	9.13	9.12	9.10	9.07	9.04	8.88	9.05	9.13
400	9.00	8.95	8.90	8.85	8.76	8.71	8.45	9.05	9.00
1765 0	9.04	9.00	8.96	8.91	8.83	8.78	8.54	9.07	9.04
200	8.67	8.57	8.48	8.41	8.32	8.21	7.87	8.83	8.66
400	8.01	7.86	7.73	7.62	7.46	7.36	6.93	8.30	7.99
1766 0	8.14	8.00	7.88	7.78	7.62	7.52	7.11	8.40	8.13
200	7.26	7.06	6.91	6.78	6.59	6.47	5.97	7.65	7.25
400	6.13	5.89	5.71	5.56	5.32	5.19	4.64	6.63	6.11
1767 0	6.34	6.11	5.92	5.78	5.56	5.42	4.88	6.83	6.31
200	5.04	4.77	4.56	4.40	4.15	4.00	3.40	5.61	5.00
400	3.56	3.26	3.03	2.85	2.59	2.43	1.81	4.20	3.52
1768 0	3.82	3.52	3.30	3.12	2.85	2.71	2.09	4.45	3.78
200	2.23	1.91	+1.68	+1.49	+1.22	+1.06	+0.43	2.93	2.20
400	0.55	0.23	−0.01	−0.20	−0.47	−0.63	−1.25	1.30	0.52
1769 0	+0.84	+0.52	+0.27	+0.08	0.19	0.35	0.96	−1.58	−0.80
200	−0.86	−1.18	−1.42	−1.61	1.87	2.02	2.62	+0.11	+0.89
400	2.54	2.85	3.08	3.26	3.51	3.65	4.19	1.80	2.57
1770 0	2.26	2.57	2.80	2.98	3.23	3.38	3.93	1.52	2.29
200	3.88	4.17	4.39	4.55	4.78	4.91	5.39	3.17	3.90
400	5.37	5.63	5.82	5.97	6.17	6.28	6.68	4.71	5.39
1771 0	5.12	5.40	5.59	5.74	5.94	6.05	6.42	4.46	5.16
200	6.47	6.71	6.87	6.99	7.16	7.26	7.57	5.88	6.50
400	7.60	7.79	7.92	8.01	8.13	8.20	8.42	7.10	7.62
1772 0	7.43	7.62	7.76	7.86	7.99	8.06	8.29	6.92	7.45
200	8.34	8.48	8.58	8.64	8.73	8.78	8.89	7.94	8.35
400	8.96	9.05	9.10	9.13	9.17	9.18	9.18	8.69	8.97
1773 0	8.88	8.98	9.04	9.08	9.11	9.13	9.16	8.58	8.89
200	9.24	9.28	9.29	9.30	9.28	9.28	9.18	9.08	9.25
400	9.29	9.26	9.23	9.19	9.14	9.10	8.89	9.26	9.29
1774 0	9.30	9.29	9.26	9.24	9.19	9.15	8.96	9.25	9.30
200	9.08	9.00	8.93	8.87	8.77	8.70	8.41	9.17	9.07
400	8.53	8.40	8.29	8.20	8.05	7.96	7.56	8.77	8.52
1775 0	8.65	8.52	8.42	8.33	8.19	8.11	7.72	8.86	8.63
200	7.87	7.68	7.54	7.41	7.24	7.13	6.65	8.20	7.85
400	6.80	6.57	6.40	6.25	6.03	5.90	5.37	7.25	6.78
1776 0	6.99	6.77	6.60	6.47	6.25	6.12	5.59	7.39	6.98
200	5.73	5.48	5.28	5.12	4.88	4.73	4.10	6.29	5.72
400	−4.29	−4.00	−3.78	−3.60	−3.34	−3.18	−2.57	+4.93	+4.26

Tab e VIII.—*Lunar Nutution in Declination*—Continued.

$\Delta_{\Omega}\delta$	Geminorum.							Draconis.	
Sid. Day.	ι	η	μ	ν	γ	ε	ζ	β	γ
	"	"	"	"	"	"	"	"	"
1777 0	—4.54	—4.26	—4.04	—3.87	—3.61	—3.45	—2.84	+5.17	+4.52
200	2.98	2.67	2.43	2.25	1.98	1.81	—1.19	3.67	2.95
400	1.31	0.99	0.75	0.56	0.29	0.13	+0.50	2.05	1.28
1778 0	—1.60	—1.28	—1.04	—0.85	—0.57	—0.41	0.21	2.33	+1.57
200	+0.10	+0.42	+0.66	+0.85	+1.12	+1.28	1.88	+0.66	—0.12
400	1.78	2.10	2.33	2.51	2.76	2.92	3.47	—1.02	1.80
1779 0	1.50	1.82	2.06	2.23	2.50	2.65	3.21	0.75	1.53
200	3.13	3.44	3.66	3.82	4.06	4.20	4.71	2.40	3.13
400	4.65	4.92	5.12	5.27	5.48	5.60	6.02	3.96	4.67
1780 0	4.40	4.69	4.89	5.04	5.26	5.38	5.83	3.71	4.42
200	5.79	6.04	6.21	6.35	6.52	6.63	6.99	5.16	5.81
400	6.98	7.18	7.32	7.44	7.57	7.65	7.92	6.43	6.99
1781 0	6.79	7.01	7.16	7.27	7.41	7.49	7.77	6.24	6.81
200	7.79	7.96	8.06	8.14	8.25	8.31	8.48	7.33	7.79
400	8.52	8.64	8.73	8.76	8.82	8.84	8.90	8.18	8.53
1782 0	8.42	8.54	8.62	8.67	8.74	8.78	8.85	8.06	8.43
200	8.92	8.98	9.02	9.04	9.06	9.07	9.03	8.68	8.92
400	9.12	9.13	9.12	9.11	9.08	9.05	8.92	9.03	9.13
1783 0	9.11	9.13	9.13	9.13	9.10	9.08	8.95	8.99	9.11
200	9.07	9.03	8.98	8.95	8.87	8.83	8.59	9.09	9.07
400	8.73	8.63	8.55	8.48	8.36	8.28	7.94	8.88	8.72
1784 0	8.80	8.72	8.64	8.57	8.46	8.39	8.08	8.94	8.80
200	8.22	8.08	7.97	7.87	7.72	7.61	7.20	8.48	8.21
400	7.36	7.17	7.02	6.89	6.70	6.58	6.10	7.76	7.35
1785 0	7.53	7.34	7.20	7.08	6.89	6.77	6.30	7.90	7.52
200	6.46	6.23	6.05	5.91	5.70	5.56	5.02	6.94	6.45
400	5.18	4.91	4.71	4.54	4.30	4.15	3.57	5.76	5.17
1786 0	5.41	5.15	4.95	4.78	4.55	4.40	3.82	5.98	5.39
200	3.98	3.68	3.46	3.29	3.02	2.86	2.25	4.62	3.96
400	2.40	2.08	1.85	1.67	1.39	1.23	0.61	3.12	2.38
1787 0	2.67	2.36	2.13	1.94	+1.67	+1.51	+0.89	3.38	2.66
200	+1.02	+0.69	+0.45	+0.26	—0.01	—0.17	—0.79	1.76	—1.00
400	—0.68	—1.01	—1.24	—1.43	1.70	1.86	2.44	0.09	+0.69
1788 0	0.40	0.72	0.96	1.15	1.41	1.57	2.17	—0.37	0.41
200	2.09	2.40	2.64	2.82	3.07	3.22	3.77	+1.32	2.10
400	3.71	4.01	4.23	4.39	4.62	4.76	5.24	2.98	3.72
1789 0	3.45	3.75	3.97	4.13	4.37	4.51	5.01	2.71	3.46
200	4.98	5.25	5.45	5.60	5.81	5.92	6.35	4.28	4.99
400	6.34	6.58	6.75	6.87	7.04	7.13	7.47	5.73	6.35
1790 0	6.13	6.36	6.55	6.67	6.84	6.94	7.29	5.49	6.14
200	7.32	7.52	7.66	7.76	7.89	7.97	8.21	6.78	7.32
400	8.26	8.41	8.51	8.58	8.67	8.71	8.85	7.83	8.26
1791 0	8.12	8.28	8.38	8.45	8.55	8.60	8.76	7.67	8.12
200	8.83	8.93	8.99	9.03	9.08	9.09	9.13	8.51	8.83
400	9.22	9.27	9.28	9.28	9.28	9.27	9.19	9.04	9.22
1792 0	9.18	9.23	9.25	9.27	9.27	9.27	9.21	8.97	9.18
200	9.31	9.30	9.28	9.26	9.20	9.18	9.00	9.25	9.31
400	9.12	9.05	8.98	8.92	8.83	8.76	8.47	9.20	9.12
1793 0	9.17	9.11	9.06	9.00	8.92	8.85	8.59	9.23	9.17
200	8.71	8.60	8.49	8.41	8.28	8.18	7.81	8.92	8.71
400	—7.95	—7.78	—7.64	—7.53	—7.35	—7.24	—6.78	+8.29	+7.94

Table VIII.—*Lunar Nutation in Declination*—Continued.

$\Delta_{\Omega}\delta$	Geminorum.							Draconis.	
Sid. Day.	ι	η	μ	ν	γ	ε	ζ	β	γ
	″	″	″	″	″	″	″	″	″
1794 0	8.10	−7.94	−7.81	−7.69	−7.52	−7.41	−6.97	+8.41	+8.09
200	7.11	6.90	6.73	6.59	6.38	6.25	5.74	7.56	7.10
400	5.88	5.63	5.43	5.27	5.03	4.89	4.31	6.43	5.87
1795 0	6.10	5.86	5.65	5.50	5.27	5.12	4.53	6.63	6.09
200	4.70	4.42	4.20	4.03	3.77	3.62	3.01	5.33	4.69
400	3.14	2.84	2.61	2.42	2.15	1.99	1.36	3.85	3.14
1796 0	3.41	3.11	2.88	2.69	2.43	2.27	−1.64	4.11	3.41
200	1.77	−1.45	−1.21	−1.02	−0.75	−0.59	+0.04	2.52	1.77
400	0.08	+0.25	+0.49	+0.68	+0.95	+1.11	1.71	0.85	0.07
1797 0	−0.36	−0.04	0.20	0.39	0.66	0.82	1.43	+1.14	+0.36
200	+1.33	+1.65	1.89	2.07	2.32	2.48	3.05	−0.55	−1.33
400	2.97	3.27	3.49	3.66	3.91	4.05	4.56	2.21	2.97
1798 0	2.70	3.01	3.23	3.40	3.65	3.79	4.31	1.94	2.70
200	4.25	4.54	4.74	4.89	5.12	5.24	5.69	3.54	4.25
400	5.66	5.91	6.09	6.22	6.41	6.51	6.88	5.00	5.66
1799 0	5.43	5.69	5.87	6.01	6.17	6.31	6.69	4.76	5.43
200	6.68	6.90	7.04	7.16	7.31	7.40	7.68	6.10	6.68
400	7.70	7.87	7.99	8.06	8.18	8.23	8.41	7.22	7.70
1800 0	7.54	7.72	7.85	7.93	8.05	8.11	8.31	7.04	7.54
200	8.35	8.48	8.56	8.61	8.69	8.73	8.82	7.97	8.35
400	8.88	8.95	8.99	9.02	9.04	9.04	9.02	8.63	8.88
1801 0	8.81	8.89	8.94	8.97	9.00	9.01	9.01	8.54	8.81
200	9.10	9.12	9.13	9.12	9.10	9.08	8.97	8.96	9.10
400	9.09	9.05	9.01	8.98	8.91	8.86	8.64	9.08	9.09
1802 0	9.11	9.09	9.05	9.02	8.97	8.92	8.72	9.09	9.11
200	8.85	8.77	8.69	8.63	8.52	8.46	8.14	8.96	8.84
400	8.30	8.16	8.05	7.95	7.80	7.70	7.31	8.55	8.29
1803 0	8.41	8.28	8.17	8.08	7.94	7.85	7.46	8.65	8.41
200	7.62	7.45	7.30	7.19	7.00	6.88	6.42	7.99	7.63
400	6.58	6.36	6.19	6.04	5.82	5.69	5.16	7.07	6.59
1804 0	6.78	6.56	6.38	6.25	6.04	5.91	5.38	7.24	6.78
200	5.55	5.29	5.09	4.93	4.69	4.54	3.97	6.12	5.55
400	4.13	3.84	3.62	3.44	3.19	3.03	2.42	4.79	4.13
1805 0	4.38	4.10	3.87	3.71	3.45	3.30	2.69	5.03	4.38
200	2.84	2.53	2.29	2.11	1.84	+1.68	+1.06	3.56	2.85
400	1.19	0.86	0.63	0.44	0.17	0.00	−0.61	1.95	1.19
1806 0	+1.47	+1.15	+0.91	+0.72	+0.45	+0.28	0.33	2.23	−1.47
200	−0.22	−0.54	−0.79	−0.97	−1.24	−1.40	2.00	−0.57	+0.21
400	1.91	2.23	2.47	2.64	2.90	3.06	3.61	+1.12	1.91
1807 0	1.63	1.95	2.18	2.37	2.62	2.78	3.35	0.83	1.62
200	3.28	3.59	3.81	3.97	4.21	4.35	4.86	2.52	3.27
400	4.83	5.12	5.31	5.45	5.66	5.78	6.22	4.11	4.82
1808 0	4.58	4.86	5.06	5.21	5.43	5.56	6.00	3.84	4.57
200	5.99	6.24	6.42	6.55	6.72	6.83	7.18	5.34	5.98
400	7.21	7.41	7.55	7.66	7.79	7.87	8.13	6.64	7.19
1809 0	7.02	7.23	7.38	7.48	7.63	7.72	7.99	6.44	7.01
200	8.03	8.20	8.30	8.38	8.48	8.53	8.70	7.57	8.03
400	8.76	8.88	8.94	8.98	9.03	9.06	9.11	8.43	8.77
1810 0	8.66	8.78	8.85	8.90	8.96	8.99	9.05	8.30	8.66
200	9.15	9.21	9.24	9.25	9.25	9.26	9.21	8.92	9.14
400	−9.31	−9.32	−9.29	−9.27	−9.23	−9.20	−9.04	+9.23	+9.32

TABLE IX.—*Annual Term in Declination for Epoch* 1770.

	$\Delta_{\odot}\delta$	ι Geminorum.			η Geminorum.			μ Geminorum.			ν Geminorum.			γ Geminorum.		
	Sid. Day.	1770.	10 d.	100 y.	1770.	10 d.	100 y.	1770.	10 d.	100 y.	1770.	10 d.	100 y.	1770.	10 d.	100 y.
		″			″			″			″			″		
0	Jan. 0	−0.20	+ 6	−21	−0.62	+ 1	−21	−0.89	− 1	−21	−1.19	−16	−19	−1.57	−42	−14
10	10	−0.10	12	22	0.58	7	22	0.87	+ 6	22	1.32	8	20	1.96	32	15
20	20	+0.03	15	22	0.48	12	23	0.77	12	24	1.36	− 1	21	2.22	22	18
30	30	0.19	16	19	0.35	15	22	0.64	16	22	1.34	+ 5	21	2.41	14	17
40	Feb. 9	+0.35	+15	−19	−0.19	+15	−20	−0.46	+18	22	−1.27	+ 9	20	2.50	6	−19
50	19	0.50	13	18	−0.04	14	18	0.30	17	20	1.16	11	20	2.54	− 1	18
60	Mar. 1	+0.60	+ 8	−16	+0.09	+11	−16	−0.14	+14	−17	−1.05	+11	18	2.52	+ 3	18
70	11	0.65	+ 2	13	0.17	6	14	−0.03	9	15	0.95	9	17	2.47	6	16
80	21	0.63	− 4	10	0.21	+ 1	12	+0.04	+ 4	13	0.88	6	15	2.42	6	13
90	31	0.56	10	8	0.19	− 5	10	+0.05	− 2	10	0.84	+ 2	12	2.35	7	12
100	April 10	+0.43	−15	− 5	+0.11	−10	− 8	0.00	− 7	− 7	−0.83	0	10	2.27	+ 8	−11
110	20	0.26	18	3	0.00	13	6	−0.09	10	5	0.84	− 3	9	2.18	10	11
120	30	+0.06	20	1	−0.14	14	4	0.21	12	4	0.88	3	6	2.08	12	9
130	May 10	−0.13	−19	− 1	−0.29	−14	− 2	−0.35	−13	− 1	−0.91	− 2	− 5	−1.96	+14	− 8
140	20	0.31	16	0	0.42	12	2	0.47	11	2	0.92	0	5	1.80	18	7
150	30	0.44	11	1	0.52	8	2	0.58	8	2	0.91	+ 3	4	1.60	22	7
160	June 9	−0.52	− 5	− 2	−0.57	− 3	− 2	−0.64	− 4	− 2	−0.85	+ 8	− 5	−1.34	+28	− 8
170	19	0.54	+ 2	3	0.57	+ 3	4	0.67	+ 1	2	0.75	13	5	1.05	32	8
180	29	0.48	9	6	0.51	9	6	0.63	6	5	0.60	17	7	0.71	36	9
190	July 9	−0.36	+15	− 8	−0.39	+14	− 8	−0.56	+10	− 6	−0.41	+21	− 9	−0.32	+39	−11
200	19	−0.18	21	12	0.23	18	11	0.43	14	11	−0.18	24	13	+0.06	40	11
210	29	+0.04	24	15	−0.03	21	14	0.29	16	13	+0.06	24	15	0.46	38	14
220	Aug. 8	+0.29	+25	−19	+0.18	+21	−18	−0.13	+16	−16	+0.29	+22	−18	+0.82	+34	−16
230	18	0.54	24	23	0.39	20	22	+0.03	14	20	0.50	18	21	1.15	28	21
240	28	0.77	22	27	0.58	16	26	0.15	10	25	0.65	12	24	1.38	18	23
250	Sept. 7	+0.97	+18	−32	+0.71	+10	−30	+0.22	+ 4	−28	+0.74	+ 5	−28	+1.51	+ 7	−25
260	17	1.12	12	37	0.77	+ 4	35	0.24	− 2	33	0.75	− 5	32	1.52	− 6	29
270	27	1.20	+ 5	41	0.79	− 2	39	0.18	10	38	0.65	14	35	1.42	20	34
280	Oct. 7	+1.22	− 1	−46	+0.72	−10	−44	+0.05	−16	−43	+0.48	−23	−41	+1.14	−34	−37
290	17	1.18	7	52	0.59	16	48	−0.15	22	47	+0.20	32	45	0.75	45	40
300	27	1.08	11	56	0.40	21	53	0.41	26	50	−0.16	38	48	+0.25	55	43
310	Nov. 6	+0.96	−13	−62	+0.17	−23	−57	−0.69	−29	−55	−0.56	−42	−53	−0.34	−62	−46
320	16	0.82	14	65	−0.07	23	61	0.99	28	57	1.00	44	56	0.99	66	50
330	26	0.69	12	68	0.30	21	65	1.27	25	61	1.43	41	59	1.66	66	53
340	Dec. 6	+0.59	− 8	−71	−0.50	−17	−68	−1.50	−20	−66	−1.82	−37	−64	−2.31	−63	−57
350	16	0.53	− 2	72	0.64	11	71	1.68	13	68	2.16	31	67	2.93	57	58
360	26	0.54	+ 4	74	0.71	− 4	73	1.78	− 6	72	2.44	22	68	3.46	49	62
370	36	+0.61	+ 9	−75	−0.71	+ 3	−74	−1.80	+ 2	−75	−2.60	−12	−72	−3.91	−39	−63

TABLE IX.—*Annual Term in Declination for Epoch* 1770—Continued.

	$\Delta_{\odot}\delta$	ε GEMINORUM.			ζ GEMINORUM.			β DRACONIS.			γ DRACONIS.		
	Sid. Day.	1770.	10 d.	100 y.	1770.	10 d.	100 y.	1770.	10 d.	100 y.	1770.	10 d.	100 y.
		″			″			″			″		
0	Jan. 0	1.56	+11	−24	−2.30	−24	−22	−5.33	352	+18	−3.51	−352	+18
10	10	1.40	20	26	2.48	−11	22	8.78	334	17	6.99	339	18
20	20	1.16	27	27	2.53	0	23	12.00	310	16	10.27	314	17
30	30	0.86	32	27	2.48	+10	24	14.86	264	15	13.28	279	18
40	Feb. 9	−0.53	+33	−26	−2.35	+16	−25	−17.27	−214	+14	−15.84	−231	+14
50	19	−0.19	33	25	2.16	20	24	19.10	153	10	17.91	183	12
60	Mar. 1	+0.12	+28	−22	−1.94	+22	−23	−20.34	91	+9	−19.39	117	+8
70	11	0.38	22	20	1.71	22	22	20.93	26	5	20.25	−53	6
80	21	0.57	16	17	1.51	19	19	20.86	+40	3	20.46	+12	4
90	31	0.68	+6	15	1.33	16	18	20.14	+102	+1	20.02	76	+2
100	April 10	+0.70	−2	−12	−1.20	+11	−15	−18.83	+159	−1	−18.94	+136	−1
110	20	0.63	10	8	1.10	7	14	16.96	210	3	17.31	188	3
120	30	0.50	16	6	1.05	4	11	14.63	253	5	15.17	236	4
130	May 10	+0.31	−20	−3	−1.03	+2	−9	−11.92	+286	−6	−12.61	+273	−6
140	20	+0.10	22	2	1.02	0	8	8.94	308	5	9.73	300	6
150	30	−0.13	23	2	1.03	0	7	5.78	321	5	6.61	319	7
160	June 9	−0.36	−22	−1	−1.02	+1	−7	−2.53	+325	−5	−3.37	+327	−6
170	19	0.58	21	1	1.01	2	7	+0.70	318	5	−0.10	326	4
180	29	0.77	17	2	0.98	4	7	3.82	303	3	+3.14	316	5
190	July 9	−0.92	−14	−4	−0.94	+5	−9	+6.76	+281	−3	+6.22	+297	−4
200	19	1.04	11	7	0.89	4	11	9.42	251	0	9.08	273	−2
210	29	1.14	10	10	0.85	+3	−13	11.78	216	0	11.66	242	0
220	Aug. 8	−1.24	−10	−13	−0.83	0	−15	+13.75	+176	+3	+13.89	+206	+2
230	18	1.34	11	17	0.86	−4	17	15.30	132	5	15.75	166	4
240	28	1.46	13	20	0.92	11	22	16.40	85	8	17.17	118	7
250	Sept. 7	−1.60	−16	−26	−1.09	−21	−24	+17.01	+36	+10	+18.12	+73	+10
260	17	1.79	21	30	1.34	30	27	17.13	−14	12	18.59	+21	12
270	27	2.02	26	35	1.67	38	31	16.73	66	15	18.54	−30	15
280	Oct. 7	−2.30	−30	−40	−2.10	−49	−34	+15.82	−116	+21	+17.98	−81	+19
290	17	2.62	33	45	2.61	56	40	14.41	165	22	16.92	132	21
300	27	2.96	34	50	3.20	61	44	12.52	212	26	15.35	180	26
310	Nov. 6	−3.30	−33	−56	−3.83	−64	−48	+10.17	−255	+28	+13.31	−226	+29
320	16	3.63	30	60	4.47	63	53	7.43	292	31	10.83	267	32
330	26	3.89	24	65	5.09	59	57	4.34	323	34	7.98	301	33
340	Dec. 6	−4.11	−16	−68	−5.65	−52	−61	+0.99	−345	+35	+4.81	−329	+35
350	16	4.21	−6	73	6.13	43	65	−2.54	357	37	+1.42	346	37
360	26	4.22	+6	75	6.51	31	68	6.13	358	37	−2.10	353	38
370	36	−4.13	+15	−77	−6.76	−19	−71	−9.68	−348	+37	−5.63	−348	+38

TABLE X.—*Approximate Corrections of Quadrant from* 1767 *to* 1787.

Star's name.	Declination.		Correction.			Annual Variation.
	1767.	1787.	1767.	1777.	1787.	
	°	°	″	″	″	″
β Draconis	52.48	52.47	. .	. .	. .	. .
γ Draconis	51.52	51.52	. .	. .	. .	. .
Capella	45.73	45.76	. .	. .	. .	. .
α Cygni	44.46	44.53	. .	. .	. .	. .
α Lyræ	38.58	38.59	. .	. .	. .	. .
α Geminorum	32.38	32.34	−2.27	−3.40	−4.53	−0.113
β Geminorum	28.57	28.52	2.00	3.00	3.99	0.100
β Tauri	28.39	28.41	1.99	2.98	3.98	0.100
α Andromedæ	27.80	27.91	1.95	2.93	3.91	0.098
α Coronæ	27.51	27.44	1.93	2.88	3.84	0.096
ε Geminorum	25.34	25.31	−1.77	−2.66	−3.54	−0.089
Π Geminorum	23.25	23.26	1.63	2.44	3.25	0.081
μ Geminorum	22.61	22.60	1.58	2.37	3.16	0.079
η Geminorum	22.55	22.55	1.58	2.37	3.16	0.079
α Arietis	22.35	22.45	1.56	2.35	3.14	0.079
ζ Geminorum	20.90	20.84	−1.46	−2.19	−2.92	−0.073
α Bootis	20.40	20.30	1.43	2.14	2.84	0.071
ν Geminorum	20.34	20.33	1.42	2.14	2.85	0.071
γ Geminorum	16.58	16.55	1.16	1.74	2.32	0.058
α Tauri	16.02	16.07	1.12	1.69	2.25	0.056
β Leonis	15.87	15.76	−1.11	−1.66	−2.21	−0.055
α Herculis	14.67	14.65	1.03	1.54	2.05	0.051
α Pegasi	13.95	14.06	0.98	1.47	1.97	0.050
γ Pegasi	13.89	14.00	0.97	1.46	1.96	0.049
ξ Geminorum	13.13	13.11	0.92	1.38	1.84	0.046
α Leonis	13.10	13.00	−0.92	−1.37	−1.82	−0.045
α Ophiuchi	12.75	12.73	0.89	1.34	1.78	0.044
γ Aquilæ	10.06	10.11	0.70	1.06	1.42	0.036
α Aquilæ	8.27	8.32	0.58	0.87	1.16	0.029
α Orionis	7.34	7.35	0.51	0.77	1.03	0.026
α Serpentis	7.17	7.11	−0.50	−0.75	−1.00	−0.025
β Aquilæ	5.84	5.89	0.41	0.62	0.82	0.021
Procyon	5.80	5.76	0.41	0.61	0.81	0.020
α Ceti	3.16	3.24	0.22	0.34	0.45	0.011
β Virginis	3.08	2.96	−0.22	−0.32	−0.41	−0.009
α Aquarii	−1.44	−1.35	+0.10	+0.15	+0.19	+0.005
α Hydræ	7.66	7.74	0.54	0.81	1.08	0.027
β Orionis	8.49	8.46	0.59	0.89	1.18	0.030
α Virginis	9.94	10.04	0.70	1.05	1.41	0.035
α Capricorni	13.23	13.17	0.93	1.39	1.84	0.046
α Libræ	−15.04	−15.12	+1.05	+1.58	+2.12	+0.053
Sirius	16.41	16.44	1.15	1.73	2.31	0.058
Antares	25.89	25.94	1.81	2.72	3.63	0.091
α Piscis Austrini	−30.85	−30.75	+2.16	+3.23	+4.30	+0.107

TABLE XI.—*Observations of Fundamental Stars, 1765 to 1812, corrected approximately* for Errors of the Quadrant.*

Star's name.	Period 1.			Period 2.			Period 3.			Period 4.		
	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe
	$''$		$''$	$''$		$''$	$''$		$''$	$''$		$''$
α Aurigæ	42.43	7	3.0	42.7	1	. .	. . .	.	. .	39.77	6	14.0
α Cygni	40.18	5	2.7	38.61	8	15.9	40.0	1	. .	40.12	5	5.1
α Lyræ	39.07	7	14.0	42.13	3	5.7	41.57	9	9.9	44.87	3	2.3
α Geminorum	. . .	.	. .	. . .	.	. .	39.78	3	6.9	. . .	.	. .
β Geminorum	. . .	.	. .	43.54	1	. .	40.67	3	3.5	. . .	.	. .
β Tauri	. . .	.	. .	. . .	.	. .	39.68	3	3.1	. . .	.	. .
α Andromedæ	. . .	.	. .	40.89	1	. .	36.39[5]	3	7.3	37.35	3	3.0
α Coronæ	40.90	3	3.2	40.40	1	. .	. . .	.	. .	38.78	1	. .
α Arietis	. . .	.	. .	40.85	1	. .	36.13	2	2.2	. . .	.	. .
α Bootis	43.15	15	20.0	. . .	.	. .	39.83	5	5.3	38.93	1	. .
α Tauri	41.17	1	. .	40.51	6	8.1	38.81	3	5.9	. . .	.	. .
β Leonis	. . .	.	. .	. . .	.	. .	39.55	3	9.0	. . .	.	. .
α Herculis	43.35	1	. .	39.84	2	3.0	. . .	.	. .	. . .	.	. .
α Pegasi	. . .	.	. .	38.34	2	1.1	. . .	.	. .	. . .	.	. .
γ Pegasi	. . .	.	. .	. . .	.	. .	37.75	3	5.1	39.00	2	1.7
α Leonis	42.60	2	2.5	. . .	.	. .	41.53	2	1.9	. . .	.	. .
α Ophiuchi	39.73	2	1.3	37.83	2	2.5	41.76	1	. .	38.44	1	. .
γ Aquilæ	41.13	3	7.9	. . .	.	. .	. . .	.	. .	. . .	.	. .
α Aquilæ	40.56	7	7.7	43.04	6	5.6	39.57	13	14.5	41.75	5	2.5
Orionis	42.23	3	4.4	39.23	2	9.2	36.80	6	4.3	36.75	1	. .
β Aquilæ	40.32	1	. .	40.12	2	2.6	. . .	.	. .	. . .	.	. .
α Serpentis	39.59	3	2.2	. . .	.	. .	. . .	.	. .	. . .	.	. .
α Canis Minoris	42.52[1]	1	. .	. . .	.	. .	40.16	2	4.3	. . .	.	. .
α Ceti	. . .	.	. .	. . .	.	. .	39.09	3	4.2	. . .	.	. .
β Virginis	. . .	.	. .	. . .	.	. .	40.39	1	. .	. . .	.	. .
α Aquarii	. . .	.	. .	. . .	.	. .	. . .	.	. .	. . .	.	. .
α Hydræ	. . .	.	. .	39.96	3	2.9	39.21	3	2.2	. . .	.	. .
β Orionis	. . .	.	. .	. . .	.	. .	37.78	4	4.5	. . .	.	. .
α Virginis	41.50	10	14.1	41.73[3]	6	10.5	41.26	1	. .	. . .	.	. .
α^1, α^2 Capricorni	. . .	.	. .	. . .	.	. .	38.73	8	4.2	36.51	1	. .
α^1, α^2 Libræ	40.45	7	7.1	. . .	.	. .	39.10	1	. .	36.56	1	. .
α Canis Majoris	39.51	6	8.0	37.87	2	2.8	41.54	4	13.8	37.25	1	. .
α Scorpii	41.10	6	7.8	38.07	1	. .	40.23	2	3.6	. . .	.	. .
α Piscis Austrini	. . .	.	. .	39.31[4]	1	. .	40.12	7	13.2	. . .	.	. .
Mean by weights[2] [from $+33°$ to $-31°$]	41.19	.	. .	40.36	.	. .	39.40[6]	.	. .	38.46	.	. .

* Except for stars from α Aurigæ to α Cygni.

[1] 1765, June 2, gives 51″.2, or corrected, 50″.82, and is rejected.

[2] 1–4 observations wt. = 1.

[2] 5–15 observations wt. = 2.

[3] An omitted observation gives 47″.55, and was made 1765, July 25.

[4] Another observation, July 11, gives 31″.51.

[5] The single observations give 37″.86, 38″.56, 32″.76; excluding the last, the mean will be 38″.21. This value was used in taking the mean at the foot of the column.

TABLE XI.—*Observations of Fundamental Stars*, 1765 *to* 1812, *etc.*—Continued.

Star's name.	Period 5[1].			Period 16[a2].			Period 19[b4].			Period 20.		
	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe
	″		″	″		″	″		″	″		″
α Aurigæ	. . .	.	. .	42.8	2	0.0	. . .	.	. .	. . .	.	. .
α Cygni	41.8	1	. .	. . .	.	. .	37.5	1	. .	38.62	5	5.9
α Lyræ	. . .	.	. .	. . .	.	. .	32.4	1	. .	38.5	1	. .
α Geminorum	37.09	2	1.4	39.22	3	2.3	. . .	.	. .	34.17	1	. .
β Geminorum	39.06	3	2.8	36.93	2	2.1	. . .	.	. .	33.06	2	4.7
β Tauri	. . .	.	. .	. . .	.	. .	. . .	.	. .	. . .	.	. .
α Andromedæ	. . .	.	. .	. . .	.	. .	31.44	2	4.1	28.19	1	. .
α Coronæ	. . .	.	. .	37.50	2	2.0	34.39	3	5.7	. . .	.	. .
α Arietis	. . .	.	. .	. . .	.	. .	35.36	1	. .	35.06	1	. .
α Bootis	38.24	3	2.3	40.57	3	0.3	33.26	5	7.2	35.06	1	. .
α Tauri	. . .	.	. .	. . .	.	. .	. . .	.	. .	. . .	.	. .
β Leonis	. . .	.	. .	39.82	3	1.5	34.59	1	. .	. . .	.	. .
α Herculis	. . .	.	. .	40.79	3	0.9	31.70	2	0.7	. . .	.	. .
α Pegasi	. . .	.	. .	. . .	.	. .	. . .	.	. .	32.93	1	. .
γ Pegasi	. . .	.	. .	. . .	.	. .	. . .	.	. .	(19.84)	1	. .
α Leonis	37.67	2	6.9	39.28	3	1.1	34.58	3	4.2	. . .	.	. .
α Ophiuchi	. . .	.	. .	40.01	3	2.9	30.77	2	0.7	35.22	1	. . .
γ Aquilæ	. . .	.	. .	. . .	.	. .	30.08	1	. .	32.28	1	. .

[1] This period has been extended to include September 27, but the observations of β Geminorum on September 25 and 26 have been excluded. They give 32″.33 and 29″.73, respectively.

After this period follows a time of great irregularity. [a] The fundamental stars give the following values, in which are included the few observations made in period 6. Some 'prentice hand seems to be here indicated.

		″			″
1773, Mar. 24	α Tauri	23.3	1773, Mar. 24	α Cygni	26.1
	α Aurigæ	26.7	Aug. 24	α Aurigæ	47.2
	α Orionis	35.8	Sept. 6	α Aurigæ	41.7
	α Canis Minoris	17.5		α Canis Majoris	39.2
	β Geminorum	15.1		α Canis Minoris	38.5
	α Leonis	20.4	Nov. 1	α Tauri	43.8
	α Aquilæ	18.14			

[2] The number of fundamental stars up to period 16 is small; they give results as follows, when properly reduced:

	Name of Star.		No. obs.	
		″		
1775, Dec. 12.	α Leonis	38.98	1	. .
1776, Mar. 31.	α Leonis	36.26	1	10
Nov. 17–24	α Aurigæ	33.0	2	11
1777, Jan. 11–28	α Canis Majoris	34.58	4	12
Aug. 9–Oct. 4	α Aurigæ	33.65	4	13
1778, June 24–28	α Bootis	34.66	4	14
July 8–Aug. 8	α Herculis	33.96	5	14
1779, April 20–24	α Canis Minoris	35.24	2	14
April 20	β Orionis	34.16	1	14
April 20	β Tauri	34.89	1	14
April 20–24	β Geminorum	33.67	2	14
1781, Sept. 25.	α Geminorum	41.3	1	15
	β Geminorum	41.5	1	15

TABLE XI.—*Observations of Fundamental Stars, etc.*—Continued.

Star's name.	Period 5[1].			Period 16a[2].			Period 19b[4].			Period 20.		
	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe
	″		″	″		″	″		″	″		″
α Aquilæ	39.50	2	1.1	. .	.	. .	29.64	1	. .	30.98	9	15.5
α Orionis	. .	.	. .	41.77	2	2.2	33.47	2	4.0	38.27	1	. .
β Aquilæ	. .	.	. .	. .	.	. .	31.38	1	. .	33.63	2	. .
α Serpentis	. .	.	. .	39.63	2	1.0	31.43	3	2.5	. .	.	. .
α Canis Minoris	39.80	4	3.9	39.33	3	1.5	33.49	1	. .	31.89	1	. .
α Ceti	. .	.	. .	. .	.	. .	. .	.	. .	. .	.	. .
β Virginis	40.00	2	4.3	39.29	3	1.5	. .	.	. .	. .	.	. .
α Aquarii	. .	.	. .	. .	.	. .	. .	.	. .	27.59	1	. .
α Hydræ	. .	.	. .	38.71	3	1.7	. .	.	. .	. .	.	. .
β Orionis	. .	.	. .	41.77	1	. .	. .	.	. .	. .	.	. .
α Virginis	. .	.	. .	38.46	3	1.5	34.39	5	2.7	. .	.	. .
α¹, α² Capricorni	36.94	1	. .	. .	.	. .	30.56	1	. .	30.91	2	. .
α¹, α² Libræ	37.83	1	. .	38.03	3	3.2	30.37	2	3.7	. .	.	. .
α Canis Majoris	. .	.	. .	40.02[3]	3	11.3	31.16	4	3.2	25.91	1	. .
α Scorpii	. .	.	. .	. .	.	. .	30.43	4	2.4	. .	.	. .
α Piscis Austrini	. .	.	. .	. .	.	. .	34.60	1	. .	34.30[8]	2	. .
										19.85	2	. .
	38.46	.	. .	39.48	.	. .	32.48	.	. .	32.38	.	. .

For the 15 observations of 6 stars in period 14, we find as mean by weights, 34″.36 wt. = 7. The probable error for weight = 1 is ± 0″.48.

I have divided period 16, ending the first division 16a, on September 26, 1783. This is of some consequence. The few observations between September 26, 1783, and September 9, 1784, do not agree well among themselves. They are these:

			″
1783,	Nov. 24	α Lyræ	47.0
	Nov. 28	α Lyræ	46.5
	Dec. 8	α Aquilæ	38.43
		α Cygni	45.7
1784,	Jan. 3	α Pegasi	40.58
	Jan. 5	α Pegasi	42.18
	Mar. 16	α Arietis	45.98
		α Ceti	45.5
	Sept. 9	α Scorpii	43.3

[3] The three values, of which this is the mean, are, respectively, 34″.4, 42″.3, 43″.4.

[4] The instrument seems to have changed irregularly between 1787, March 6, and 1787, June 14. I have begun period 19b with the latter date. The observations before this time are:

		Name of Star.	p'	N.
			″	
1787,	Mar. 6–7	α Coronæ	29.06	2
	Mar. 20–May 3	α Aurigæ	37.58	5
	May 8–14	α Aurigæ	45.90	3
	May 16	α Aurigæ	39.07	1
	May 28–June 2	α Coronæ	30.06	3

MASKELYNE altered the position of the quadrant June 14.

TABLE XI.—*Observations of Fundamental Stars, etc.*—Continued.

Star's name.	Period 25[5].			Period 26.			Period 27.			Period 28[8].		
	p'	N.	Σe	p'	N.	Σe	p°	N.	Σe	p'	N.	Σe
	$''$		$''$	$''$		$''$	$''$		$''$	$''$		$''$
α Aurigæ	. .	.	. .	37.83	3	0.7	37.8	2	0.6	40.36	5	7.6
α Cygni	41.1	1	. .	39.33	3	0.9	41.05[8]	8	16.4	38.3	1	. .
α Lyræ	. .	.	. .	38.96[6]	5	5.0	39.40	3	2.2	40.2	1	. .
α Geminorum	34.17	1	. .	31.77	2	0.6	32.85	8	11.6	33.17	1	. .
β Geminorum	35.52	2	7.6	32.01	2	1.0	31.36	6	6.5	33.11	1	. .
β Tauri	. .	.	. .	28.92	1	. .	31.64	8	10.0	31.42	1	. .
α Andromedæ	35.59	1	. .	30.29	1	. .	31.54	4	2.1	32.39	1	. .
α Coronæ	35.96	2	2.8	31.81	2	1.7	32.76	4	4.4	34.06	2	0.6
α Arietis	. .	.	. .	31.36	2	0.8	31.66	5	3.4	. .	.	. .
α Bootis	34.17	11	9.3	32.98	10	8.0	34.27	7	8.7	35.86	3	2.2
α Tauri	31.60	2	1.7	32.95	2	0.2	33.43	6	6.5	. .	.	. .
β Leonis	31.89	1	. .	31.89	1	. .	33.62	3	0.2	35.19	4	1.4
α Herculis	33.35	3	1.6	29.33	10	17.4	. .	.	. .	33.25	3	2.8
α Pegasi	. .	.	. .	30.43	3	2.4	30.08	2	0.9	31.23	2	0.4
γ Pegasi	. .	.	. .	31.94	2	1.0	. .	.	. .	. .	.	. .
α Leonis	30.88	3	7.2	32.38	4	2.2	32.56	8	5.2	. .	.	. .
α Ophiuchi	33.04	4	5.5	31.58	7	5.9	. .	.	. .	31.58	7	5.9

[5] In period 21 α Tauri was once observed, 1792, October 31, giving 33″.85. Period 15 extends over 3½ years, viz, from 1796, July 6, to 1799, December 31. The stars in Gemini, during this time, exhibit slight variations only, not at all comparable with the irregularities of the fundamental stars; and these last sometimes contradict the former, so that, with some hesitation, I have retained the period as a whole.

The few observations of period 23, inclusive, are as follows:

	Name of Star.	q'		Name of Star.	q'
		$''$			$''$
1794, Mar. 7	α Tauri	34.65	1795, Jan. 22–25[3]	α Canis Majoris	27.68
June 20	α Coronæ	32.66	Feb. 2–25[b]	α Canis Majoris	31.67
	α Serpentis	33.00	Aug. 5	α Canis Minoris	31.89
	α² Libræ	32.12	Aug. 6	α Coronæ	33.26
	α Scorpii	31.13		α Bootis	35.36
June 23	α Bootis	33.76		α Serpentis	37.5
	α Virginis	33.21		α Virginis	32.31
Nov. 8	α Tauri	30.85		α² Libræ	39.72
				α Scorpii	32.23
			Aug. 17	α Aurigæ	45.1

It appears from this table that the mean value of p' for the period 23 is about 32″.7 ± 0″.4 by the fundamental stars, but that three observations of α Canis Majoris during January, 1795, which gave, respectively, 29″.3, 26″.4, 27″.3, seem to indicate a change in the instrument, but are immediately contradicted by most of those in February, which give in their order 31″.2, 29″.0 32″.0, 33″.1, 33″.0. The temperature in January was exceptionally low.

[6] Excluding 1800, July 13, which gives 46″.2.

[7] There is a discrepancy between the three observations 1801, January 4, December 27, and 1802, January 1; their mean is 43″.63, and that of the five others 39″.50.

TABLE XI.—*Observations of Fundamental Stars, etc.*—Continued.

Star's name.	Period 25[5].			Period 26.			Period 27.			Period 28[10].		
	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe
	$''$		$''$	$''$		$''$	$''$		$''$	$''$		$''$
γ Aquilæ	. . .	.	. .	34.05	3	0.3	. . .	.	. .	30.50	4	1.9
α Aquilæ	33.99	2	0.5	32.34	6	2.6	32.94	3	. .	32.19	2	1.1
α Orionis	32.95[6]	5	5.8	30.07	1	. .	33.33	10	10.8	. . .	.	. .
β Aquilæ	. . .	.	. .	33.31	3	2.3	. . .	.	. .	32.18	6	4.8
α Serpentis	35.10	3	2.6	34.60	1	. .	32.98	5	2.7	32.95	2	2.1
α Canis Minoris	32.79	3	4.0	30.46	3	2.5	32.44	6	11.8	30.99	2	0.0
α Ceti	. . .	.	. .	33.75[8]	1	. .	31.18	4	2.9	. . .	.	. .
β Virginis	. . .	.	. .	. . .	.	. .	33.36	3	1.1	33.29	2	0.2
α Aquarii	32.44	2	6.5	31.32	3	1.1	31.89	3	4.2	. . .	.	. .
α Hydræ	33.28	1	. .	32.68	2	2.2	33.38	7	8.6	. . .	.	. .
β Orionis	32.58	1	. .	30.88	1	. .	32.00	6	8.1	. . .	.	. .
α Virginis	31.51	3	4.6	33.41	2	0.0	30.06	2	0.9	32.51	2	1.6
α^1, α^2 Capricorni	33.86	2	. .	32.22	8	7.6	. . .	.	. .	30.17	12	16.7
α^1, α^2 Libræ	. . .	.	. .	. . .	.	. .	32.62	2	2.8	32.06	11	12.1
α Canis Majoris	33.83[3]	6	4.1	34.41	1	. .	33.12	13	20.1	31.71	1	. .
α Scorpii	28.73	1	. .	30.85	4	5.5	31.33	2	0.4	33.80	3	3.3
α Piscis Austrini	(43.20)	1	. .	32.10[9]	2	0.2	32.70	1	. .	31.30	3	8.8
Mean by weights	33.29	.	. .	31.90	.	. .	32.49	.	. .	32.34	.	. .

[8] Excluding 1800, November 29, which gives 23″.0.

[9] With some hesitation I have excluded 1800, July 9, which gives 24″.3. The probable error of one refraction is here ± 1″.1; but on the whole this deviation is more likely to arise from the plumb-line than from the atmosphere.

[10] During the periods which follow 28, that is, between 1803 and Maskelyne's decease in 1811, we have the following results from fundamental stars, the northernmost three uncorrected as before.

	Name of Star.	p'	N.
		$''$	
Period 29. 1803, Jan. 8—1804, Dec. 13	α Bootis	33.14	22
	α Canis Majoris	31.67	3
Period 30. 1804, Dec. 14—1805, Mar. 17	α Bootis	31.81	2
Period 31. 1805, June 24—Sept. 29	α Bootis	34.64	6
	γ Aquilæ	29.18	2
	α Aquilæ	30.24	3
	β Aquilæ	31.71	3
Period 32. 1805, Oct. 2—1806, Feb. 27	α Cygni	38.4	1
	α Lyræ	41.12	6
	α Ophiuchi	32.58	5
	α Aquilæ	28.85	6
	Fomalhaut	29.90	1
Period 33. 1806, May 25—Dec. 12	α Aurigæ	38.0	1
	α Cygni	37.36	10
	α Lyræ	41.19	8
	α Bootis	31.79	6

In the last period α Lyræ was also observed twice, 1806, August 23 and August 24, with results 52″.9 and 51″.3, respectively; but there are here either errors of 10″ or of 13″.2, or some derangement of the plumb-line. If, in addition, the observation of August 31 be excluded (it gives 46″.1) the average of the remaining seven observations would be 40″.49.

TABLE XI.—*Observations of Fundamental Stars, etc.*—Continued.

Star's name.	Period 36. p'	Period 36. N.	Star's name.	Period 36. p'	Period 36. N.
	$''$			$''$	
α Aurigæ	41.75	34	γ Aquilæ	37.64	40
α Cygni	42.96	53	α Aquilæ	37.78	60
α Lyræ	43.53	29	α Orionis	38.08	31
α Geminorum	38.45	33	α Serpentis	37.75	35
β Geminorum	37.24	31	β Aquilæ	37.61	35
β Tauri	37.06	25	α Canis Minoris	37.70	35
α Andromedæ	35.93	30	α Ceti	38.27	18
α Coronæ	38.08	44	α Aquarii	37.62	41
α Arietis	37.45	29	α Hydræ	37.90	30
α Bootis	37.47	70	β Orionis	38.14	31
α Tauri	37.83	31	α Virginis	38.45	47
β Leonis	39.04	28	α^1, α^2 Capricorni	36.33	$31\alpha^1.34\alpha^2$
α Herculis	37.88	33	α^1, α^2 Libræ	36.35	12
α Pegasi	36.51	33	α Canis Majoris	37.52	37
γ Pegasi	36.30	28	α Scorpii	38.27	41
α Leonis	37.34	31	α Piscis Austrini	37.69	41
α Ophiuchi	37.26	34	Mean + 33° to — 31°	37.56	..

This period, not discussed by OLUFSEN, contains the observations under POND'S direction in the years 1811 and 1812. I have not subdivided it, although this would have improved the agreement. The general accordance south of + 33° shows that the adopted correction for this region is sufficient for our purpose. For the three northern stars no correction has been applied.

TABLE XII.—*Observations of Stars in Gemini, 1767 to 1806, corrected approximately for Errors of the Quadrant.*

No. of Period.	ε Geminorum.			ι Geminorum.			μ Geminorum.			η Geminorum.		
	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe
	″		″	″		″	″		″	″		″
4	37.29	10	16.7	38.36	8	5.9	37.97	12	16.1	38.19	10	12.2
5	38.07	10	12.5	38.01	6	9.5	39.33	10	12.0	37.76	8	11.0
6	40.73	3	1.4	41.23	2	0.8	40.48	4	5.0	38.98	5	3.7
7	44.94	4	11.5	43.52	3	2.1	45.13[5]	5	15.5	43.43	4	5.3
8	49.39[1]	4	6.8	48.64[2]	1	. .	49.12[6]	5	6.3	49.33	3	2.3
9	30.13	5	5.7	32.30	3	2.1	30.78	5	3.5	31.70	4	3.4
10	33.46	11	10.8	34.20	5	5.6	35.38	11	17.5	34.55	8	10.4
11	. . .	.	. .	33.18	2	2.2	. . .	.	. .	32.45	1	. .
12	32.72	5	5.4	33.71	5	3.2	33.76	5	3.2	33.46	5	3.4
13	30.96	4	4.9	32.18	5	4.1	32.00	4	2.3	32.27	4	4.4
14	33.81	10	8.9	32.27	10	9.5	34.81	11	6.9	33.61	10	9.2
15	40.96	5	10.2	43.70[3]	5	9.7	42.77	5	6.5	42.10[8]	4	1.4
16[a]	37.43	6	3.3	37.76	7	11.7	39.97	7	3.9	38.83	7	2.8
17	31.89	3	1.3	34.50	3	0.8	34.95	3	0.3	32.98	3	3.0
18	34.70	3	1.7	34.59	3	1.9	36.78	3	1.3	36.71	3	1.2
20	31.16	10	10.2	31.74	9[4]	13.0	32.94	10	14.2	31.81	10	9.3
21	33.56	5	3.8	32.20	2	1.5	34.74	5	9.6	34.92	5	8.3
22	28.06	3	2.2	30.92	4	5.1	28.46	4	4.5	29.11	3	2.1
23	30.30	14	12.8	31.80	14	18.2	32.20	14	19.5	32.09	13	24.0
24	29.40	5	2.8	29.75	5	3.8	29.77[7]	3	0.9	29.00	5	4.4
25	32.72	9	5.2	33.13	5	1.9	34.31	10	8.1	33.12	9	9.2
26	29.73	3	2.9	30.88	3	2.1	31.47	3	1.9	29.17	3	3.3
27	. . .	.	. .	. . .	.	. .	. . .	.	. .	. . .	.	. .
28	. . .	.	. .	. . .	.	. .	. . .	.	. .	. . .	.	. .
29	. . .	.	. .	. . .	.	. .	. . .	.	. .	. . .	.	. .
30	33.22	5	1.0	32.73	5	6.3	33.96	5	8.5	32.26	5	7.7
31	31.76	2	0.0	30.75	2	0.0	31.29	2	1.5	31.49	2	2.1
32	32.74	4	5.5	32.48	4	8.7	34.86	4	6.1	33.14	4	3.8
33	31.56	1	. .	38.95	1	. .	30.74	1	. .	. . .	.	. .

[1] Omitting 1774, October 10, which gives 39″.2.

[2] Omitting 1774, which gives 40″.0.

[3] Omitting 1781, October 3, which gives 47″.9, we obtain 42″.65, 4.

[4] Four observations 1788, October 4, March 4, November 9, November 15, give in mean 44″.13. The last observation comes in period 21. OLUFSEN had remarked the discrepancy as probably due to the plumb-line.

[5] Respective data, 48″.5, 49″.6, 43″.8, 43″.4, 40″.4.

[6] 1774, October 10, gives 42″.5, and is excluded.

[7] 1795, October 24, gives 35″.8, and is excluded with hesitation.

[8] 1781, October 3 gives 48″.95, and is excluded.

TABLE XII.—*Observations of Stars in Gemini, etc.*—Continued.

No. of Period.	ζ GEMINORUM.			ν GEMINORUM.			γ GEMINORUM.			ξ GEMINORUM.		
	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe	p'	N.	Σe
	″		″	″		″	″		″	″		″
4	. .	.	. .	36.80	11	15.7	36.68	11	13.4	37.15	7	12.4
5	41.30	4	2.8	35.75	8	8.8	37.09	10	15.4	36.62	9	9.3
6	. .	.	. .	40.30[1]	3	3.8	40.45	3	4.4	40.57	2	1.6
7[6]	42.49	3	1.9	41.76	2	3.2	42.90	6	5.5	42.55	1	. .
8	50.84	3	4.5	49.07[2]	4	2.5	49.09[5]	4	5.2	48.54[7]	4	5.9
9	34.29	4	3.5	30.05[3]	4	3.1	31.49	5	3.1	30.46	5	3.2
10	37.18	4	3.7	33.74	5	6.1	34.91	12	12.4	34.50	8	12.1
11	34.85	3	1.7	31.61	5	2.2	. .	.	. .	32.78	2	0.9
12	34.98	5	2.3	32.17	5	2.7	34.25	5	2.0	33.63	5	5.7
13	34.25	1	. .	30.36	4	4.4	33.12	4	6.2	31.74	4	2.8
14	34.91	8	7.7	32.67	10	9.4	34.22	10	9.2	33.34	10	9.9
15	42.71	4	6.2	41.90	5	9.6	43.28	5	9.0	43.45[8]	4	5.4
16[illegible]	36.79	5	2.5	37.09	7	5.9	38.47	7	4.5	36.70	5	2.2
17	33.21	3	2.8	32.01	3	1.5	33.38	3	0.8	32.64	3	2.0
18	34.74	3	0.6	36.14	3	1.5	35.60	3	0.1	35.97	3	0.9
20	31.57	8	5.9	31.17	10	9.4	31.63	10	11.0	31.24	9	9.8
21	33.83	4	1.6	33.23	4	1.1	34.20	5	4.7	33.66	4	6.4
22	30.08	1	. .	28.60	4	2.2	28.26	4	2.7	26.21	2	0.3
23	32.22	12	11.9	31.67[4]	12	14.0	30.92	14	19.9	30.98	12	12.7
24	30.43	4	3.1	29.03	5	7.3	29.48	5	6.0	29.62	5	9.0
25	34.12	8	6.3	33.69	9	9.4	33.38	9	4.6	34.15	8	5.3
26	31.45	3	2.7	30.85	3	4.2	30.15	3	2.9	30.33	3	3.1
30	34.88	5	5.0	32.73	5	5.5	33.14	5	1.6	33.28	5	7.1
31	. .	.	. .	28.45	1	. .	30.48	2	0.2	30.36	2	1.2
32	32.48	5	5.6	33.29	5	11.0	31.40	4	4.3	31.54	4	5.1
33	31.88	1	. .	29.85	1	. .	28.78	1	. .	26.46	1	. .

[1] 1773, March 24, gives 31″.5, and is excluded.
[2] 1774, October 10, gives 36″.4, and is excluded.
1776, February 17, gives 39″.0, and is excluded.
[4] 1795, March 6, gives 38″.95, and is excluded.
[5] 1774, October 10, gives 41″.5, and is excluded.
Period extended to 1774, March 14, two days.
1774, October 10, gives 38″.1, and is excluded.
[8] 1771, October 28, gives 36″.6, and is excluded.

TABLE XIII.—*Results of Quadrant Observations of β and γ Draconis, uncorrected*

No. of Period.	β Draconis.			γ Draconis.			Combined.	
	p'	N.	Wt.	p'	N.	Wt.	p'	Wt.
	″			″			″	
4	36.65	11	3	39.20	10	2	37.67	5
11	32.53	3	1	31.84	11	3	32.01	4
13	32.44	9	2	33.16	19	3	32.87	5
17	32.88	4	1	35.27	12	3	34.67	4
18	36.77	3	1	37.83	3	1	37.30	2
19	37.42	13	3	36.95	15	3	37.18	6
25	. .	.	.	38.70	5	2	38.70	2
26	37.79	11	3	37.70	42	3	37.74	6
27	. .	.	.	37.51	37	3	37.51	3
28	37.55	4	1	37.29	57	3	37.36	4
29	. .	.	.	36.34	26	3	36.34	3
30	. .	.	.	35.25	4	1	35.25	1
31	. .	.	.	34.84	32	3	34.84	3
32	. .	.	.	36.27	6	2	36.27	2
33	. .	.	.	34.80	23	3	34.80	3
34	. .	.	.	36.41	36	3	36.41	3
35	. .	.	.	41.65	67	3	41.65	3
36	. .	.	.	41.85	100	3	. .	.

The values from this table, beginning with period 19, have been corrected by −4″.8, to form the corresponding column in Table XIV. The differences between the numbers in the last column but one, above, and the results for p' from the fundamental stars, except the northernmost three, combined with the stars in Gemini, are as follows:

Period.	Stars in Draco. *minus* other stars.	Period.	Stars in Draco. *minus* other stars.
	″		″
4	−0.25	27	+5.02
11	−0.96	28	+5.02
13	+0.78	30	+1.97
17	+1.47	31	+4.19
18	+1.65	32	+3.53
19	+4.69	33	+4.67
25	+5.30	36	+4.29
26	+6.11		

TABLE XIV.—*Consolidated Results of Quadrant Observations.*

[The degrees and minutes are everywhere 51° 28′.]

No. of Period.	Fundamental Stars.*		Stars in Gemini.			β and γ Draconis.		Final result.	
	p'	Wt.	p'	Wt.	Σe	p'	Wt.	p'	Wt.
	″		″		″	″		″	
1	41.18	23	. . .	.	. .	. . .	.	41.18	23
2	40.36	19	. . .	.	. .	. . .	.	40.36	19
3	39.41	30	. . .	.	. .	. . .	.	39.41	30
4	38.46	11	37.49	14	4.09	37.67†	(5)	37.92	25
5	38.46	9	37.99	16	9.49	. . .	.	38.16	25
6	. . .	.	40.39	7	3.01	. . .	.	40.39	7
7	. . .	.	43.34	8	7.32	. . .	.	43.34	8
8	. . .	.	49.25	8	3.60	. . .	.	49.25	8
9	. . .	.	31.40	8	8.36	. . .	.	31.40	8
10	. . .	.	34.74	16	6.50	. . .	.	34.74	16
11	. . .	.	32.97	5	4.16	32.01†	(4)	32.97	5
12	. . .	.	33.58	16	4.82	. . .	.	33.58	16
13	. . .	.	32.11	8	6.76	32.89†	(4)	32.11	8
14	. . .	.	33.70	8	5.86	. . .	.	33.70	8
15	. . .	.	42.61	8	5.73	. . .	.	42.61	8
16[a]	39.47	17	37.88	16	7.26	34.67†	4	38.70	33
17	. . .	.	33.20	8	6.52	37.30†	2	33.20	8
18	. . .	.	35.65	8	5.97	. . .	.	35.65	8
19[b]	32.49	22	. . .	.	. .	32.38	6	32.57	28
20	32.38	17	31.66	16	3.04	. . .	.	32.03	33
21	. . .	.	33.79	8	5.04	. . .	.	33.79	8
22	. . .	.	28.71	8	7.94	. . .	.	28.71	8
23	. . .	.	31.52	16	4.74	. . .	.	31.52	16
24	. . .	.	29.56	16	2.66	. . .	.	29.56	16
25	33.29	25	33.58	16	3.92	33.90	2	33.43	43
26	31.90	34	30.50	8	5.27	32.94	6	31.80	48
27	32.49	38	. . .	.	. .	32.71	3	32.51	41
28	32.34	28	. . .	.	. .	32.56	4	32.37	32
29	. . .	.	. . .	.	. .	31.54	3	31.54	3
30	. . .	.	33.28	16	4.60	30.45	1	33.11	17
31	. . .	.	30.65	7	5.35	30.04	3	30.47	10
32	. . .	.	32.74	8	6.13	31.47	2	32.49	10
33	. . .	.	30.19	5	7.35	30.00	3	30.08	8
34	. . .	.	. . .	.	. .	31.61	3	31.01	3
35	. . .	.	. . .	.	. .	36.85	3	36.85	3
36	37.56	90	. . .	.	. .	37.05	3	37.54	93

* Omitting α Aurigæ, α Cygni, and α Lyræ.

† These stars are not corrected till 1787, and do not enter into the result before that time. The probable error to weight = 1 from the fundamental stars = $\pm$ 1″.17. From the stars in Gemini the same probable error is $\pm$ 0″.76 only. The probable reason for this is the fact that the fundamental stars were so infrequently observed that the plumb-line behaved worse, while the stars in Gemini were continuously observed. The column Σe contains here the sums of the individual discrepancies of the stars in Gemini—7 discrepancies for period 4 for example; each of which has a weight $\doteq$ 2. I have, in these cases, not discriminated between the stars, but given a weight = 2, where the average number of observations was 5 or more, to each star.

TABLE XV.—*Concluded Constants for Correcting the Quadrant, compared with* OLUFSEN'S *Constants.*

No. of Period.	Period.	OLUFSEN'S— p	OLUFSEN'S— q_1	OLUFSEN'S— q_2	N.	SAFFORD'S— p'	SAFFORD'S— q'
		° ′ ″	″	″		° ′ ″	″
1	1765, May 10–July 6	51 28 40.27	1.54	1.55	87	51 28 41.18	+0.065
2	1765, July 7–Aug. 30	39.87	1.55	1.56	50	40.36	+0.065
3	1765, Sept. 1–1766, July 10	39.26	1.56	1.63	86	39.41	+0.067
4	1767, Oct. 1–1769, Oct. 12	38.94	1.73	1.90	97	37.92	+0.076
5	1770, Jan. 23–1772, Sept. 25	39.09	1.92	2.14	70	38.16	+0.085
6	1773, Sept. 6–Nov. 1	41.85	2.22	2.33	22	40.39	+0.094
7	1774, Feb. 20–Mar. 12	45.01	2.26	2.26	26	43.34	+0.095
8	1774, Sept. 26–Oct. 24	51.01	2.30	2.31	24	49.25	+0.097
9	1776, Feb. 12–Mar. 2	33.34	2.41	2.42	29	31.40	+0.102
10	1776, Mar. 11–Sept. 23	36.48	2.42	2.47	56	34.74	+0.103
11	1776, Oct. 15–Dec. 30	32.33	2.47	2.49	26	32.97	+0.105
12	1777, Jan. 11–Mar. 1	35.38	2.49	2.50	39	33.58	+0.105
13	1777, Aug. 9–Oct. 4	32.44	2.54	2.55	53	32.11	+0.107
14	1777, Nov. 7–1780, Mar. 25	35.32	2.56	3.02	83	33.70	+0.112
15	1781, Sept. 25–Nov. 3	44.24	4.18	4.27	34	42.61	+0.122
16	1782, July 20–1784, Sept. 9	39.36	4.82	6.47	104	38.70	+0.128
17	1785, Mar. 11–Nov. 23	32.66	6.85	7.40	33	33.20	+0.135
18	1785, Dec. 29–1786, July 31	35.85	7.49	7.94	25	35.65	+0.138
19	1787, Mar. 6–Aug. 28	30.41	8.40	8.76	87	32.47	+0.140
20	1787, Sept. 9–1788, Nov. 10	31.52	8.81	9.70	100	32.03	+0.140
21	1788, Nov. 15–1793, Mar. 11	33.30	9.71	11.52	30	33.79	+0.140
22	1793, Sept. 24–Nov. 1	28.63	11.42	11.39	23	28.71	+0.140
23	1794, Mar. 7–1795, Oct. 8	31.03	11.32	11.00	121	31.52	+0.140
24	1795, Oct. 9–Oct. 24	29.18	11.00	11.98	33	29.56	+0.140
25	1796, July 6–1799, Dec. 31	32.65	11.83	10.11	124	33.43	+0.140
26	1800, April 26–Dec. 31	30.90	10.04	9.90	159	31.80	+0.140
27	1801, Jan. 4–1802, Feb. 26	31.52	9.90	9.66	182	32.51	+0.140
28	1802, Mar. 12–Dec. 27	31.38	9.65	9.48	143	32.37	+0.140
29	1803, Jan. 8–1804, Dec. 13	31.36	9.48	9.07	50	31.54	+0.140
30	1804, Dec. 14–1805, Mar. 17	33.08	9.07	9.02	41	33.11	+0.140
31	1805, June 24–Sept. 29	30.20	8.96	8.91	56	30.47	+0.140
32	1805, Oct. 2–1806, Feb. 27	33.48	8.90	8.82	51	32.49	+0.140
33	1806, May 25–Dec. 12	31.22	8.77	8.65	52	30.08	+0.140
34	1807, April 25–1808, May 19	30.94	8.65	8.65	36	31.61	+0.140
35	1808, June 26–1810, Aug. 31	36.02	8.65	8.65	67	36.85	+0.140

OLUFSEN'S formula for correction, $p + q \sin \delta$.
SAFFORD'S formula for correction, $p' + q' \delta^\circ$.
S.—O. $= p' - p + q' \delta^\circ - q \sin \delta$.
The numbers q_1 q_2 denote the values of q for the beginning and end of each period.
In applying this table declinations between $+ 33^\circ$ and $- 31^\circ$ alone are to be included.

APPENDIX 1.

Values of p' for the Quadrant Observations of 1824–25.

1824—	p'	No. of Stars.	Weight.
	° ′ ″		
Jan. 24–30	51 28 25.34	4	4
31–Feb. 2	29.89	6	6
Feb. 3–11	34.26	6	6
14–21	32.96	4	5
March	29.95	7	11
April	29.78	6	10
May	28.94	6	8
June 1–6	30.56	3	3
7–18	25.55	5	7
25–July 20	30.52	7	11
July 22–Aug. 9	25.90	7	8
Aug. 11–Sept. 7	25.05	6	11
Oct. 21–Nov. 15	25.38	5	5
Nov. 21–Dec. 11	24.07	4	5
Dec. 13–Jan. 2 (1825)	51 28 20.31	5	5

These are derived from all the fundamental stars observed during 1824–25 with the brass quadrant.

The quantity

$$p' + 0''.140\,\delta^{\circ}$$

takes the place of OLUFSEN's

$$p + q \sin \delta$$

and the correction for index-error which AIRY gives during the intervals, 1824, Jan. 24–Sept. 5; and Oct. 25–Dec. 29. That is, during these intervals, AIRY's index-error is made $= 0$, when this table is used.

APPENDIX 2.

Results of Discussion of Fundamental Star Transits during 1811 *and* 1812; *compared with observations of the Pole star during the same interval.* The object of this is to show that in the usual position of the transit instrument in the latter part of MASKELYNE's time the correction $+0^{s}.025$ to the n derived from polar stars, as obtained by Professor AUWERS, is still valid. POND's observations, with MASKELYNE's transit, *in its usual position,* were made in 1811 and following years; during one period the transit was reversed and gives results which are not so regular; during this period planetary observations extend from 1812, May 6 to August 20, but are few in number.

In the following table Δ_n denotes the mean difference obtained by subtracting

the n derived from Polaris corrected according to AUWERS from that derived from various pairs of fundamental stars; and N the number of observations of each pair. The weights are roughly estimated.

Fundamental Stars.	Δn	N.	Wt.
	s.		
Capella—Rigel	+ 0.058	24	3
Sirius—Pollux	+ 0.012	13	1
Spica—Arcturus	— 0.079	29	1
α Coronæ—Antares	+ 0.027	17	2
α Coronæ—α Libræ	— 0.110	9	1
α Lyræ—Antares	+ 0.112	8	2
α Cygni—α Aquilæ	— 0.154	5	1
α Lyræ—α¹ Capricorni	+ 0.021	13	2
α Cygni—α² Capricorni	+ 0.104	25	3
α Cygni—α Aquarii	— 0.053	19	2
α Cygni—Fomalhaut	— 0.020	5	1
α Pegasi—Fomalhaut	+ 0.076	9	1
α Andromedæ—Fomalhaut	+ 0.045	13	2
Mean by weights	+ 0.023 ± 0ˢ.014		

The correction to the values of n derived from Polaris should then be further increased by $+0^{s}.023$; the observations of 1811–13 were not accessible to Professor AUWERS when he discussed MASKELYNE's transits. But the correction is not very certain, and can be omitted.

APPENDIX 3.

Corrections to BESSEL'S *refractions to reduce to the Pulcova tables.*

As the Pulcova refractions are less than BESSEL'S (Tabulæ Regiomontanæ) the effect of the change to the former will be to increase the (north) declinations of stars south of the zenith.

Declination.	Correction.
°	″
30	+0.07
20	0.10
10	0.15
0	0.21
—10	0.29
—20	0.44
—30	+0.75

The difference is that of mean refraction only; which is sufficient for the present purpose.

MEASURES

OF THE

VELOCITY OF LIGHT

MADE UNDER DIRECTION OF

THE SECRETARY OF THE NAVY

DURING

THE YEARS 1880-'82.

BY

SIMON NEWCOMB,

PROFESSOR, U. S. NAVY.

PREFACE.

In planning the following determination of the velocity of light, in which the result depends upon the measure of an arc sometimes exceeding eight degrees, it was hoped to reach a concluded value of which the probable error would be between five and ten kilometers, and which, if repeated and verified, might serve to test the invariability of our standard of length. For reasons fully discussed in the paper, this degree of precision was not reached. Believing, however, that the result is amply accurate for all astronomical purposes, the writer does not himself intend to repeat the measures. But he supposes it possible that the degree of precision originally aimed at can be reached without any radical change in the instrument, and would be happy to co-operate with any physicist who may desire to utilize it for farther researches.

TABLE OF CONTENTS.

MEASURES OF THE VELOCITY OF LIGHT.

CHAPTER I.

INTRODUCTION.

When it became clearly understood that vision was not an immediate perception of objects by the eye, but was produced by the passage of an entity called light from the object to the eye, the question of the time which might possibly be required for this passage became one of interest to physical investigators. The first proposal for an experimental investigation of this question is due to GALILEO.* He suggested that two observers, each holding a lantern, should be stationed at a distance apart, in sight of each other. Each should be supplied with a screen, by which he could, in a moment, cover or uncover his lantern. One observer should then uncover his lantern and the other uncover the other the moment he perceived the light from the first lantern The interval which elapsed after the first uncovered his light, until he perceived the light of the second, would be the interval required for the light to go and come, plus the time required for the second observer to perceive the light and make the required movement. This experiment was tried by the Florentine Academy, and of course resulted in a conclusion that the time required was insensible, since we now know that it was far below any interval that could have been detected by so rude a method.

It is, however, interesting to notice that, rude though this experiment was, the principle on which it was based is the same which underlies one of the most celebrated methods used in recent times for the attainment of the same object. Two very simple improvements which we might have imagined the academicians to make in their experiments are these:

Firstly, to dispense with the second observer, and in his place to erect a mirror, in which the first observer could see the image of his own lantern by reflection. The time required for the second observer to perceive the light and uncover his lantern would then have been eliminated from the problem. The interval sought would have been that between the moment at which the observer uncovered his lamp and the moment at which he perceived the reflection.

Secondly, to use the same screen with which he uncovered his own lamp to cut off the returning ray from the distant mirror, and thus obviate the necessity of an uncertain estimate of the interval between his muscular effort in removing the screen and his perception of the return flash of light. If the image was perceived before he

*POGGENDORFF *Geschichte der Physik*, p. 402, where reference is made to the *Saggi* of the Florentine Academy.

could cover his own eye with the screen removed from the lamp, it would show that the interval of passage was less than the time required to make a motion with the screen. This interval might have been reduced almost indefinitely by having both lines of sight as near together as possible.

Had these improvements been made the academicians would have had, in principle, FIZEAU'S method of measuring the velocity of light by the toothed wheel, a tooth being represented by the screens. To realize the principle more fully, the two lines of sight should have been rendered absolutely coincident by reflection through a telescope. It does not, however, appear that any effort to put the question to a severer test was made until the subject was approached from a different point of view. It was probably considered that the passage was absolutely instantaneous, or, at least, that the velocity was above all powers of measurement.

The subject was next approached from the astronomical side. In 1676 ROEMER made his celebrated communication to the French Academy, claiming that observations of the eclipses of the first satellite of Jupiter did really prove that light required time to pass through the celestial spaces.* He found 11^m to be the time required for light to pass over a distance equal to the radius of the earth's orbit. DOMINIQUE CASSINI, while admitting that the hypothesis of ROEMER explained the observed inequality, contested its right to reception as an established theory, on the ground that the observed inequality might be a real one in the motion of the satellite itself.†

Continued observation showed that the time assigned by ROEMER for the passage of light between the earth and sun, or "the light equation" as it is briefly called, was somewhat too great. In 1809 it was fixed by DELAMBRE at $493^s.2$ from an immense number of observations of eclipses of Jupiter's satellites during the previous 150 years. This number has been received as a definitive result with a degree of confidence not at all warranted. In 1875, GLASENAPP, then of Pulkowa, from a discussion of all available eclipses of Jupiter's first satellite between 1848 and 1870, showed that results between 496^s and 501^s could be obtained from different classes of these observations by different hypotheses.‡

As not a trace of DELAMBRE'S investigation remains in print, and probably, not in manuscript, it is impossible to subject it to any discussion.§

The discovery of aberration by BRADLEY afforded an independent and yet more accurate method of determining the light equation. We call to mind that the latter constant, and that of aberration, are not to be regarded as independent of each other, but only as two entirely distinct expressions of the same result. The constant of aberration gives a relation between the velocity of light and the velocity of the earth in its orbit from which, by a very simple calculation, the time required for light to pass from the sun to the earth may be deduced.

It is remarkable that the early determinations of the constant of aberration

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* *Paris Memoirs*, tome i, page 212, and tome x, page 575.

† *Ibid*, tome viii, page 47. POGGENDORFF (*Geschichte der Physik*, page 656) quotes MARALDI as also contesting ROEMER'S explanation on the ground that a similar inequality should be found depending on the position of Jupiter in his orbit. The ground here taken was quite correct, the only fallacy being the assumption that such an inequality did not exist.

‡ This paper of GLASENAPP'S was published only in the Russian language as an inaugural dissertation, and in consequence has never become generally known.

§ The author could find no remains of this investigation among DELAMBRE'S papers at the Paris Observatory.

agreed with DELAMBRE's determination of the light equation, although we now know they were both in error by an amount far exceeding what was, at the time, supposed probable. STRUVE's value, $20''.445$, determined in 1845 from observations with the prime vertical transit of Pulkowa, has been the standard up to the present time. The recent determinations by NYRÉN being founded on a much longer series of observations than those made by STRUVE, and including determinations with several instruments, must be regarded as a standard at present. His result is:*

Definitive value of the constant of aberration $= 20''492 \pm 0''.006$.

At the time STRUVE's result was published there was an apparent difference of one per cent. between its value and that of the light equation determined by DELAMBRE. The question then naturally arose whether the light equation, deduced on the hypothesis that the tangent of the angle of the constant of aberration was the ratio of the velocity of the earth in its orbit to the velocity of light, might not need correction or modification. This question cannot yet be considered as definitely settled, since the modifications or corrections might arise from a variety of causes. One of these causes is connected with a very delicate question in the theory of the luminiferous medium; a question which can be most clearly understood when placed in the following form: It is a result of optical principles that a ray falling perpendicularly upon the bounding surface of a refracting medium retains its direction unaltered. Now, if this surface is carried along by the motion of the earth, and the light comes from a star, and it is desired that this surface shall be so directed that there shall be no refraction, must it be placed perpendicular to the *true* direction of the star as freed from aberration, or to its *apparent* direction as affected by aberration? The difference of the two directions may exceed $20''$, and since the index of refraction of glass exceeds 1.5, there will be a difference of more than $10''$ in the direction of the refracted ray, according as we adopt one or the other hypothesis. Assuming that the standard direction would be perpendicular to the true or absolute direction of the star, it is easily shown that the constant of aberration determined in the usual way would be too large by a quantity depending on the ratio of the thickness of the objective to the focal length of the telescope. In an ordinary telescope the difference would be nearly one hundredth of the total value of the aberration, and would, therefore, closely correspond to the discrepancy between DELAMBRE's result from the satellites of Jupiter and the modern determinations of the constant of aberration. The question of this particular cause was set at rest by AIRY's experiments with a telescope filled with water, which showed that the result was independent of the thickness of the objective, and, therefore, that the apparent direction of the star was that on which refraction depended.

If, in accordance with the undulatory theory of light, we suppose the hypothetical entity called "the luminiferous medium" to be a substance, each part of which has its own definite and fixed location in space, then we must conceive that another unknown quantity may enter into the problem, namely, the motion of the heavenly bodies through this medium. We have relative motions in the solar system, exceeding 50 kilometers per second, and possibly greater relative motions among the stars. Now

*Mémoires de l'Académie Impériale des Sciences de St.-Pétersbourg, vii série, tome xxxi, No. 9.

it is clear that the heavenly bodies cannot all be at rest relative to the medium, but must move through it with velocities at least of the order of 50 kilometers per second, and possibly greater without limit, since it is conceivable that the whole visible universe might be moving in a common direction relative to the medium.

It is easily seen that if we suppose the velocity of the earth, through the medium, to have a small ratio, α, to the velocity of light, then the observed constant of aberration may be altered by an amount found by multiplying its value by a quantity of the order of magnitude of α. This alteration would be entirely insensible if the earth does not move through the medium with any greater velocity than it does around the sun, since the value would then be only $\frac{1}{10,000}$. It is remarkable that so far as yet investigated every optical effect arising from such a motion, which could be measured on the surface of the earth, is of the order of magnitude of the square of α. Thus, no phenomenon has yet been discovered which can be traced to the motion in question.

Assuming that there is no general motion of the solar system through the ether of a higher order of magnitude than that of the relative motions of the fixed stars to each other, and that the ordinary theory of aberration is correct, there will be three constants between which a relation exists, such that when any two are found the third can be determined. These constants are:

1. The distance of the sun in terrestrial units of measure;
2. The velocity of light in units of the same measure; and
3. The constant of aberration, or, which is supposed to be equivalent, the light equation.

Until our own time the first and third constants were used to determine the second. From the fact that light required about 500 seconds to traverse the distance from the sun to the earth, and that the distance of the sun was, as supposed, 95,000,000 of miles, it was concluded that light moved 190,000 miles per second. The hopelessness of measuring such a velocity by any means at the command of physicists was such that we find no serious attempt in this direction between the date of the futile effort of the Florentine Academy, and that of the researches of WHEATSTONE, ARAGO, FIZEAU, and FOUCAULT nearly two centuries later. One of the most curious features presented by the history of the subject is that two entirely distinct methods, resting on different principles, were investigated and put into operation almost simultaneously. The revolving mirror of WHEATSTONE, and its application to determine the duration of the electric spark and the velocity of electricity, come first in the order of time. But, before this ingenious instrument had been applied to the actual measurement of the velocity of light, FIZEAU had invented his toothed wheel, by which the same object was attained.

FIZEAU'S paper on the subject was presented to the Academy of Sciences July 23, 1849.* We have already shown that his method and that of GALILEO rest fundamentally upon the same principle. The arrangement of his apparatus was substantially as follows:

A telescope was fixed upon a house at Surésne pointing to the hill Montmartre. On this hill was a second fixed telescope looking directly into the first, the distance between them being about 8,633 meters. In the focus of this second telescope was fixed a small reflector, so that a beam of light from the first would be reflected directly

* *Comptes Rendus*, vol. xxix, 1849, p. 90.

back to it. By means of a transparent glass, fixed in the eye-piece at an angle of 45°, a beam of light was sent from the first telescope to the second, and, on its return through a total distance of 17 kilometers, could be seen as a star by an eye looking through the first. Alongside the eye-piece of the latter a revolving wheel, with 720 teeth cut upon its circumference, was fixed in such a way that the beam of light both in going and coming had to pass between the teeth. When the wheel was set so that the tooth was in the focus, the beam would be entirely cut off in its passage through the telescope. Changing the position of the wheel through half the space between the middles of two consecutive teeth, the light would go and come freely between the teeth. When the wheel was set in revolution a succession of flashes would be sent out. If, on the return of each flash, a tooth was interposed, it would be invisible to the eye looking through the telescope. FIZEAU found that with a velocity of 12.6 turns per second each flash which went out was on its return cut off by the advancing tooth. With a velocity twice as great as this it was seen on its return through the opening next following that through which it went. With three times this velocity it was caught on the second tooth following, and so on.*

This experiment of FIZEAU was soon followed by the application of the revolving mirror of Sir CHARLES WHEATSTONE. Shortly after measuring the duration of the electric spark this investigator called attention to the fact that the same system could be applied to determine the velocity of light, and especially to compare the velocities through air and through water. In 1838 the subject was taken up by ARAGO, who took pains to demonstrate that it was possible, by the use of the revolving mirror, to decide between the theory of emission and that of indulations by determining the relative velocities in air and in a refracting medium.†

The difficulties in the way of securing the necessary velocities of the mirror and of arranging the apparatus were such that ARAGO never personally succeeded in carrying out his experiments. This seems to have been done almost simultaneously by FOUCAULT and FIZEAU about the beginning of 1850. Both experimenters seem to have proceeded substantially on the same principle and to have reached the same result, namely, that the motion of light through water was slower than through air in the inverse proportion of the indices of refraction of the two media.‡

An important and most necessary modification of ARAGO's plan was made by these experimenters. As originally proposed, the plan proposed was to send an instantaneous flash of light through water and through the air and to receive it on the revolving mirror and determine the relative deviations in the positions of the images produced by the two rays. This system would, however, be inapplicable to the measurement of the actual time of transmission, owing to the impossibility of making any comparison between the time at which the flash was transmitted and that at which it was received on the mirror. This circumstance would, indeed, have rendered

*It is curious that the author's account of this remarkable experiment, which forms an epoch in the history of physical science, is contained within the limits of two pages, and terminates without any definite discussion of the results. It is merely stated that the result is 70,948 leagues of 25 to the degree, but the velocity, in kilometers, which must have been that first obtained, is not given, nor is it stated what length the degree was supposed to have in the computation.

† *Comptes Rendus*, 1838, vol. vii, page 954; *Œuvres de François Arago*, vol. vii, page 569.

‡ *Comptes Rendus*, xxx, 1850, pages 551 and 771.

the actual realization of ARAGO's project nearly impossible for the reason that the flashes of light, seen through the water, would have reached the mirror at every point of its revolution; and only an exceedingly small fraction of them could have been reflected to the eye of the observer.

This difficulty was speedily overcome by FOUCAULT and FIZEAU by a most ingenious arrangement, of equal importance with the revolving mirror itself. Instead of sending independent flashes of light to be reflected from the mirror, a continuous beam was first reflected from the revolving mirror itself to a fixed mirror, and returned from the fixed mirror back on its own path to the revolving one. A succession of flashes was thus emitted as it were from the fixed mirror, but their correspondence with a definite position of the revolving mirror was rendered perfect. Moreover, by this means, the image was rendered optically continuous, since a flash was sent through and back with every revolution of the mirror, and after the velocity of the latter exceeded 30 turns per second, the successive flashes presented themselves to the eye as a perfectly continuous image.

It was not until 1862 that this system was put into operation by FOUCAULT for the actual measurement of the velocity of light through the atmosphere. A new interest had in the mean time been added to the problem by the discovery that the long accepted value of the solar parallax was too small, and that the measurement of the velocity of light afforded a method of fixing the value of that constant. The central idea of the method adopted by FOUCAULT was that already applied in comparing velocities through different media. The element sought is made to depend upon the amount by which the revolving mirror rotates while a flash of light is passing from its surface to the distant reflector, and coming back again. As the details of FOUCAULT's method will be best apprehended by a comparison of them with those adopted in the present investigation, a complete description of his apparatus will here be passed over. It may, however, be remarked, that what he sought to observe was not the simple deviation of a slit, but the deviation of the image of a reticule. The deviation actually measured was 0.7 millimeter, and the system adopted was to determine at what distance, with a definite velocity, this amount of deviation could be obtained. His result for the velocity of light was 298,000 kilometers per second.

The next measures of the element in question were those of CORNU. The method which he adopted was not that of the revolving mirror, but FIZEAU's invention of the toothed wheel. His earlier measures, made in 1870, and communicated to the French Academy in 1871, led to a result nearly the same as that of FOUCAULT.* This result was, however, not so satisfactory that the author could record it as definitive. He, therefore, in 1874, repeated the determination on a much larger scale and with more perfect apparatus. The distance between the two stations was nearly 23 kilometers, and, therefore, much greater than any before employed. He was thus enabled to follow the successive appearances and extinctions of the reflected image to the thirtieth order; that is, to make fifteen teeth of his wheel pass before a flash returned from the distant reflector, and to have it stopped by the sixteenth tooth.

This method has a defect, the result of which is evident by an examination of

* *Comptes Rendus*, vol. lxxiii, 1871, p. 857.

CORNU's numbers. It is that the extinctions and reappearances of the light as the wheel changes its speed are not sudden phenomena, occurring at definite moments, but are so gradual that it is difficult to fix the precise moment at which they occur. Of this defect the able experimenter was fully conscious, and his discussion of the disturbing causes which come into play, and of the amount of error due both to the apparatus, the observer, and to the method of eliminating them, form altogether one of the most exhaustive discussions of a physical problem.* But the uncertainties are not of a kind which admit of complete investigation, and it now appears that although his result was far superior in point of accuracy to that of FOUCAULT, it was nevertheless in error by about 0.0015 of its whole amount. It was, in fact, when reduced to a vacuum, 300,400 kilometers per second, while we may now regard it as well established that the true velocity is less than 300,000.

The next determination of the velocity of light was that of MICHELSON,† whose result was 299,910 kilometers per second. His investigation being a part of the first volume of the present series need not be here discussed, but it is worth while to remark that his method seems far superior in reliability to any before applied.

An attempt has been made by Messrs. JAMES YOUNG and GEORGE FORBES to improve FIZEAU's method, by diminishing the uncertainty arising from the gradual extinction of the visible image.‡ By the method of these experimenters the result depends, not upon the moment when the image disappears, but when two images, side by side, are equal in brightness. This is effected by employing two reflectors, at unequal distances, but nearly in the same line from the telescope, to return the ray. Each reflector then forms its own image in the field of view of the sending telescope. With a regularly increasing velocity of the toothed wheel, each image goes independently through the same periodic series of changes as when only one mirror is used; but owing to the unequal distance the period is not the same. If the speed of the mirror be carried to such a point that the difference of phase in the two images is half a period, then one image will be increasing while the other is diminishing, and the stage at which the two images are equal would appear to admit of fairly accurate determination.

The distant reflectors were separated from the observing telescope by the Firth of Clyde. The distances were respectively 16,835 feet, and 18,212 feet. A study of the printed descriptions of their experiments gives the impression that the performance of the subsidiary parts of the apparatus was not such as to do justice to the method. The resulting velocity of light was 301,382 kilometers per second, and the difference between the extreme results of twelve separate determinations was 4,000 kilometers.

The most important result of the work of these gentlemen, could it be accepted, would be the establishment of a difference between the velocities of differently-colored rays. We may regard it as quite certain, from the absence of any change in the color of the variable star, β Persei, while it is increasing and diminishing, that the difference between the times required by red and by blue rays to reach us from that star

* Annales de l'Observatoire de Paris. Mémoires. Tome xiii.

† Astronomical Papers of the American Ephemeris, vol. i, part iii. Owing to an error in applying one of the corrections the result was given as 299,942 kilometers.

‡ Philosophical Transactions for 1882, p. 231.

cannot exceed a moderate fraction of one hour. It is quite improbable that its parallax is more than $0''.1$, and, therefore, probable, that its distance is 2,000,000 or more astronomical units. The possible difference between the velocities in question can, therefore, only be a small fraction of the hundred thousandth part of either of them. No apparatus yet devised would suffice for the measurement of a difference so minute, and we are justified in concluding that the phenomena observed by Messrs YOUNG and FORBES arose from some other cause than a difference between the velocities of red and blue rays.

The present determination had its origin as far back as 1867. In his "Investigation of the distance of the Sun," published in that year, the author introduced some remarks upon FOUCAULT's method, and pointed out the importance to the determination of the solar parallax of repeating the determination of FOUCAULT on a much larger scale, with a fixed reflector placed at a distance of three or four kilometers.*

From that time forward the subject excited the attention of American physicists, several of whom formed plans, more or less definite, for executing the experiments. As, up to the year 1878, no important steps in this direction had been taken, the author, in April of that year, brought the subject before the National Academy of Sciences, with the view of eliciting from that body an expression of opinion upon the propriety of asking the Government to bear the expenses of the work. The subject was referred to a select committee, who, in January, 1879, made a favorable report on the subject, which was communicated to the Secretary of the Navy. On the recommendation of the Secretary, Hon R. W. THOMPSON, Congress, in March following, made an appropriation of $5,000 for the purpose, and the author was charged by the Department with the duty of carrying out the experiments.

In the mean time it became known that Mr. MICHELSON had made preparations for repeating FOUCAULT's determination at his own expense, with the desirable improvement of placing the fixed reflector at a considerable distance. But before the reliability of Mr. MICHELSON's work had been established, the preparations for the present determination had been so far advanced that it was not deemed advisable to make any change in them on account of what Mr. MICHELSON had done. The ability shown by the latter was, however, such that, at the request of the writer, he was detailed to assist him in carrying out his own experiments, and acted in this capacity until September, 1880, when he accepted the professorship of physics in the Case Institute, Cleveland, Ohio. After the departure of Mr. MICHELSON his place was taken by Ensign J. H. L. HOLCOMBE, U. S. N., who assisted in every part of the work to the entire satisfaction of the projector until its close.

In addition to the results above mentioned, it is well to add those of MICHELSON in 1882, which are published in connection with the present paper. The occasion for this repetition of Professor MICHELSON's investigation is described hereafter.

As the form of apparatus finally adopted was the result of a long study of the conditions of the problem, a discussion of those conditions may not be without value to any future experimenter who may occupy himself with the determination of the physical constant in question. A separate chapter will now be devoted to this subject.

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* Washington Observation, 1865. Appendix ii.

Chapter II.

CONDITIONS OF THE PROBLEM AND FORM OF APPARATUS DEVISED TO MEET THEM.

For the sake of clearness, we begin by calling to mind the essential features of the process. A ray of light, or a pencil of parallel rays, moving in a definite direction strikes a movable mirror. It is thence reflected to a distant concave mirror, of any size, having its center of curvature in the position of the movable mirror. In consequence the ray is reflected back on its own path to the movable mirror, which again reflects it in the direction of its origin. Although we should change the position of the movable mirror, and thus send the ray of light out in different directions, all such motion is eliminated by the return reflection, so that the return ray, if there be any, is always reflected from the movable mirror in an invariable direction.

Let the reader imagine himself looking along a fixed line into the moving mirror. If he suitably turns the latter he will, at certain times, see the image of the fixed mirror move across his field of view. If it be of considerable size it may more than fill the field, and it may be a sensible time in moving across. If a ray of light be sent out in such a way that on its return it will enter the eye of the observer, the latter will see it as a bright point on the surface of the image of the fixed mirror. If the moving mirror then be slowly turned, the bright point will move in an opposite direction relatively to the image of the fixed mirror, so as to be seen constantly in the same position in the field of view. In other words, if the observer imagines the return image to be always existent, its direction will be fixed, and as the one mirror turns, the image of the other mirror will simply move past the return image of the ray. The latter will, of course, in reality be visible only while the distant mirror is passing it.

In what precedes it is assumed that the revolving mirror, which we may call A, is in the same position when any ray is sent out, and when the same ray is received on its return after reflection. If, however, it moves with any finite velocity, the return ray will be deviated by twice the amount of motion of mirror A while the ray is going and coming. The amount of deviation will, however, be perfectly constant so long as the velocity remains unchanged.

The management of the ray of light, as a mathematical line, being impracticable, a lense or a system of lenses must be used in realizing the plan. It is necessary that a luminous slit, or the line of a reticle be observed, that this slit and the concave reflector be in conjugate foci of the lens, and that the revolving mirror be interposed in the path of the ray.

When a single lens is used, two of its possible arrangements are shown in the accompanying figure. Here S is the slit, or other fiducial fixed object, the image of which is to be observed. L is an achromatic lens, A the revolving mirror, and M the distant concave spherical mirror, having its center coincident with A. S and M are

in conjugate foci of the lens. These we may regard as the primary and fundamental parts and conditions of the apparatus which, however, admit of amplification and variety of arrangement as details are added.

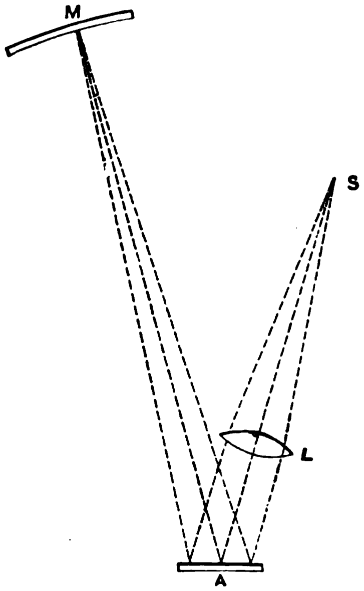

FIG. 1.

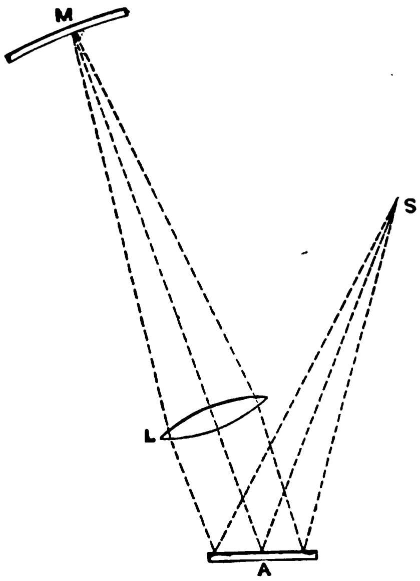

FIG. 2.

The figure shows two possible positions of the lens. In the one it is between the slit and the revolving mirror and in the other between the revolving mirror and the fixed mirror. It is quite obvious that in the first form and with the arrangement shown in the figure only small deviations can be accurately measured, since a wide deviation will send the return ray through a different part of the lens from the outgoing ray, and uncertainties will be produced by the obliquity of the course.

To see how, starting from these fundamental appliances, that system which may be supposed to give the most accurate results is to be reached, we must begin by considering the difficulties which are met as we proceed step by step into details. The problem is to measure the angle through which the mirror is rotating between the moment at which any flash of light falls upon it from the slit *S* and the moment at which the same flash is reflected on its return from *M*. This angle is half the angle between the outgoing and incoming pencils of rays. The object aimed at is to make such an arrangement that this angle shall be as great as possible. With a given velocity this involves the requirement that the concave mirror, *M*, shall be as far away as possible. It is true that, so far as the linear displacement of the return image is concerned, the same result may be reached by placing the slit at a great distance from the revolving mirror. But practically, it is well known to astronomers that the advantage thus gained soon reaches its limit, because there is a limit to the accuracy of any angular measure, which cannot be extended by merely lengthening the radius. The reason of this is that in lengthening the radius we increase the diffusion of the image, atmospheric vibration, effects of difference of temperature, and other sources of error, in a ratio which soon equals that in which the radius is increased. The astronomical

limit of accuracy may be considered as an important fraction of a second of arc, and the arrangement should be such that this error shall bear the smallest possible ratio to the angle measured.

In FOUCAULT'S measures, the object of which the image was formed and observed was not an illuminated slit, but the line of a reticule. An accurate image of such a line could not, however, be formed at any great distance, and thus we find that all his measures were made within the limits of a single room. With so small a distance a large angular deviation was out of the question. To employ a greater distance the image must be that of an illuminated slit. No limit can then be set to the distance at which an image of the slit may be formed. But when the image is returned and reflected from the rapidly revolving mirror, its brilliancy is necessarily diminished in the ratio of the diameter of the distant mirror to the circumference of a circle whose radius is the distance of that mirror from the revolving one. Besides this, a very large fraction of the light will be lost by reflection, absorption, and dispersion, in passing over a great distance. In order, therefore, that a considerable distance may be practicable, it is essential that the arrangement shall be such that the feeblest possible return ray shall be visible. This, again, requires that the field of view in which the image is sent shall be as dark as possible. To trace out these requirements let us see what fraction of the light would be reflected when the apparatus is arranged as in Fig. I. A concave mirror, of which the diameter should be 1 decimeter for each kilometer of distance, would receive approximately $\frac{1}{60,000}$ part of the light reflected from the revolving mirror. Should the distance be several kilometers, the difficulty of securing such curvature of the mirror that every part of its surface should be normal to the lines from the station would be nearly insurmountable, though no difficulty would arise in making some part of its surface fulfill the required condition. Joining with this cause the numerous sources of loss in transmission and reflection, we may readily see that in practice only a small fraction of the 60,000 part of the light would really be utilized. Now, when the apparatus is in the form II, the lens admits of being utilized, so as to greatly increase the brilliancy of the return image. Were it admissible to place the revolving mirror instead of the slit in the conjugate focus of the lens, then a return pencil would be reflected from the distant mirror during the whole time that the outgoing ray fell upon the lens. Although this arrangement is inadmissible, since it is the slit and not the mirror, A, which must be in the conjugate focus, yet, by increasing the distance, $A\ L$, between the lens and the mirror, and diminishing the distance, $A\ S$, between the mirror and slit, it is possible to approximate indefinitely to this state of things; that is, to increase the brilliancy of the return image approximately in the ratio which the angular diameter of the mirror subtended at the center of motion by the lens bears to that subtended by the distant mirror.

But every step we take in thus increasing the brilliancy of the return image is accompanied by a drawback from another source. At a distance of several kilometers atmospheric diffusion and vibration form the most serious defect in a return image. It will readily be seen that this defect increases in the same ratio as the length, $S\ A + A\ L$, which is the effective length of the telescope with which the observation is made. For the defect consists in this, that the light, which forms the image, in-

stead of going accurately along a definite line, $M L$, from the distant mirror to the lens, is scattered through a certain angle, which, to fix the ideas, we may suppose to be $1''$. The result will be that some part of the image will be formed at the focus, in a position $1''$ in error, as seen from the lens. This error, when expressed in linear measure, will be proportional to the focal length of the lens. To the practiced telescopic observer this explanation will hardly be necessary, since it amounts to nothing more than explaining the familiar fact that with a given magnifying power, the images of stars are not improved by lengthening the telescope, and that the angular accuracy with which a micrometer wire can be set on such an image, is not increased by increasing the magnifying power beyond a certain limit. This amounts to saying that if we employ a very long telescope we shall find, even though we use an eye-pieec of the same invariable short focus, the probable linear error of setting a micrometer wire upon the image of a star will, beyond certain limits of length, increase in the same ratio as the length of the telescope.

Returning now to Fig. 2, it follows from what has just been said that, under the best conditions, the probable linear error of measuring the division of the slit will increase in the ratio of the focal length $S A + A L$ of the lens. But the resulting error in measuring the angular deviation will be proportional to the linear error divided by the distance $S A$ of the slit from the revolving mirror. Thus, as we increase the focal length of the lens, the error from atmospheric tremulousness, dispersion, and from the imperfection of the lens itself will be increased in the ratio $S A + A L : S A$. It is true that by choosing moments of fine atmospheric definition this source of error will be diminished. But general experience with the telescope shows that it cannot be increased indefinitely, while the necessity of confining observations to exceptional periods of time renders the execution of the work correspondingly difficult. The result is that we cannot increase the brilliancy of the image many fold by elongating the focal distance of the lens without increasing too much this source of error.

But there is another possible drawback to the use of the lens in the position shown in Fig. 2. The image of the lens, or some part of it, will necessarily pass through the field of view with every revolution of the mirror, and there will thus be a certain amount of illumination of the field

It is not easy, without actual trial, to estimate the greatest distance at which a return image can be seen on this system, without making the image too bad for good observations. There was no difficulty in computing that, on the other system, a distance of three or four kilometers could be employed; it was therefore decided to adopt it, and to look into the revolving mirror with a telescope.

It is evidently a desideratum that the illumination of the field by the lens, L, should be avoided. This was effected, and certain other advantages gained, by using separate telescopes for sending a ray out and receiving the return flash. In order that the two reflections should, however, take place from the same part of the revolving mirror, the two telescopes would require to be placed in the same horizontal plane. But, by this arrangement, not only would the measurement of small deviations have been impossible, because one telescope would have been in the way of the other, but, as experiments show, the desired end of darkening the field would not have been

reached, owing to the quantity of light irregularly reflected from the mirror when the sun's rays shone upon it. Although all these difficulties may not be insurmountable, yet it was deemed best to avoid them by placing the two telescopes one above the other and elongating the mirror so as to receive the rays from each telescope. An important condition of accurate measurement was thus obtained through the elimination of the zero point, and the feasibility of comparing deviations in opposite directions. As we have defined the angle from which the velocity is concluded, it is that between the outgoing and the incoming ray. A measure of this sort is subject to constant error, arising from the different natures of the two objects between which the measure is made, an error analogous to the index error of the old quadrant. With the new arrangement the result depends upon the difference of deviations with two different velocities of the mirror. Theoretically the position could be observed with any attainable velocity—positive, negative, or zero. But practically, the observations were confined to high positive and negative velocities.

Besides thus eliminating the necessary uncertainty in the measurement from a zero point, the observer can on this system make the field of his telescope almost as dark as he pleases by shutting out extraneous light, and can thus be enabled to see a much fainter image.

The method of doing this will be shown more fully in the detailed description of the apparatus, but it may be summarily mentioned here that it was secured by completely darkening the room in which the observations were made with the exception of one opening for admitting the ray of light to the slit of the sending telescope, and another for sending and receiving the reflected light. In order to prevent the latter opening from acting as an illuminated slit the light was sent through a small opening in a dark box, about fifty yards distant on the path of the ray. The result of this arrangement was that, supposing the observer to look into the receiving telescope and slowly turn the mirror, he would see no light whatever except while the mirror was turning through an angle of a very few minutes of arc. This light would come from the region surrounding the distant reflector, and, if necessary, could have been entirely cut off by surrounding that region with a blackened surface. But as the ordinary illumination of daylight was reduced to, perhaps, its thousandth part without this additional auxiliary, the latter was not deemed necessary. As the experiments were actually conducted the illumination was just sufficient to permit the spider line in the telescope to be seen on a faint illuminated back ground. With a fainter light bright wires would have been necessary.

This arrangement necessitates the use of a mirror sufficiently long to receive the outgoing and incoming rays on separate portions of its surface. Its form is therefore rectangular, which, with high velocities, may be less advantageous than the circular form. But whichever of the two forms of mirror might be used, a larger one than had previously been employed was absolutely necessary to secure the required telescopic power. The problem of the structure of the mirror was therefore altogether one of the most embarrassing ones presented.

The plan first proposed was to employ for the mirror a rectangular piece of glass, 6 or 8 centimeters in breadth, and double that amount in length, silvered on both

sides, and held in a rectangular frame. This arrangement would have the advantage of permitting the use of both surfaces as reflectors, and thus doubling the amount of light received from the distant image. It was, however, seen that the problem of constructing a frame which would remain stable at speeds exceeding 200 turns per second, was an extremely difficult one, because the centrifugal force would be such as to produce a considerable curvature in the most rigid material that could be used. Moreover, owing not only to the resistance of the air, but to the possible effect of the air vortices upon the ray of light, which would injure the definition, it would have been necessary to place the revolving mirror in a vacuum. While still in doubt how the difficulties could be best overcome, the writer was fortunate enough to interest Prof. HENRY A. ROWLAND, of Baltimore, in the question. This gentleman suggested to dispense with the frame entirely and employ a single piece of polished steel as the mirror, which could then be made of sufficient thickness to guard against lateral flexure. It was soon seen that not only could this plan be adopted to advantage, but that the mirror could be thickened into a regular prism, with four polished sides, thus doubling again the amount of light which could be employed, and lessening the air vortex in consequence of the oblique motion of these reflecting surfaces. The use of the vacuum could therefore be dispensed with

It is true that this construction would make the mirror very heavy in comparison with any which had before been driven at high velocities, and would proportionately increase the friction which might be produced by any want of coincidence between the axis of inertia and that of rotation. The jarring produced by this cause was one of the difficulties encountered by FOUCAULT in securing high velocities with a mirror only weighing a few grammes, and unless great perfection in construction could be procured the difficulty would be greatly increased when the revolving body weighed 1 or 2 kilograms. It was, however, deemed advisable to diminish the dimensions of the mirror to 1½ or 2 English inches in breadth, as the diminution of optical power would probably be more than compensated by the more rapid rotation which would be attainable.

As the transmitting telescope must send its rays in a direction approximately the same as that from which they were received, an inconvenience would arise from having the telescopes directly above each other, owing to the observer being near the transmitted ray of light. The transmitting telescope was, therefore, bent at a right angle as near as practicable to the objective, and a reflector inserted at the angle to deflect the light. The observer was thus completely out of the way of the transmitted ray.

A difficult problem was the method of driving the mirror. If gear-wheels of any sort were used for this purpose with so heavy a mirror, not only would they be speedily worn out by the attrition, but a jarring, which might seriously interfere with the quality of the image, was to be expected. It was, therefore, almost an absolute necessity that the power should be applied directly to the mirror. The application of electricity for this purpose was suggested by some of the physicists consulted, but it appeared to the projector that the air turbine was the most certain in its action. Various forms of turbine were taken into consideration, though the conditions of the problem greatly narrowed the choice.

Being an essential part of the plan that the mirror could be turned in either direction without any change of arrangement, it was necessary either to use a reversible turbine, or to employ two turbines, one for each direction of motion. In the latter case one would be placed at the top of the mirror and the other at the bottom. It was, however, found extremely difficult to apply any of the usual principles of the turbine wheel without weakening the supports of the opening in which the pivots of the mirror must turn. It was at length concluded that the most certain and convenient arrangement would be a pair of simple fan-wheels, one at each end of the mirror, to be driven by an air-blast. The reversal of the direction of the blast was a very simple problem. By having blasts on the opposite sides of each wheel all pressure on the axis was avoided. It is true that this system involved a considerable waste of power, and that a stronger blast was necessary than would have been required for a regular turbine; but this drawback was believed to be more than compensated by the simplicity of the apparatus. The exact structure of the fan-wheel can be seen by reference to the plates and to the detailed description of the apparatus. As a matter of fact a speed of 250 turns per second was attained without difficulty, and this was believed to approach the limit of safety. From the above account of the adopted system, it will be seen that the solution of the problem consists fundamentally in the determination of the angles between the directions in which the return ray was reflected from the revolving mirror when the latter moved with various velocities in both directions. The avoidance of systematic error in the measurement of this angle required careful attention, and a firm mounting of the whole apparatus. The mirror was mounted on a firm brick foundation, and made to revolve around a vertical axis, in order that the advantage of measuring a horizontal arc might be attained, and the coincidence between the direction in which the telescope swung between one extreme and the other, and the direction of motion of the mirror might be secured. The observing telescope was supported on a frame, which could be swung around a vertical axis, coincident with the axis of rotation of the mirror, the outer end of this frame resting upon a horizontal arc, with a radius of 2.4 meters. This arc was connected with the support of the revolving mirror by a trussed frame, and to avoid all spring one end of it was firmly screwed to a stone pier, on which the whole arc rests. Room was thus left to allow contraction and expansion by variations of temperature without detracting from the firmness of the mounting. On the upper surface of this arc a set of fine divisions were set, and were read by a pair of microscopes attached to the moving-frame.

The measurement of the speed of the mirror was another problem, and not free from difficulty. A perfect chronographic record of each revolution of the mirror was no doubt attainable by galvanic methods. But the reading of such a record would have entailed much unnecessary labor, since what was really wanted was not the velocity at any moment, but the mean velocity through the period of each experiment. A system of wheel work was therefore attached to the mirror, the axis of which carried a small pinion. This pinion geared into a larger wheel, on the axis of which a second pinion geared into a second wheel; the latter made 28 turns to each turn of the mirror, and broke a circuit at each revolution. Thus every twenty-eighth turn of the mirror was recorded on a chronograph along with the beats of a sidereal break-circuit chronometer.

Chapter III.

DETAILED DESCRIPTION OF PHOTOTACHOMETER.

The general features of the instrument may be seen by reference to Plates I and II. The plates following show details of the several parts of the instrument. For convenience we shall first describe the essential parts, as shown in the first two plates. They are:

First. The sending telescope F (Plate I), into which the light of the sun, thrown from a heliostat, enters at the slit S, passes down the tube, and is reflected by a plane mirror at the elbow, through the objective J.

Second. The revolving mirror in the box m, which receives the light from the objective J, and reflects it to the distant mirror along the line Z.

Third. A stiff frame, $MMMM$, firmly screwed into stone caps surmounting the brick piers V, W, and having fixed to its end a horizontal divided arc, A. A receiving telescope, L, of which the objective is immediately below J, is mounted in adjustable Y's, on a frame N, moving horizontally around a vertical axis coincident with the axis of m, and carrying at its further end a pair of microscopes, p, h, for reading the divisions on the arc below. That portion of the light which each face of the mirror reflects in a certain definite direction, Z, toward the distant mirror is returned by the latter on the same line and again reflected from the lower part of the mirror into the receiving telescope L. If the mirror remains immovable during the time of passage of the light, the direction of the return ray will be defined as parallel to the ray which passed from J to the mirror; but if the mirror is moved during the interval, the direction of the reflected ray will be altered by double this angular motion, and this change of direction is measured by the motion of the telescope KL, when the latter is brought into such a position that the return ray is seen through it upon the micrometer wire. The extreme range of motion is about 8°.

In arranging the detailed description it will, perhaps, be convenient to follow the course of the ray, as in the above general description.

The slit and accessory apparatus of the sending telescope, Plates I and III.—An elevation of the slit S and the accessory apparatus is shown in Plate III. The brass plates qq, whose adjacent ends form the edges of the slit, can be moved horizontally so as to change the width of the slit from zero to 6 or 7 millimeters, by turning a thumb-screw, h, above the plate. The motion is such that the two sides move equally and simultaneously, leaving the center of the slit unmoved when its width is altered. The motion is effected by two slightly inclined pieces moving in corresponding inclined slits ss, in a plate moved by the screw H.

A low-power microscope, R, forming, in fact, an eye-piece, can be turned around the

axis, *A*, in such a manner as to look into the slit. This has a twofold object: First, to set the slit longitudinally in the focal plane of the telescope, and, second, to adjust the slit and sending telescope so as to send the light to the distant mirror. To effect the latter object the slit is opened as widely as possible. The eye-piece, *R*, is pointed into it, and the revolving mirror is set so as to permit the distant mirror to be viewed by reflection along the line, *Z*. If the image of that mirror does not appear near the line passing horizontally through the middle of the slit, the sending telescope, *F*, is turned on its own axis until the image of the mirror is seen on that line. This adjustment renders it certain that a ray of light passing through the slit in such a direction as to fill the objective, *J*, will be reflected to the distant mirror when the revolving mirror is in its proper position.

To adjust the focus the slit is attached to a sliding tube, which can be drawn in and out until the slit, *S*, is found to be in the conjugate focus of the distant mirror.

As shown in the figures (Plates II, IV, and V), and as the apparatus was used during the first two seasons, the sending telescope is above the receiving one. But in 1882 a change was made so that either telescope could be used above.

The Revolving Mirror (*Plates V and VI*).—The revolving mirror, with such of its accessories as are firmly connected with it, is shown in perspective in Plate VI, Fig. 5. Its essential part is a rectangular prism of polished steel, of which a horizontal section is 37.5 millimeters square, and of which the height is 85^{mm}. Its vertical faces (of which only two, *A*, *B*, are shown in the drawing) are nickel plated and form the surface from which the light is reflected. Although the polish is not equal to that of silver, it commends itself by its durability, not having suffered materially during the three summers through which the experiments extended. The makers did not succeed in making the faces perfectly flat to their edges. In consequence the faces, instead of being planes, approach more nearly to semi-elliptical cylinders, of which the horizontal section is a semi-ellipse, differing exceedingly little from a straight line. This figure was indicated by the fact that the best focus when the distant mirror was viewed through the sending telescope was slightly shorter for vertical than for horizontal lines, the difference being a quantity of the order of magnitude of one two-hundredth of the focal length. This would indicate a figure of which the mean radius of curvature was about 4,000 feet. The fact that the observations had to be made through three kilometers of air, more or less, and with an objective only about 35^{mm} in diameter, made impossible the production of an image of sufficient sharpness to accurately investigate the deformation.

To the top and bottom of the mirror are fastened pairs of circular plates, *C* and *D*, each pair holding between them a set of fans, 12 in number, forming a species of fan-wheel on which an air-blast impinges. Owing to the great centrifugal force to which the wings are subjected, they are held between two horizontal circular plates, to each of which they are soldered as well as to the interior cylinder, *SC*, behind them. A section of these fan-wheels is shown in Plate VI, Fig. 3, which shows a horizontal section of the mirror in the box which contains it while running.*

The top and bottom of the axis of the mirror terminate in pivots, one of which is

*It is an error that the figure is drawn as if the wings of the fan-wheel were contained within a cylindrical surface.

seen in Fig. 1, Plate V. These pivots are slightly conical, and the bottom one terminates in a nearly flat horizontal surface, about 2^{mm} in diameter, which rests upon a diamond in a way to be described presently.

Mirror Box (Plates IV, V, VI).—The mirror is contained in a heavy cylindrical box, *M*. This box is held by firm standards, *SS*, which screw into the bed-plate of the instrument, permitting an adjustment by which the box can be truly leveled. It was stated by the makers to have been turned in a lathe, so that its different planes and lines should be truly rectangular.

The bottom of the box is pierced by a screw-hole, through which passes a hollow cylinder, having both its internal and external surfaces cut into a screw. This screw turns by a head, *R*, and can be adjusted in height so as to bring the revolving mirror vertically into the proper position. Its opening widens conically at the top so that the conical surface of the lower pivot of the mirror fits accurately into the top of the opening.

An interior screw, turned by the head *T*, passes through the opening. This screw terminates on the top in a diamond, whose upper surface is slightly convex.

When the screw *R* has been properly adjusted, *T* is screwed in until the diamond touches the end of the steel pivot of the revolving mirror, so as just to support the weight of the latter. The proper position is indicated by the freedom with which the mirror can turn on its axis, and the adjustment is made with a nicety which is limited only by the difficulty of giving a sufficiently minute motion to the screw.

The two additional screw-heads belong to binding screws, which clamp both the other screws when adjusted.

Near its top and bottom the box expands into square plates, which, however, perform no vital function, being merely placed there for the convenience of using eight openings, four of which are shown at *a a a a*, Plate IV. These openings terminate in short cylinders, over which can be fitted rubber air-tubes, as will be more fully described hereafter. The height and position of these openings are such that, when the revolving mirror is in position, a blast of air through them strikes upon the fan-wheels, as shown in Fig. 3, Plate VI.

The top of the box is closed by a circular brass plate, *KK* (Plate V, Fig 1), on the bottom and near the circumference of which a slight shoulder is turned, so that the interior fits into the top of the box, which it enters to perhaps the depth of a millimeter. It is then fastened in place by three screws, one of which is shown at *q*. Its center is perforated to receive the vertical screw, whose head is shown at *r*, and whose axis is perforated through and through by an opening. This opening is slightly conical at its lower end, where it fits over the upper pivot of the revolving mirror. In use the head *r* is turned until it fits accurately around the pivot, which is shown by increased friction in the motion of the mirror. It is then fixed in place by a clamp, *t*.

Apparatus for recording revolutions.—Around the upper pivot of the mirror fits a pinion, *b*, Plate V, Fig. 1, and Plate VI, Fig. 5, containing 16 teeth. This pinion gears into a toothed wheel, *c*, containing 64 teeth, whose axis passes through the plate *KK*, and gears, by a ten-toothed pinion at the top, into a wheel, *e*, of 70 teeth, which thus makes one revolution to every 28 revolutions of the mirror. A knob, *k*,

on this wheel, Plate VI, Figs. 1, 2, at every revolution, touches a spring, which forms part of an electric circuit passing through the standard h, and thus breaks the circuit once in every 28 revolutions of the mirror.

Chronograph Record.—One of the cylinder chronographs used by the Transit of Venus Commission was altered so as to perform one revolution in 10 seconds. Its pen was connected with a closed circuit passing through a break-circuit chronometer, and through the spring just described on the box of the revolving mirror. Thus two series of indentations appeared in the line marked by the pen on the chronograph, the one at intervals of a second and the other at the intervals in which the mirror performed 28 revolutions.

Air-blasts for turning the mirror.—As above remarked the mirror is turned by a blast of air impinging on the wings of the fan-wheels CC. After a careful examination of the different methods of generating a blast of sufficient power with a small machine, it was found that the ROOT blower offered the greatest advantages for this special purpose. A blower of sufficient accuracy to secure a high pressure was therefore specially constructed by the ROOT Company. It was operated by a portable engine loaned by the Bureau of Steam Engineering of the Navy Department. The blast from the blower passes through the pipe D, Plates I and V, which divides at H into two branches. These branches can be separately closed by globe valves TT, whose rims are cut into a groove to receive the endless cord xx. Beyond the valves each branch is subdivided into four others, which can be separately connected by rubber tubes with the openings *a a a a*, Plate VI, Fig. 3, the arrangement of course being such that the four blasts from one pipe all conspire to turn the mirror in the same direction.

Mode of applying the blasts.—To keep the mirror revolving with any speed, at pleasure, one of the valves, T, is opened; the other valve is closed and connected with the endless cord x. When the observer at the eye-piece of the telescope finds the mirror to attain the desired speed, he slightly opens the other valve by means of the cord, so as to produce a minute counter current of air. The speed can thus be governed with exceeding delicacy. It was remarked that the higher the velocity the more delicately could the adjustment be made.

Mode of reading off the position of the receiving telescope L.—The end of the frame N terminates in an arc, BB, holding the bearings of the reading microscopes L, R, as shown in Plate VII. The microscopes can read either of two sets of fine divisions on the upper surface of the fixed arc AA. The existence of two sets of divisions was purely accidental, arising from the fact that the makers were not fully satisfied with the first set which they cut. Practically, only one set was used.

Reading microscopes.—It was not judged necessary to supply the microscopes with micrometers; in lieu thereof each eye-piece was supplied with four parallel wires whose distance apart corresponded to $2''.4$ of arc on the divided limb. In using, the telescope was set so that some division was nearly midway between the central wires of the microscope R. After the observation was made, the position of the image of the division north or south of the middle was estimated for each microscope in a measure whose unit was the distance between two adjacent wires.

The outer face of the fixed arc A was supplied with a set of coarse divisions

corresponding to those on its upper face, which could be read off by a pointer, and which were numbered. The movable arc B could be clamped in any position by the clamping-screw C, and then accurately adjusted by the slow motion D.

The eye-piece of the receiving telescope.—The eye-piece micrometer E was fitted into the end of the sliding tube H. Its frame was supplied with a fixed vertical thread and a pair of movable ones distant about 30″. The value of one revolution and the other constants are considered in connection with the determination of the unit of arc on the divided limb.

The fixed mirrors and their adjustments.—In order to strengthen the return image and to avoid loss from any slight displacement of a single mirror, two concave mirrors, each about 40 c. m. in diameter, were used side by side to reflect the ray. The radius of curvature was found to be about 3,000 meters, but was not determined with great precision. Two different locations were used for the mirrors, the one on the grounds of the Naval Observatory, and the other near the base of the Washington Monument, as will be more fully described hereafter.

The mirrors were mounted on cast-iron stands, each consisting of a triangular base, about a meter in length, with an upright ring at the larger end to receive the mirror. The opening through this ring was somewhat less than the diameter of the mirror. A fine screw, passing vertically through the small end of the base, and resting upon a plate set in the top of the stone pier, served to adjust the normal of the mirror vertically; and a pair of antagonistic screws, acting horizontally, served to direct it horizontally. A detailed description of this motion is not deemed necessary, because a system admitting of finer adjustments should be used in any subsequent observations.

At each station the base of the mirror consisted of a solid brick pier rising some 10 feet above the ground, surmounted by a stone cap, and covered by a frame building with an elevated floor. The accurate adjustment of the mirror, so that the normal of its surface should be directed toward the revolving mirror, was a somewhat delicate and troublesome operation, owing partly to the fact that special apparatus of the necessary delicacy was not expressly constructed for the purpose. In the first series of experiments a coarse adjustment was made by means of a reversible collimator set upon fixed adjustable Y's. The latter were so adjusted that the coarse wires of the collimator looking at the station should be coincident with the image of the opening through which passes the light from the revolving mirror. The collimator was then reversed end for end, so as to look into the revolving mirror, and the latter was adjusted so that the reflected images of the coarse wires of the collimator should coincide with the wires themselves. By this means the adjustment could be made with sufficient precision to enable the observer at the sending station to find the return image of the sun reflected from the revolving mirror, but not precise enough for use in the experiments. To make the final adjustments the observer at the station, after finding the image, signaled to the assistant at the fixed mirror in what direction to turn the adjusting screws, and thus, by a series of trials, each mirror was got into position.

It being found difficult to see the reflected image of the collimator wire, a different system was used in the set of measures from the Washington Monument. An attempt

was made to set the mirror with such precision that no signals should be required from the observing station. The method was as follows:

Two signals, each in the form of a rhombus, plainly visible with a telescope from the Monument, were set up in the grounds of Fort Myer in such positions that the line from the fixed to the revolving mirrors bisected the angle formed by the lines from the fixed mirror to the signals. A few meters distant from the pier A supporting the reflectors, on the line toward the station, a second pier, B, was built to about an equal height, with Y's for two pairs of collimating telescopes, one pair for each mirror. A collimating telescope C was pointed upon the mirror and so directed that its cross wires should coincide with the image of the left-hand signal coming from direction P, and reflected by the mirror, C remaining unmoved, a second collimator, also looking into the mirror, was placed at D, on the line from P, and so adjusted that the image of its wires, seen in C by reflection from the mirror, should coincide with the wires of C. The line of sight of the collimator D was then known to coincide with the right line from the signal P. The collimator C was then removed, and the mirror M so adjusted that the image of the second signal, coming from the direction Q, and reflected from the mirror, should coincide with the cross wires of the collimator D. The result of this adjustment would be that a ray of light from one signal, reflected from the mirror, would pass through the other signal. To insure accuracy, the adjustment was repeated alternately with the two collimators until it was found perfect for both. The normal to the mirror should then pass through the revolving mirror.

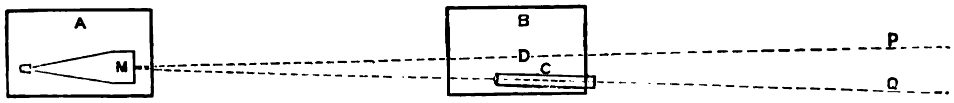

FIG. 3.

As a matter of fact, however, it was found that owing to the insufficient optical power of the collimators, and the want of a sufficiently fine movement of their supports, the final adjustment of the reflector had to be made by signals from the station as before.

Course of the reflected ray from the revolving mirror.—Owing to the small fraction of the incident light from the revolving mirror which could be reflected from the distant mirror, owing also to the necessary loss by dispersion and absorption in the atmosphere, it was necessary to take special precautions for darkening the field of view of the telescope in order that the return image might be distinctly seen. Since no light should enter the receiving telescope of the phototachometer except by reflection from the revolving mirror, it was necessary, as far as possible, to prevent any light falling upon the latter in a horizontal direction, except what came from the distant reflector. The following precautions were taken to secure this end:

1. The house in which the phototachometer was placed was darkened during each experiment, and a hole was made in the wall just large enough for the passage of the outgoing and the returning rays.

2. At a distance of some 30 meters, on the line towards the distant reflector, a wooden box, in the form of an elongated parallelopiped, open at the end towards the phototachometer, was mounted upon frame-work. It was made of such size that no

light from around it could enter the opening in the side of the house so as to fall upon the revolving mirror. An elongated opening, 4 inches high, was cut through its further end to admit light from the mirror.

The theory of this device, which may be found useful when all extraneous light is to be shut out from a telescope, is this: The light may be shut out simply by interposing a perfectly black screen at the proper point, with an opening in it no larger than is necessary to view the object, or to admit the desired light. But no screen can be made perfectly black. It is, therefore, necessary to cut off as much daylight as possible from the screen without interposing another screen between the first one and the telescope. This is effected by building out from the screen a box in the direction of the telescope, which will cut off a portion of the light, which will be larger as the box is longer. By arranging a series of such boxes, whose distances should, in theory, increase as the terms of a geometrical progression, the field of view surrounding the point of observation may be darkened to any extent, how brightly soever the sun may be shining upon the boxes

The result of this arrangement was that if the observer looked into the receiving telescope, and the revolving mirror was slowly turned, he saw no appreciable amount of light during the revolution of the mirror except when his line of sight passed through the opening in the dark box. Then that part of the Monument in the immediate neighborhood of the reflector was visible, brightly illuminated by the sun. Had a yet greater degree of darkness been necessary, it could have been secured by erecting a black wall around the reflector. As the arrangement was actually made, the illumination of the field when the mirror was in rotation was no greater than was necessary for distinct vision of the spider lines. Hence, had the utmost degree of darkness been secured, illuminated spider lines would have been necessary. As a matter of fact, however, the reflected image was, under all favorable conditions, so clear and distinct that there was no need of further diminution of the light.

It may be of interest to note that although, from the known curvature of the distant mirrors, all the light reflected from them should have been collected on its return to the station within a circle of ten or fifteen centimeters radius, yet twinkling and flickering rays were scattered to three or four times this distance. It would seem from this that a single ray of light cannot be sent through seven kilometers of air in the daytime without portions of its light suffering deviations of from sixty to eignty centimeters, or, say, the ten-thousandth part of the distance.

CHAPTER IV.

DETERMINATION OF THE ANGULAR VALUE OF THE DIVISIONS OF THE ARC.

From the description of the instrument, it will be seen that the final result for the velocity of light depends upon the arc through which a horizontal arm, turning on a vertical axis coincident with that of the mirror, was moved in different observations. The angular position of this arm was determined by reading a set of divisions cut on the upper and horizontal plane surface of the arc. The readings were made by two microscopes, one on each side of the eye-piece of the telescope. The angular distance between the divisions was not fixed so as to give any definite value in arc. The method of making the readings with the two microscopes has already been described in Chapter III, and need not be repeated.

In order to eliminate the effect of any possible deformation of the supports of the telescope, as it was moved from side to side, it was deemed an indispensable condition that the measures of the value of the divisions should be referred immediately to the optical axis of the telescope used in actual observation. This precludes all determinations resting upon measures of the linear distance between the divisions, and reduces the problem to that of determining the angle through which the optical axis of the telescope itself moved when the reading of the microscope was changed by definite amounts. Two distinct and entirely independent methods were employed. The one rested on the computed angular distance between the divisions of certain linear steel scales, placed at a distance from the center of motion, and the other upon measures of the motion of the optical axis of the telescope by means of one of the great theodolites of the Coast Survey. These measures will be described separately.

DETERMINATION BY MEANS OF SCALES.

In applying this method large wooden posts were set firmly in the ground in the line of the instrument, the one about five meters west from the revolving mirror, in the direction in which the telescope looked when at the middle of its arc, and the other nine meters east of the mirror. These distances were determined by the condition that the western post should be in the focus of the telescope when a certain lens, supplied by the makers for the purpose, was placed over the objective, and the other distance was determined by the condition that when the telescope was reversed in its bearings, its conjugate focus should be found at the surface of the post towards the south.

The linear measures were supplied by means of DARLING, BROWN and SHARP'S steel scales. The scales employed for reading with the telescope were each one meter in length and divided to millimeters. The divisions, although coarse as compared with those of an astronomical instrument, were very well cut. On top of the posts already described was placed a horizontal plank, and on this plank the meter-

scale rested in such a position that the perpendicular upon it from the revolving mirror fell within 1 or 2 centimeters of its middle point. When the adjustments were all made, and the revolving mirror removed, the divisions on the westernmost scale were in the focus of the eye-piece of the telescope. The arm supporting the latter was then moved into different positions, and the microscopes read exactly as in the observations on the deflected ray of light. At each setting one or more of the millimeter divisions were read with the eye-piece micrometer. These readings were repeated along those parts of the scale most used in observations of the deflected ray.

To read the east scale the telescope was taken up and reversed in its bearings Since, however, the bearing near the revolving mirror was not in a position to admit of the observer looking through the eye-piece when the telescope was reversed without changing its middle point, a vertical Y was made, which could be fixed in the supporting frame of the telescope, near its middle. Then when the telescope was reversed, the end near the eye-piece rested upon this additional Y, and the object end projected several feet beyond the eye-piece Y. As the latter bearing was then near the center of gravity of the telescope, the eye end of the latter was fastened to the frame by an elastic piece of twine in order to prevent accidental displacement. Readings were then made on the scale as before.

The two scales were arranged in this manner in order to eliminate any want of coincidence of the center of motion of the telescope with the axis of the revolving mirror. The determinations from the east scale would be more accurate owing to the longer arc used, but they are subject to some objection owing to the reversed position of the telescope. Any error from this cause could arise only from some twisting or bending motion in the supporting frame of the telescope. Although the motion was so easy and smooth as not to suggest any serious error from this source, there was a possible source of error of this nature which had to be considered.

While the experiments were in progress it was my habit from time to time to place a spirit-level on the moving frame carrying the receiving telescope in order to ascertain that every part of the frame moved in a horizontal circle. It was found that when the spirit-level was in a radial position its readings were unchanged as the frame was moved. But when placed tangentially it showed that as the frame moved towards the north its north end was a little elevated, and that the south end was relatively elevated when moving towards the south. The effect was the same as if the telescope, instead of being supported by a horizontal circular arc, were suspended from a point of support some 2,000 feet vertically above the instrument. Two hypotheses might be made respecting this motion; one that the whole frame turned without changing its shape; in other words, that the motion was round a slightly inclined axis, and the other that the change could be entirely represented by a slight motion of different parts of the frame in the vertical direction, thus producing a strain. In the first case the angular motion of the telescope would be different from that of the base of the frame, while on the second it would not.

To decide between the hypotheses the spirit-level was placed on top of the Y supporting the telescope. It was then found that the torsion did not change the verticality of the Y, the top of which preserved its horizontality, thus showing that the

second hypothesis was the true one, and that the value of one division was not thus affected. I conclude from this that no serious error resulted from torsion of the frame when the telescope was reversed in its Y's.

It was necessary at each determination to measure the distance between the middle point of the scale and the center of motion. This measure was made by various steel rules and straight edges procured from DARLING, BROWN and SHARP. These rules were cut and squared with a precision far beyond all the requirements of the problem. To employ them a frame-work of plank was constructed between the instrument and the supports of the scales on each side, and the rules were set on edge on the support by holding them in adjustable wooden blocks. When the rules were in position each one was separately leveled by a spirit-level placed on its top, and care was taken that the ends should touch so closely that no noticeable amount of light could be seen between them. The vertical alignment was secured by sighting along from the outer end to that next the revolving mirror. As the adjustment could not be made so that one end of a rule should always terminate at the face of the mirror, the space between the inner end and the mirror was measured by an additional rule with the aid of some finely divided millimeter scales. To avoid error it was common to use two separate scales, the one divided to millimeters, the other to small fractions of an inch. The proper position of the mirror was determined by the condition that the observer looking into it along the lines of the scales saw the latter reflected in an apparently straight line with that of the scales themselves. The radius of the mirror being added to the measured distance gives the perpendicular distance between the center of motion and the divided scale.

During the first season these measures were found very troublesome owing to lack of stability in the wooden supports. The latter were, therefore, removed and replaced by solid brick walls. The walls supporting the divided scale were covered with stone caps, in which were set metal supports to keep the scale in the same invariable position.

Effect of errors in placing the separate measures.—The system of measuring, just described, would lead to erroneous results unless the different rules formed an accurately straight line. If we consider the measuring scales as lines, then any deviation from the line of measurement will result in the apparent measured distance being too great. The alignment was made with the unaided eye, by which it could be assured that no end of a rule deviated from the direct line more than 3 or 4 millimeters. If two metal bars were placed zigzag to such an extent that each pair of adjacent ends deviated by 4^{mm} from the right line joining the farther ends, the distance measured would, on the hypothesis of no thickness, require to be multiplied by $\cos \frac{4}{1000} = \cos 004 = 1 - .000008$. Therefore, in this extreme case, the error would be less than the hundred thousandth part.

A simple calculation will show that, if the bars were placed along the arc of a circle, so that in the space of 10 meters the deviation at the middle point was 6 millimeters, the error would be less than the millionth part.

But we have to take into account the fact that the bars are really rectangular parallelopipedons, having a thickness of from 2½ to 3 millimeters. Now, if the bars

were so placed that the alternate vertical edges should be in a right line, the measured distance would be too small by a large fraction of a hundred-thousandth part.

Since the two sources of error act in opposite directions, and neither can with due care amount to the hundred-thousandth part, I conclude that there is little danger of serious error from this source.

An error in the vertical direction would be much more serious, since the vertical breadth of the bars ranged from 1 to 1½ inches, or from 24 to 60^{mm}. But, by leveling each bar its inclination to the horizon was generally assured to 1′ of arc, and the additional precaution of seeing that no light could penetrate between two contiguous ends was generally taken.

Notwithstanding all the care taken the separate results are more discordant than I should have anticipated. The mean deviation of each one from the general mean is about the ten-thousandth part.

Lengths of the scales.—In the methods just described, it is essential that the lengths of the rules which were used as common measures to determine the distance between the center of motion and the dividing scale should be referred directly to the distance between the millimeter divisions on the divided scales used in observation. It was deemed best to do this independent of any hypothesis respecting the absolute length of the scales. The method was as follows:

One of the meter scales, represented by *A* in the diagram, was laid flat on a horizontal surface, and a true straight-edge, *B*, placed firmly against one end of it, as shown in the figure.

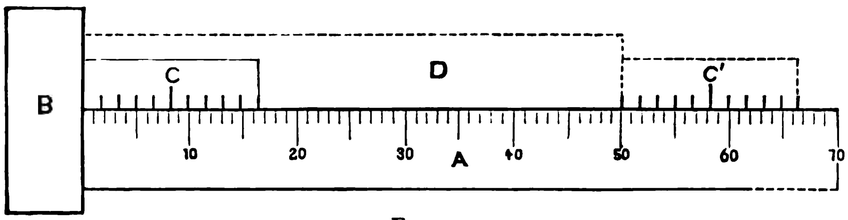

Fig. 4.

The meter scale, it will be recollected, was divided into millimeters on each edge. In the corner, at *C*, was placed a two-inch measure, divided to twenty-fifths of an inch. The difference between 1 millimeter and $\frac{2}{50}$ of an inch is 0.016^{mm}. The divisions on *C* thus serve the purpose of a vernier, by which the reading in millimeters of some definite line on *C* can be determined within 0.01^{m} or 0.02^{m}. The measure *C* was then removed, and a rule or straight-edge, *D*, which was to be used as an end measure in determining the distance, was placed against the meter scale and the straight-edge, as shown by the dotted outline *D*. The measure *C* was then placed in the position *C′*, and again read against the millimeter divisions as a vernier. The difference of the readings in the two positions, *C* and *C′*, give the motion of *C* in millimeters while passing from its original position to *C′*; that is, the length of *D* when used as an end measure, expressed directly in terms of the divisions on the millimeter scale.

The sum of the lengths of the two meter scales, themselves referred to the same scale of millimeters, was found by pressing the two meter scales together, end to end, and then, by means of a smaller scale, reading the distance between some two divisions, one on each side of the touching point. The probable error of this determination was of the order of magnitude of the hundredth of a millimeter.

The uniformity of the divisions on the meter-scale itself is here of capital importance. The measures show no sensible deviation from perfect uniformity, from which I infer that the errors could not exceed two or three hundredths of a millimeter. It is also to be remarked that the readings of the small scale C were made near the ends of the meter scale, and that when the east scale was read the value of one division on the arc of the phototachometer depended upon reading near the ends, so that errors of the intermediate divisions would be eliminated.

The lengths of the several scales as end measures were thus found to be as follows. As an example the separate results for the yard measures are first given:

The best determinations of the three yard measures were:

	mm.	
Yard A:	914.546	
Yard B:	914.545	
Yard C:	914.524	
Sums of lengths	2743.615,	wt. 4
The other results were	.612	2
	.604	1
	.639	4
	.610	4
General mean	2743.619	

The lengths of the yards may be supposed equal. The following are the concluded lengths of all the measures in terms of the divisions of the meter-scale:

	mm.
Each yard measure	914.540
Each meter-scale	1000.103
Straight-edge X	918.340
Plain straight-edge	917.567

As a matter of interest, though not affecting the work of the present paper, the divided inch on the yard measures was compared with the divisions on the meter-scale with the results:

		mm.		in.
On yard A:	34 inches =	863.653;	one meter =	39.3676
On yard B:	"	.652	"	.3677
On yard C:	"	.639	"	.3684

The standard ratio is 39.37043. The difference of 0.0025 corresponds to the change in length produced by a change of about 10° Fahr. in temperature, and is such that if τ is the temperature at which the meter-scale divisions are correct, the yard divisions are correct at $\tau - 10^\circ$.

Comparison of the four-meter bars with meter-scales.—On some occasions the four-meter bars used in measuring the base line for the triangulation were also employed for the measures under consideration.

In order to refer the mean length of these bars to the divisions of the meter-scale without any hypothesis respecting the uniformity of the various measures, the four-meter bars were measured in the following way: A stand was erected on which the bars should rest side by side in the same manner that they rested on their trestles in actually executing measurements. The distance apart of the bars was 7.15 inches at one end and 7.20 at the other. The steel measuring rules, described above, were then placed end to end between the bars along their length and carefully aligned and leveled. At one end the measuring rule was brought into fiducial contact with a straight-edge extending horizontally across the contiguous agate ends of the two bars and in contact with each of them. At the other end a similar straight-edge was also brought into contact with both agate points, and the 50^{mm} scale pressed against it while resting upon the scale below, which, at this point, was yard-measure C.

The temperature in all the measures was about 70°, but as all the bars are supposed to be of the same material, no account of temperature need be taken in the comparison.

The separate results for excess of the mean length of the four-meter bars above the combined length of the two straight-edges and the two meter-scales are as follows. The small suffixes indicate the weights assigned:

1881.		mm.	
April 5.	E =	164.42_1.	
		164.30_1.	Rod not certainly in contact.
		164.35_1.	Rod perhaps pressed too hard.
		164.41_1.	
April 6.		164.70_0.	Adjustment of rods not tested.
		164.30_4	Adjustments carefully tested.
		164.29_4.	Adjustments carefully tested.
		164.27_4.	Adjustments carefully tested.
	Mean	$164.31 \pm 0^{mm}.02$.	
2 meter-bars		2000.21	
2 straight-edges		1835.90	

$4000.42 \pm 0^{mm}.03$ = mean length of the two bars in terms of the Darling, Brown & Sharp meter-scale divisions

The mean length of the bars is reported by the Coast Survey to be correct at 42° F. The comparison would indicate that the divisions on the D., B. & S. meter-scales are correct in length at a temperature of 58° F.

Effect of temperature.—If the absolute distance between the center of motion and the meter-scale were required, it would be necessary to reduce the measures to some uniform temperature. But assuming the temperature of the divided scale and the measures to remain the same, both would be expanded in the same ratio by a rise of temperature, and the angular result would be independent of the temperature. The precaution was generally taken, when necessary, to protect the scales from the direct rays of the sun, and it is not supposed that any appreciable systematic error has entered into the results in consequence of any possible different temperatures of the scale and the measuring rods.

Observations and results.—In the first of the following tables, column S gives the division on which the left-hand microscope was set. Column L gives the reading of the left microscope, and column R the reading of the right one, which, it will be noted, was set upon a division greater by 206 than the one in column S. The signs are such that when *positive* the scale-reading S should be increased. The column scale reading shows the reading on the meter-scale of an arbitrary zero point in the field of view of the telescope. At each setting the micrometer was set upon 5 different divisions of the scale, and its reading taken on each division. The mean of these 5 readings was reduced to the arbitrary zero point, which was taken as near as possible to the general mean position of the micrometer.

The next column gives the corrections to the scale reading for the mean of the microscope corrections. Each unit of reading is 2″.4 in arc. The corresponding lengths in millimeters on the scales are:

	″	mm.
East scale - - - - - - -	2.4 =	0.11
West scale - - - - - - -	2.4 =	0.07

The sum of the readings is to be multiplied by the half of the factor for the scale, and applied to the scale reading. It would seem that, through inadvertence, the factor for the west scale was used for the east scale also. A minute correction to the results will therefore be necessary.

After several repetitions of the readings the telescope was brought back to its original setting. If a marked change had occurred in the scale reading it was assumed to have gone on continuously, owing to change of temperature and other causes. The correction given on this hypothesis is shown in the next column, and the corrected scale reading is thus obtained. In commencing the reading that point of the scale was found which would be the foot of the perpendicular from the center of motion upon the scale.

The distance of the point of the scale on which the telescope was set from the perpendicular point is given in the next column. This distance is the tangent of the arc, whose radius is the length of the perpendicular upon the scale, and with this length, shown at the bottom of the table, the arc was computed.

In the tables which follow the data are condensed; but it is believed that they need no further explanation. The methods are too simple to render any details necessary.

[1880, August 18. East scale. Telescope reversed. Observer, MICHELSON.]

S.	Microscopes—		Scale readings.	Correction for—		Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
	R.	L.		Mic.	Change.			
d.			mm.			mm.	mm.	"
0	0.0	+ 1.1	7.62	— .04	— .03	7.55	— 512.45	— 11300.9
195	0.0	— 0.1	995.82	+ .00	— .03	995.79	+ 475.79	+ 10493.7
0	0.0	+ 0.7	7.54	— .02	— .03	7.49	— 512.51	— 11302.2
195	0.0	— 0.2	995.83	+ .01	— .03	995.81	+ 475.81	+ 10494.1
0	0.0	+ 1.2	7.55	— .04	— .02	7.49	— 512.51	— 11302.2
1	0.0	+ 0.3	12.53	— .01	— .02	12.50	— 507.50	— 11192.2
2	0.0	+ 0.7	17.68	— .02	— .02	17.64	— 502.36	— 11078.5
3	0.0	+ 0.2	22.70	— .01	— .02	22.67	— 497.33	— 10968.3
4	0.0	+ 0.6	27.82	— .02	— .01	27.79	— 492.21	— 10855.5
5	0.0	+ 0.4	32.84	— .01	— .01	32.82	— 487.18	— 10744.5
10	0.0	+ 0.3	58.25	— .01	— .01	58.23	— 461.77	— 10185.2
15	0.0	+ 0.9	83.66	— .03	— .01	83.62	— 436.38	— 9626.0
20	0.0	+ 0.6	109.02	— .02	.00	109.00	— 411.00	— 9064.7
25	0.0	+ 0.8	134.38	— .03	.00	134.35	— 385.65	— 8508.5
50	0.0	+ 0.6	261.15	— .02	.00	261.13	— 258.87	— 5713.0
100	0.0	+ 0.4	514.42	— .01	.00	514.41	— 5.59	— 123.4
150	0.0	+ 0.2	767.68:	— .01	.00	767.67:	+ 247.67:	+ 5466.0
170	0.0	+ 0.1	869.04	.00	.00	869.04	349.04	7701.4
175	0.0	+ 0.2	894.36	.00	+ .01	894.37	374.37	8259.8
180	0.0	— 0.5	919.70:	+ .02	+ .01	919.73:	399.73	8818.7
185	0.0	— 0.4	944.97	+ .01	+ .01	944.99	424.99	9375.2
190	0.0	+ 0.2	970.40	— .01	+ .01	970.40	450.40	9934.7
191	0.0	0.0	975.49	.00	+ .02	975.51	455.51	10047.5
192	0.0	+ 0.1	980.56	.00	+ .02	980.58	460.58	10158.7
193	0.0	0.0	985.65	.00	+ .02	985.67	465.67	10271.2
194	0.0	0.0	990.72	.00	+ .02	990.74	470.74	10382.9
195	0.0	— 0.2	995.78	+ .01	+ .03	995.82	+ 475.82	+ 10494.3
0	0.0	+ 1.3	7.49	— .04	+ .03	7.48	— 512.52	— 11302.4

Foot of perpendicular, 520^{mm}.
Length of perpendicular from center of mirror to scale:

	mm.
6 yard measures	5487.24
2 straight-edges	1835.91
2 meter scales	2000.21
5 hundredths of an inch + △	1.27
Radius of mirror	19.28
Length of perpendicular	9343.91

Results August 18.

	"		"
Length of arc from 0^d to 195^d	21795.9,	or, 1^d =	111.774
General mean reading for 0^d to 5^d	— 11023.6		
" " 190^d to 195^d	+ 10214.9		
Length of arc for 190^d	21238.5,	1^d =	111.782

[August 19. West scale. Observer, MICHELSON.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	"
0	0.0	+ 0.6	165.91	— .02	. . . .	165.89	— 334.11	— 11866.6
200	0.0	— 0.3	795.40	+ .01	. . . .	795.41	+ 295.41	+ 10494.6
0	0.0	+ 0.6	165.92	— .02	. . . .	165.90	— 334.10	— 11866.3
1	0.0	+ 0.3	169.06	— .01	. . . .	169.05	— 330.95	— 11754.4
2	0.0	+ 0.4	172.19	— .01	. . . .	172.18	— 327.82	— 11643.7
3	0.0	+ 0.3	175.36	— .01	. . . .	175.35	— 324.65	— 11531.3
4	0.0	+ 0.5	178.51	— .01	. . . .	178.50	— 321.50	— 11419.7
5	0.0	+ 0.5	181.66	— .01	. . . .	181.65	— 318.35	— 11308.0
200	0.0	— 0.4	795.40	+ .01	. . . .	795.41	+ 295.41	+ 10494.6
199	0.0	— 0.5	792.21	+ .01	. . . .	792.22	+ 292.22	+ 10381.5
198	0.0	. . . .	789.08	.00	. . . .	789.08	+ 289.08	+ 10270.1
197	0.0	— 0.1	785.94	.00	. . . .	785.94	+ 285.94	+ 10158.8
196	0.0	— 0.3	782.82	+ .01	. . . .	782.83	+ 282.83	+ 10048.6
195	0.0	— 0.3	779.63	+ .01	. . . .	779.64	+ 279.64	+ 9935.3
0	0.0	+ 0.5	165.91	— .01	. . . .	165.90	— 334.10	— 11866.3
200	. . .	— 0.5	795.31	+ .01	. . . .	795.32	+ 295.32	+ 10491.4

Foot of perpendicular at 500^{mm}.
Length of perpendicular from center of mirror upon scale:

	mm.
2 meter-bars	2000.21
3 yards	2743.62
1 straight-edge	918.34
4 inches	101.60
18^{mm}	18.00
Radius of mirror	19.28
	5801.05

Results, August 19.

		"
Mean reading for - - - - - - - - - -	$2^d.5$ —	11587.3
Mean reading for - - - - - - - - - -	$197^d.5$ +	10214.8
Mean value of 195 div. - - - - - - -		21802.1
Mean value of 1 div. - - - - - - -		111.806

[August 20. West scale. Observer, MICHELSON.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	"
0	0.0	— 0.6?	164.18	(?)	. . . .	164.18:	— 335.82	— 11929.6
200	0.0	— 0.5	793.57	+ .02	. . . .	793.59	+ 293.59	+ 10432.1
0	0.0	+ 0.4	164.18	— .01	. . . .	164.17	— 335.83	— 11929.9
10	0.0	+ 0.4	195.75	— .01	. . . .	195.74	— 304.26	— 10810.4
11	0.0	+ 0.2	198.86	— .00	. . . .	198.86	— 301.14	— 10700.0
12	0.0	+ 0.5	202.02	— .01	. . . .	202.01	— 297.99	— 10587.7
13	0.0	+ 0.4	205.20	— .01	. . . .	205.19	— 294.81	— 10475.4
14	0.0	+ 0.4	208.32	— .01	. . . .	208.31	— 291.69	— 10364.8
15	0.0	+ 0.7	211.49	— .02	. . . .	211.47	— 288.53	— 10253.0
16	0.0	+ 0.5	214.66	— .01	. . . .	214.65	— 285.35	— 10140.0
17	0.0	+ 0.5	217.78	— .01	. . . .	217.77	— 282.23	— 10029.3
18	0.0	+ 0.4	220.92	— .01	. . . .	220.91	— 279.09	— 9917.7
19	0.0	+ 0.4	224.09	— .01	. . . .	224.08	— 275.92	— 9805.3
20	0.0	+ 0.5	227.24	— .01	. . . .	227.23	— 272.77	— 9693.5
180	0.0	— 0.4	730.56	+ .01	. . . .	730.57	+ 230.57	+ 8195.6
181	0.0	— 0.5	733.71	+ .01	. . . .	733.72	+ 233.72	+ 8307.4
182	0.0	— 0.5	736.86	+ .01	. . . .	736.87	+ 236.87	+ 8419.2
183	0.0	— 0.5	739.99	+ .01	. . . .	740.00	+ 240.00	+ 8530.4
184	0.0	— 0.5	743.13	+ .01	. . . .	743.14	+ 243.14	+ 8641.7
185	0.0	— 0.5	746.31	+ .01	. . . .	746.32	+ 246.32:	+ 8754.8
186	0.0	— 0.5	749.40	+ .01	. . . .	749.41	+ 249.41:	+ 8864.3
187	0.0	— 0.5	752.59	+ .01	. . . .	752.60	+ 252.60	+ 8977.6
188	0.0	— 0.4	755.73	+ .01	. . . .	755.74	+ 255.74	+ 9089.1
189	0.0	— 0.1	758.88	+ .00	. . . .	758.88	+ 258.88	+ 9200.7
190	0.0	— 0.3	762.07	+ .01	. . . .	762.08	+ 262.08	+ 9314.5

Assume foot of perpendicular at 500^{mm}, as it seems not to have been recorded.

	mm.
3 yards	2743.62
2 straight-edges	1835.91
1^{m}	1000.10
4-in	101.65
99^{mm}	99.03
Radius of mirror	19.28
	5799.59 = length of perpendicular upon west scale.

[August 20. East scale. Telescope reversed.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	"
200	0.0	− 0.5	998.70	+ .02	. . . .	998.72	+ 498.72	+ 10997.4
0	0.0	+ 0.4	− 15.03	− .01	. . . .	− 15.04	− 515.04	− 11356.6
200	0.0	− 0.5	998.69	+ .02	. . . .	998.71	+ 498.71	+ 10997.2
0	0.0	+ 0.5	− 15.00	− .02	. . . .	− 15.02	− 515.02	− 11356.2
10	0.0	+ 0.5	35.69	− .02	. . . .	35.67	− 464.33	− 10240.3
11	0.0	+ 0.2	40.78	− .01	. . . .	40.77	− 459.23	− 10129.1
12	0.0	+ 0.5	45.86	− .02	. . . .	45.84	− 454.16	− 10016.3
13	0.0	+ 0.5	50.90	− .02	. . . .	50.88	− 449.12	− 9905.6
14	0.0	+ 0.4	55.95	− .01	. . . .	55.94	− 444.06	− 9793.9
15	0.0	+ 0.6	61.07	− .02	. . . .	61.05	− 438.95	− 9681.6
16	0.0	+ 0.4	66.13	− .01	. . . .	66.12	− 433.88	− 9569.9
17	0.0	+ 0.5	71.21	− .02	. . . .	71.19	− 428.81	− 9458.1
18	0.0	+ 0.4	76.28	− .01	. . . .	76.27	− 423.73	− 9346.4
19	0.0	+ 0.4	81.40	− .01	. . . .	81.39	− 418.61	− 9333.6
20	0.0	− 0.5	86.50	− .02	. . . .	86.48	− 413.52	− 9121.2
180	0.0	− 0.5	897.22	+ .02	. . . .	897.24	+ 397.24	+ 8762.7
181	0.0	− 0.5	902.26	+ .02	. . . .	902.28	+ 402.28	+ 8873.7
182	0.0	− 0.5	907.30	+ .02	. . . .	907.32	+ 407.32	+ 8984.8
183	0.0	− 0.5	912.40	+ .02	. . . .	912.42	+ 412.42	+ 9096.9
184	0.0	− 0.5	917.46	+ .02	. . . .	917.48	+ 417.48	+ 9208.8
185	0.0	− 0.5	922.52	+ .02	. . . .	922.54	+ 422.54	+ 9320.4
186	0.0	− 0.6	927.60	+ .02	. . . .	927.62	+ 427.62	+ 9432.0
187	0.0	− 0.5	932.66	+ .02	. . . .	932.68	+ 432.68	+ 9543.9
188	0.0	− 0.5	937.74	+ .02	. . . .	937.76	+ 437.76	+ 9655.3
189	0.0	− 0.1	942.87	.00	. . . .	942.87	+ 442.87	+ 9767.8
190	. . .	− 0.4	947.94	+ .01	. . . .	947.95	+ 447.95	+ 9862.6
200	. . .	− 0.5	998.69	+ .02	. . . .	998.71	+ 498.71	+ 10997.0

Foot of perpendicular at 500^{mm}.

	mm.
5 yards	4572.70
4 straight-edges	3671.81
1 meter	1000.10
$80^{mm}.7$	80.73
Radius of mirror	19.28
	9344.62 = length of perpendicular upon east scale.

	mm.
8 yards	7316.32
6 straight-edges	5507.72
2 meters	2000.20
321^{mm}	321.15
	15145.39 = distance between scales.

Results on August 20.

There is a discrepancy of $1^{mm}.18$ between the sum of the two separate distances, measured from the faces of the mirror, and increased by its diameter, and the direct

measure of the whole distance between the scales, made when the mirror was removed. The adjustment considered the most probable gives:

	mm.
Perpendicular upon west scale - - - - - - - - - -	5799.9
Perpendicular upon east scale - - - - - - - - - -	9345.0

We then have

	West scale. mm.	″	East scale. mm.	″
0^d - - - - - - - - - -	−335.83	−11929.81	−515.03	−11356.3
200^d - - - - - - - - - -	+293.59	+10432.1	+498.71	+10997.3
Angular value of 200 divisions ($0^d - 200^d$)		22361.9		22353.6
Angular value of 1 division - - - - - -		111 809		111.768
Mean reading for 15^d - - - - - - - -		−10252 5		− 9681.6
Mean reading for 185^d - - - - - - - -		+ 8754 1		+ 9320.5
Mean value of 170^d (15 − 185) - - -		19006.6		19002.1
Mean value of 1 division - - - - - -		111.804		111.777

[1880, October 6. East scale. Observer, NEWCOMB.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Corrections for— Mic.	Corrections for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	″
0	0.0	+ 0.6	0.85	− .02	. . . .	0.83	− 509.17	− 11294.5
200	0.0	− 0.5	1008.59	+ .02	. . . .	1008.67	+ 498.61	+ 11060.7
0	0.0	+ 0.5	− 0.79	− .02	. . . .	0.77	− 509.23	− 11296.0
200	0.0	− 0.5	1008.57	+ .02	. . . .	1008.59	+ 498.59	+ 11060.5
199	0.0	− 0.5	1003.50	+ .02	. . . .	1003.52	+ 493.52	+ 10948.2
198	0.0	− 0.5	998.41	+ .02	. . . .	998.43	+ 488.43	+ 10835.2
197	0.0	− 0.4	993.36	+ .01	. . . .	993.37	+ 483.37	+ 10723.3
196	0.0	− 0.5	988.34	+ .02	. . . .	988.36	+ 478.36	+ 10612.4
195	0.0	− 0.5	983.31	+ .02	. . . .	983.33	+ 473.33	+ 10501.0
0	0.0	+ 0.5	0.75	− .02	. . . .	0.73	− 509.27	− 11296.5
1	0.0	+ 0.3	5.80	− .01	. . . .	5.79	− 504.21	− 11184.7
2	0.0	+ 0.4	10.81	− .01	. . . .	10.80	499.20	− 11074.0
3	0.0	+ 0.3	15.88	− .01	. . . .	15.87	− 494.13	− 10961.8
4	0.0	+ 0.5	20.96	− .01	. . . .	20.95	489.05	− 10849.1
5	0.0	+ 0.5	25.97	− .02	. . . .	25.95	− 484.05	− 10738.4

Foot of perpendicular at 510^{mm}.
Distance to 578^{mm} on east scale:

	mm.
6 yards	5487.24
1 meter	1000.10
2 straight-edges	1835.91
1 straight-edge X	918.34
$28^{mm}.7$	28.73
Radius of mirror	19.28
Total	9289.60
Reduction to perpendicular	− .30
Length of perpendicular	**9289.30**

[1880, October 6. West scale. Observer, NEWCOMB.]

First set.

S.	Microscopes— R.	Microscopes— L.	Scale reading.	Corrections for— Mic.	Corrections for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	″
0	0.0	+ 0.4	789.09	+ .01	. . . .	+ 789.10	+ 314.10	+ 11212.0
200	—0.2	— 0.6	162.91	— .03	. . . .	— 162.88	— 312.12	— 11141.5
0	0.0	+ 0.5	789.11	+ .02	. . . .	+ 789.13	+ 314.13	+ 11212.8
200	0.0	— 0.5	162.90	— .02	. . . .	— 162.88	— 312.12	— 11141.5
0	0.0	+ 0.4	789.09	+ .01	. . . .	+ 789.10	+ 314.10	+ 11212.0

[Second set.]

0	—0.2	+ 0.3	211.52	.00	. . . .	— 211.52	— 313.48	— 11190.7
200	+0.2	— 0.4	837.67	+ .01	. . . .	+ 837.68	+ 312.68	+ 11161.5
0	0.0	+ 0.5	211.45	— .02	. . . .	— 211.43	— 313.57	— 11193.0
200	+0.1	— 0.4	837.68	+ .01	. . . .	+ 837.69	+ 312.69	+ 11161.8
0	0.0	+ 0.5	211.47	— .02	. . . .	— 211.45	— 313.55	— 11192.5

The first set of readings on west scale were made on the divisions at the upper edge, the second on the divisions of the lower edge, there being two sets of divisions on each face. After the reading the upper edge of the west scale was found to be ½mm nearer the telescope than the lower edge.

Foot of perpendicular, at 525mm for upper divisions.
Foot of perpendicular, at 475mm for lower divisions.
Length of perpendicular on west scale:

	mm.
4 yards	3658.16
2 straight-edges	1835.91
10in.2	259.36
Radius of mirror	19.28
	5772.71

Results,

	West scale (1).	West scale (2).	East scale.
	″	″	″
Mean reading 0^d - - - - -	+11212.3	—11192.1	—11295 5
200^d - - - - - - - - -	—11141.5	+11161.7	+11060.6
Value of 200 divisions - -	22353 8	22353.8	22356.1
Value of 1 division - - -	111.769	1167.69	111 781
Mean reading $2^d.5$ - - -			11017.4
Mean reading, $197^d.5$ - -			10780.1
Value of 195^d - - - - -			21797.5
Value of 1^d - - - - - -			111.782
General mean on October 6			111.776

[1880, October 7. West scale. Observer, HOLCOMBE.]

S.	Microscopes— R.	Microscopes— L.	Scale reading.	Corrections for— Mic.	Corrections for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	"
190	0.0	− 0.4	798.91	+ .01	....	+ 798.92	+ 288.92	+ 10317.8
0	0.0	+ 0.5	204.22	− .02	....	− 204.20	− 305.80	− 10919.5
190	0.0	− 0.4	798.91	+ .01	....	+ 798.92	+ 288.92	+ 10317.8
0	0.0	+ 0.5	204.21	− .02	....	− 204.19	− 305.81	− 10919.8
190	0.0	− 0.5	798.91	+ .02	....	+ 798.93	+ 288.93	+ 10318.0
0	0.0	+ 0.5	204.22	− .02	....	− 204.20	− 305.80	− 10919.5

Foot of perpendicular, at 510^{mm}.
Length of perpendicular on west scale:

	mm.
3 yards	2743.62
2 straight-edges	1835.91
1^{m}	1001.10
$6^{in}.74$	171.23
Radius of mirror	19.28
	5771.14

[1880, October 7. East scale. Observer, HOLCOMBE.]

0	0.0	+ 0.5	3.42	− .03	....	− 3.39	− 489.61	− 10858.6
190	0.0	− 0.3	960.82	+ .02	....	+ 960.84	+ 467.84	+ 10376.7
0	0.0	+ 0.5	3.18	− .03	....	− 3.15	− 489.85	− 10864.0
190	0.0	− 0.2	960.83	+ .01	....	+ 960.84	+ 467.84	+ 10376.7
0	0.0	+ 0.5	3.14	− .03	....	− 3.11	− 489.89	− 10865.0
190	0.0	− 0.2	960.79	+ .01	....	+ 960.80	+ 467.80	+ 10375.8

Foot of perpendicular, at 493^{mm}.
Length of perpendicular on east scale:

	mm.
6 yards	5487.24
2 straight-edges	1835.91
1^{m}	1000.10
$31^{mm}.6$	31.63
1 straight-edge	917.57
Radius of mirror	19.28
	9291.73

Results, October 7.

	West scale.	East scale
	"	"
Mean reading for 0^{d}	−10919.6	−10864.5
Mean reading for 190^{d}	+10317.9	+10376.4
Value of 190^{d}	21237.5	21240.9
Value of 1^{d}	111 776	111.794

General mean on October 7, 111''.785.

The first reading on the east scale is rejected, as there was evidently some change in the instrument.

[1880. October 9. West scale. Upper edge. Observer, HOLCOMBE.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	"
190	0.0	+ 0.1	777.19	.00	. . . .	777.19	+ 287.19	. . . . . .
0	0.0	+ 0.6	182.52	— .02	. . . .	182.50	— 308.50	. . . . . .
190	0.0	— 0.2	777.11	+ .01	. . . .	777.12	+ 287.12	. . . . . .
0	0.0	+ 0.6	182.53	— .02	. . . .	182.51	— 308.49	. . . . . .
190	0.0	— 0.2	777.11	+ .01	. . . .	777.12	+ 287.12	. . . . . .
0	0.0	+ 0.6	182.55	— .02	. . . .	182.53	— 308.47	. . . . . .

[Lower edge.]

S.	R.	L.	Scale readings.	Mic.	Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
1	0.0	+ 0.5	813.84	+ .02	. . . .	813.86	+ 303.82	. . . . . .
190	0.0	— 0.2	224.41	— .01	. . . .	222.40	— 287.58	. . . . . .
1	0.0	+ 0.5	813.82	+ .02	. . . .	813.84	+ 303.80	. . . . . .
190	0.0	— 0.1	222.41	.00	. . . .	222.41	— 287.59	. . . . . .
1	0.0	+ 0.5	813.84	+ .02	. . . .	813.86	+ 303.82	. . . . . .
190	0.0	— 0.1	222.44	.00	. . . .	222.44	— 287.56	. . . . . .

[East scale.]

S.	R.	L.	Scale readings.	Mic.	Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
190	0.0	— 0.1	988.01	.00	. . . .	988.01	+ 458.01	. . . . . .
0	0.0	+ 0.5	30.57	— .03	. . . .	30.54	— 499.46	. . . . . .
190	0.0	— 0.1	988.07	.00	. . . .	988.07	+ 458.07	. . . . . .
0	0.0	+ 0.5	30.72	— .03	. . . .	36.69	— 499.31	. . . . . .
190	0.0	— 0.2	988.16	+ .01	. . . .	988.17	+ 458.17	. . . . . .
0	0.0	+ 0.5	30.74	— .03	. . . .	30.71	— 499.29	. . . . . .
190	0.0	— 0.2	988.15	+ .01	. . . .	988.16	+ 458.16	. . . . . .
0	0.0	+ 0.5	30.74	— .03	. . . .	30.71	— 499.29	. . . . . .
190	0.0	— 0.2	988.24	+ .01	. . . .	988.25	+ 458.25	. . . . . .
0	0.0	+ 0.5	30.84	— .03	. . . .	30.81	— 499.19	. . . . . .

Foot of perpendicular, at 490^{mm} on west scale, and 530^{mm} on east scale.

	mm.
10 yards	9145.40
4 straight-edges	3671.81
1 straight-edge	917.56
1^{m}	1000.10
12^{in}.84	326.16
Excess of foot measure	.05
	15061.08
Correction for obliquity of west scale	— .23
Distance from east to west scales	15060.85

The separate distances to the two scales were not measured.

Results (from both scales).

$$190^{d} = 21236''.00$$
$$1^{d} = 111.768$$

[1880. October 12. West Scale. Lower edge. Observer, HOLCOMBE.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	"
1	0.0	+ 0.5	812.46	+ .02	. . . .	812.48	+ 294.48	+10520.83
191	0.0	— 0.2	218.02	— .01	. . . .	218.01	— 299.99	—10717.34
1	0.0	+ 0.5	812.45	+ .02	. . . .	812.47	+ 294.47	+10520.48
191	0.0	— 0.2	218.00	— .01	. . . .	217.99	— 300.01	—10718.00
1	0.0	+ 0.5	812.45	+ .02	. . . .	812.47	+ 294.47	+10520.48
191	0.0	— 0.3	218.02	— .01	. . . .	218.01	— 299.99	—10717.34
1	0.0	+ 0.5	812.44	+ .02	. . . .	812.46	+ 294.46	+10520.12
191	0.0	— 0.2	218.01	— .01	. . . .	218.00	— 300.00	—10717.69
1	0.0	+ 0.5	812.46	+ .02	. . . .	812.48	+ 294.48	+10520.83
191	0.0	— 0.3	218.02	— .01	. . . .	218.01	— 299.99	—10717.34

Foot of perpendicular, at 518mm.

	mm.
3 yards	2743.62
2 straight-edges	1835.91
1m	1000.10
6in.67	169.42
	19.28
	.05
Length of perpendicular	5768.38

Results from lower edge.

$$190\text{ d} = 21238''.09$$
$$1\text{ d.} = 111\ 779$$

[Upper edge.]

191	0.0	— 0.3	782.28	+ .01	. . . .	782.29	+ 300.29	+10728.00
1	0.0	+ 0.5	187.77	— .02	. . . .	187.75	— 294.25	—10512.54
191	0.0	— 0.2	782.28	+ .01	. . . .	782.29	+ 300.29	+10728.00
1	0.0	+ 0.5	187.78	— .02	. . . .	187.76	— 294.24	—10512.27
191	0.0	— 0.3	782.28	+ .01	. . . .	782.29	+ 300.29	+10728.00
1	0.0	+ 0.5	187.76	— .02	. . . .	187.74	— 294.26	+10513.00

Results from upper edge.

$$190\text{ d.} = 21240''.60$$
$$1\text{ d.} = 111.792$$

[1880. October 12. East scale. Observer, HOLCOMBE.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	″
1	0.0	+ 0.5	11.15	— .03	— .05	11.07	— 475.93	— 10557.9
191	0.0	— 0.2	968.35	+ .01	— .03	968.33	+ 481.33	+ 10677.0
1	0.0	+ 0.5	11.04	— .03	— .01	11.00	— 476.00	10559.4
191	0.0	— 0.2	968.31	+ .01	+ .01	968.33	+ 481.33	+ 10677.5
1	0.0	+ 0.5	10.99	— .03	+ .03	10.99	— 476.01	— 10559.7
191	0.0	— 0.2	968.30	+ .01	+ .05	968.36	+ 481.36	+ 10678.1

Foot of perpendicular, at 484^{mm}.

	mm.
6 yards	5487.24
2 straight-edges	1835.91
1 straight-edge X	918.34
1^m	1000.10
29^{mm}	29.00
Radius of mirror	19.28
Overplus of scale	.05
Length of perpendicular on east scale	9289.92

Results.

$$190\ d. = 21236''.5$$
$$1\ d. = 111.771$$

[October 14. West scale. Lower edge. Observer, HOLCOMBE.]

1	0.0	+ 0.5	817.80	+ .02	. . . .	817.82	307.82	10997.60
191	0.0	— 0.3	223.24	— .01	. . . .	223.23	286.77	10246.85
1	0.0	+ 0.5	817.80	+ .02	. . . .	817.82	307.82	10997.60
191	0.0	— 0.3	223.21	— .01	. . . .	223.20	286.80	10247.92
1	0.0	+ 0.5	817.79	+ .02	. . . .	817.81	307.81	10997.27

[Upper edge.]

1	0.0	+ 0.5	182.61	— .02	. . . .	182.59	307.41	10983.00
191	0.0	— 0.3	777.11	+ .01	. . . .	777.12	287.12	10259.33
1	0.0	+ 0.5	182.60	— .02	. . . .	182.58	307.42	10983.40
191	0.0	— 0.3	777.11	+ .01	. . . .	777.12	287.12	10259.33
1	0.0	+ 0.5	182.58	— .02	. . . .	182.56	307.44	10984.00

Foot of perpendicular, at 510^{mm} on lower edge, and at 490^{mm} on upper edge.

	mm.
3 yards	2743.62
2 straight-edges	1835.90
1^m	1000.10
6 in	152.41
$16^{mm}.45$	16.45
Radius of mirror	19.28
Overplus	.05
Length of perpendicular on west scale	5767.81

Results.

$$190\ d. = 21243''.83$$
$$1\ d. = 111.809$$

[1880. October 14. East scale. Observer, HOLCOMBE.]

[Upper edge.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	″
191	0.0	− 0.3	964.82	+ .02	. . . .	964.84	484.84	10754.68
1	0.0	+ 0.5	7.49	− .03	. . . .	7.46	472.54	10482.32
191	0.0	− 0.3	964.83	+ .02	. . . .	964.85	484.85	10754.90
1	0.0	+ 0.5	7.51	− .03	. . . .	7.48	472.52	10481.88
191	0.0	− 0.3	964.83	+ .02	. . . .	964.85	484.85	10754.90
1	0.0	+ 0.5	7.51	− .03	. . . .	7.48	472.52	10481.88

Foot of perpendicular, at 480^{mm}.

	mm.
7 yards	6401.78
2 straight edges	1835.90
1 straight edge X	918.34
4 in	101.60
$13^{mm}.4$	13.40
Overplus + radius of mirror	19.33
Length of perpendicular	9290.35

Results.

$$190\ \text{d.} = 21236''.86$$
$$1\ \text{d.} = 111''.773$$

The following were made after the brick supports were built for the scales:

[1880. December 16. West scale. Observer, HOLCOMBE.]

S.	R.	L.	Scale readings.	Mic.	Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
1	0.0	− 0.1	102.17	.00	. . . .	102.17	− 309.83	−10945.82
191	0.0	− 0.5	703.26	+ .02	. . . .	703.28	+ 291.28	+10291.57
1	0.0	− 0.3	102.20	+ .01	. . . .	102.21	− 309.79	−10944.41
191	0.0	− 0.5	703.29	+ .02	. . . .	703.31	+ 291.31	+10292.59
1	0.0	− 0.3	102.20	+ .01	. . . .	102.21	− 309.79	−10944.41
191	0.0	− 0.5	703.30	+ .02	. . . .	703.32	+ 291.32	+10292.97
1	0.0	− 0.2	102.19	+ .01	. . . .	102.20	− 309.80	−10944.80
191	0.0	− 0.5	703.28	+ .02	. . . .	703.30	+ 291.30	+10292.50
1	0.0	− 0.2	102.20	+ .01	. . . .	102.21	− 309.79	−10944.41
191	0.0	− 0.5	703.27	+ .02	. . . .	703.29	+ 291.29	+10292.15

Perpendicular point, 412^{mm}.

Length of perpendicular:

1st measurement.	mm.	2d measurement.	mm.
4 yards	3658.16	3 yards	2743.62
2 straight edges . . .	1835.91	2 straight edges . . } 1 straight edge . . }	2757.47
12.58	319.58		
	1928.28	$22^{in}.46$	316.79
			19.28
	5832.93		5833.16

Mean 5833.04

Results

$$190\ \text{d.} = 21237''.12$$
$$1\ \text{d.} = 111''.774$$

[1880. December 16. East scale. Observer, HOLCOMBE.]

S.	Microscopes—		Scale readings.	Corrections for—		Corrected readings.	Dist. from perpendicular point.	Corresponding angle.
	R.	L.		Mic.	Change.			
d.			mm.			mm.	mm.	″
1	0.0	— 0.2	992.40	— .01	. . . .	992.39	+ 526.39	+11650.00
191	0.0	— 0.5	32.99	— .03	. . . .	32.96	— 433.04	— 9587.27
1	0.0	— 0.2	992.41	— .01	. . . .	992.40	+ 526.40	+11650.21
191	0.0	— 0.5	33.01	— .03	. . . .	32.98	— 433.02	— 9586.83
1	0.0	0.0	992.47	.00	. . . .	992.47	+ 526.47	+11651.75
191	0.0	— 0.5	33.07	— .03	. . . .	33.04	— 432.96	— 9585.52
1	0.0	— 0.3	992.41	— .01	. . . .	992.40	+ 526.40	+11650.21
191	0.0	— 0.5	32.96	— .03	. . . .	32.93	— 433.07	— 9587.93
1	0.0	— 0.1	992.41	— .00	. . . .	992.41	+ 526.41	+11650.43
191	0.0	— 0.4	33.00	— .02	. . . .	32.98	— 433.02	— 9586.83

Perpendicular point 466mm.

1st measurement.

	mm.
7 yards	6401.78
2 straight-edges . . .	1835.91
Straight-edge X . .	918.34
134mm.5	134.54
	19.28
	9309.85

2d measurement.

	mm.
6 yards	5487.24
4 straight-edges	3671.81
5in.18 (+ .05)	131.62
	19.28
	9309.95

Mean = 9309.90 = length of perpendicular on east scale.

Results.

$$190\text{ d.} = 21237.39''$$
$$1\text{ d.} = 111.775$$

[1881. January 13. East scale. Observer, HOLCOMBE.]

1	0.0	— 0.2	986.65	— .01	. . . .	986.64	+ 522.64	+11567.10
191	0.0	— 0.5	27.15	— .03	. . . .	27.12	— 436.88	— 9672.10
1	0.0	— 0.1	986.64	.00	. . . .	986.64	+ 522.64	+11567.10
191	0.0	— 0.5	27.13	— .03	. . . .	27.10	— 436.90	— 9672.54
1	0.0	— 0.2	986.65	— .01	. . . .	986.64	+ 522.64	+11567.10
191	0.0	— 0.5	27.14	— .03	. . . .	27.11	— 436.89	— 9672.32
1	0.0	— 0.2	986.6[illegible]	— .01	. . . .	986.64	+ 522.64	+11567.10
191	0.0	— 0.5	27.13	— .03	. . . .	27.10	— 436.90	— 9672.54
1	0.0	— 0.1	986.65	.00	. . . .	986.65	+ 522.65	+11567.31
191	0.0	— 0.5	27.14	— .03	. . . .	27.11	— 436.89	— 9672.32

Perpendicular point, at 464mm.

	mm.
6 yards	5487.24
4 straight-edges	3671.81
5in.18	131.63
	19.28
	9309.96 = length of perpendicular on east scale.

Results.

$$190\text{ d} = 21239.50''$$
$$1\text{ d.} = 111.786$$

[1881. January 13. West scale. Observer, HOLCOMBE.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	''
1	0.0	— 0.1	103.42	.00	....	103.42	— 309.58	— 10937.65
191	0.0	— 0.5	704.46	+ .02	....	704.48	+ 291.48	+10299.26
1	0.0	— 0.1	103.44	.00	....	103.44	— 309.56	—10936.96
191	0.0	— 0.5	704.46	+ .02	....	704.48	+ 251.48	+10299.26
1	0.0	— 0.1	103.42	.00	....	103.42	— 309.58	—10937.65
191	0.0	— 0.5	704.47	+ .02	....	704.49	+ 291.49	+10299.61
1	0.0	— 0.1	103.41	.00	....	103.41	— 309.59	—10938.00
191	0.0	— 0.5	704.46	+ .02	....	704.48	+ 291.48	+10299.26
1	0.0	— 0.1	103.40	.00	....	103.40	— 309.60	—10938.35
191	0.0	— 0.5	704.46	+ .02	....	704.48	+ 291.48	+10299.26

Perpendicular point at 413mm.

	mm.
3 yards	2743.62
2 straight-edges	1835.91
1 straight-edge	917.57
12in.45	316.28
	19.28
	5832.66 = length of perpendicular on west scale.

Results.

$$190 \text{ d.} = 21237.05''$$
$$1 \text{ d.} = 111.774$$

[1881. January 15. West scale. Observer, HOLCOMBE.]

1	0.0	— 0.1	103.43	.00	....	103.43	— 309.57	—10936.80
191	0.0	— 0.5	704.60	+ .02	....	704.62	+ 291.62	+10303.73
1	0.0	— 0.1	103.41	.00	....	103.41	— 309.59	—10937.72
191	0.0	— 0.5	704.61	+ .02	....	704.63	+ 291.63	+10304.00
1	0.0	— 0.1	103.42	.00	....	103.42	— 309.58	—10937.00
191	0.0	— 0.5	704.60	+ .02	....	704.62	+ 291.62	+10303.73
1	0.0	— 0.1	103.38	.00	....	103.38	— 309.62	—10938.56
191	0.0	— 0.5	704.52	+ .02	....	704.54	+ 291.54	+10301.00
1	0.0	— 0.1	103.32	.00	....	103.32	— 309.68	—10940.70
191	0.0	— 0.5	704.51	+ .02	....	704.53	+ 291.53	+10300.57

Foot of perpendicular at 413mm.

	mm.
3 yards	2743.62
2 straight-edges	1835.91
1 straight-edge	917.57
12in.46	316.49
	19.28
	.05
	5832.92 = length of perpendicular on west scale.

Results.

$$190 \text{ d.} = 21240.56''$$
$$1 \text{ d.} = 111.792$$

[1881. January 15. East scale. Observer, HOLCOMBE.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	″
191	0.0	— 0.5	25.40	— .03	....	25.37	— 437.63	— 9688.00
1	0.0	— 0.1	985.09	.00	....	985.09	+ 522.09	+11554.32
191	0.0	— 0.5	25.39	— .03	....	25.36	— 437.64	— 9688.37
1	0.0	— 0.1	985.09	.00	....	985.09	+ 522.09	+11554.32
191	0.0	— 0.5	25.37	— .03	....	25.34	— 437.66	— 9688.80
1	0.0	— 0.1	985.07	.00	....	985.07	+ 522.07	+11553.89
191	0.0	— 0.5	25.37	— .03	....	25.34	— 437.66	— 9688.80
1	0.0	— 0.1	985.10	.00	....	985.10	+ 522.10	+11554.56
191	0.0	— 0.5	25.37	— .03	....	25.34	— 437.66	— 9688.80
1	0.0	— 0.1	985.09	.00	....	985.09	+ 522.09	+11554.32

Foot of perpendicular at 463^{mm}.

	mm.
6 yards	5487.24
4 straight-edges	3671.81
$5^{in}.2$	132.08
	19.28
	.05
	9310.46 = length of perpendicular on east scale.

Results.

$$110\text{ d.} = 21243''.83$$
$$1\text{ d.} = 111.809$$

[1881. April 19. West scale. Observer, HOLCOMBE.]

5	0.0	0.0	129.21	.00	....	129.21	297.79	10518.19
195	0.0	— 0.4	730.50	+ .01	....	730.51	303.51	10719.87
5	0.0	0.0	129.21	.00	....	129.21	297.79	10518.19
195	0.0	— 0.5	730.51	+ .02	....	730.53	303.53	10720.65
5	0.0	0.0	129.18	.00	....	129.18	297.82	10519.24
195	0.0	— 0.4	730.53	+ .01	....	730.54	303.54	10720.95

Foot of perpendicular at 427^{mm}.

1st. measurement.		2d measurement.	
	mm.		mm.
4929^{mm} =	4929.42	4945 =	4945.42
$34^{in}.875$ =	885.86	$34^{in}.25$	869.98
	.05		.05
	19.28		19.28
	5834.61		5834.73

Mean = 5834.67 length of perpendicular on west scale.

Results.

$$190\text{ d.} = 21239''.03$$
$$1\text{ d.} = 111\ 784$$

[1881. April 19. East scale. Observer, HOLCOMBE.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	″
5	0.0	0.0	23.71	.00	. . . .	23.71	501.29	11091.89
195	0.0	— 0.5	983.46	+ .03	. . . .	983.49	458.49	10146.28
195	0.0	— 0.5	983.55	+ .03	. . . .	983.58	458.58	10148.28
5	0.0	0.0	23.71	.00	. . . .	23.71	501.29	11091.89
195	0.0	— 0.5	983.57	+ .03	. . . .	983.60	458.60	10148.76

Foot of perpendicular at 525^{mm}.

	mm.		mm.
8 m. + 2 feet =	8610.49		8610.49
26^{in}.625	676.30	25^{in}.25 =	641.38
7^{mm}.	7.00	42^{mm}	42.00
	19.28		19.28
	0.05		0.05
	9313.12		9313.20

Mean = 9313.16 = length of perpendicular.

Results.

$$190 \text{ d.} = 21239.52''$$
$$1 \text{ d.} = 111.787$$

[1881. May 5. West scale. Observer, HOLCOMBE.]

10	0.0	0.0	855.00	.00	. . . .	855.00	325.00	11477.26
200	0.0	— 0.5	253.62	— .02	. . . .	253.60	276.40	9763.72
10	0.0	— 0.0	855.01	— .00	. . . .	855.01	325.01	11477.60
200	0.0	— 0.5	253.61	— .02	. . . .	253.59	276.41	9764.10
10	0.0	0.0	854.95	— .00	. . . .	854.95	324.95	11475.52
200	0.0	— 0.5	253.63	— .02	. . . .	253.61	276.39	9763.40

Foot of perpendicular at 530^{mm}.

	mm.
3 yards	2743.62
2 straight-edges	1835.91
2 feet	609.65
23 feet	584.23
42^{mm}	42.00
	19.28
	0.05
	5834.74 = length of perpendicular.

Results.

$$190 \text{ d.} = 21240\ 56''$$
$$1 \text{ d.} = 111.792$$

[1881. May 5. East scale. Observer, HOLCOMBE.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	"
10	0.0	0.0	960.92	.00	....	960.92	480.92	10641.00
200	0.0	— 0.5	1.37	— .03	....	1.34	478.66	10591.00
10	0.0	0.0	960.82	— .00	....	960.82	480.82	10638.87
200	0.0	0.5	1.08	— .03	....	1.05	478.95	10597.44
10	0.0	— 0.1	960.74	.00	....	960.74	480.74	10637.00
200	0.0	— 0.5	1.08	— .03	....	1.05	478.95	10597.44

Foot of perpendicular at 480mm.

	mm.
4 straight-edges	3671.81
5 yards	4572.70
2 feet	609.65
16 inches	406.40
34mm	34.00
	19.28
	0.05
	9313.89 = length of perpendicular on east scale.

Results.

$$190 \text{ div.} = 21234''25$$
$$1 \text{ div.} = 111\ 759$$

[1881. May 6. East scale. Observer, HOLCOMBE.]

S.	R.	L.	Scale readings.	Mic.	Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
200	0.0	— 0.5	6.89	— .03	....	6.86	478.14	10581.79
10	0.0	— 0.0	966.60	.00	....	966.60	481.60	10658.50
200	0.0	— 0.5	6.91	— .03	....	6.88	478.12	10581.39
10	0.0	0.0	966.60	.00	....	966.60	481.60	10658.50
200	0.0	— 0.5	6.90	— .03	....	6.87	478.13	10581.60
10	0.0	0.0	966.58	.00	....	966.58	481.58	:0657.80

Foot of perpendicular at 485mm.

	mm.
4 straight-edges	3671.80
5 yards	4572.70
2 feet	609.65
16 inches	406.42
32mm	32.00
	19.28
	0.05
	9311.90

Results.

$$190 \text{ d.} = 21239''.49$$
$$1 \text{ d.} = 111.786$$

[1881. May 6. West scale. Observer, HOLCOMBE.]

S.	Microscopes— R.	Microscopes— L.	Scale readings.	Correction for— Mic.	Correction for— Change.	Corrected readings.	Dist. from perpendicular point.	Corresponding arc.
d.			mm.			mm.	mm.	$''$
200	0.0	— 0.5	253.06	— .02	. . . .	253.04	284.96	10066.11
10	0.0	0.0	854.35	.00	. . . .	854.35	316.35	11173.00
200	0.0	— 0.5	253.04	— .02	. . . .	253.02	284.98	10066.74
10	0.0	0.0	854.34	.00	. . . .	854.34	316.34	11172.53
200	0.0	— 0.5	253.04	— .02	. . . .	253.02	284.98	10066.74
10	0.0	0.0	854.34	.00	. . . .	854.34	316.34	11172.53

Foot of perpendicular at 538^{mm}.	mm.
2 straight-edges	1835.91
Straight-edge X	918.34
3 yards	2743.62
$11^{in}.9$	302.26
15^{mm}	15.00
Radius of mirror and overplus	19.33
Length of perpendicular	5834.46

Results.

$$190\,\text{d.} = 21239''.15$$
$$1\,\text{d.} = 111''.785$$

*Results of the first method.**—The preceding results are collected in the following table. The results after the scales were mounted on brick supports are given separately. The weights assigned in the first series are mere estimates, founded on the number of divisions determined and the number of measures on each:

[First series. Scales on wooden supports.]

Date.	East scale.	West scale.	Both scales.
1880.	$''$	$''$	$''$
Aug. 18	111.780_3	. . . . . .	. . . . . .
19	. . . . . .	111.806_2	. . . . . .
20	111.773_3	111.805_3	. . . . . .
Oct. 6	111.781_2	111.769_1	. . . . . .
7	111.794_1	111.776_2	. . . . . .
9	. . . . . .	. . . . . .	111.768_3
12	111.771_1	111.785_2	. . . . . .
14	111.773_1	111.809_2	. . . . . .
Means	111.778_{11}	111.795_{12}	111.768_3

*In addition to the above measures three sets were made in the autumn of 1881, giving the mean result:

	$''$
From three sets on east scale	111.767
" west scale	111.801

They were accidentally omitted from the discussion, and as their mean result is in good agreement with the others, it has not been deemed necessary to add them.

[Second series. Scales on brick supports.]

Date.	East scale.	West scale.
1880.	$''$	$''$
Dec. 16	111.775	111.774
1881.		
Jan. 13	111.786	111.774
15	111.809	111.792
April 19	111.787	111.784
May 5	111.759	111.792
6	111.786	111.785
Mean	111.784	111.784

The comparison of these results does not seem to afford material for extended discussion, since the discordances show nothing systematic in their character, although in the second series they are much larger than I should have anticipated. In strictness, we should expect to get different results from measurements of different divisions, owing to errors of division. But it seems quite certain that these errors can only be fractions of a second, and would, therefore, change the final results by quantities of an order of magnitude less than $\pm 0''.005$, which would be masked by the accidental errors. Any admissible combination gives substantially the same result, namely:

$$\text{One division of the limb} = 111''.784 \pm 0''.003$$

SECOND METHOD; BY MEANS OF THE THEODOLITE.

Notwithstanding the good agreement of the means in the preceding series, the agreement of the individual results is by no means satisfactory, the mean deviation from the average corresponding to a change of 40 or 50 kilometers in the velocity of light. Another and entirely independent method of making the same determination was therefore devised, and was put in execution by the United States Coast Survey. Through the courtesy of that establishment one of the large theodolites used in triangulation was loaned for the purpose, and Assistant C. H. SINCLAIR was detailed to make the determinations. He was assisted in the work by Ensign HOLCOMBE. The method was as follows: A brick pier was erected immediately outside the building on the west, a few feet from the revolving mirror, and in the line of sight of the moving telescope of the phototachometer, when the latter was in a position for making observations. Into this pier was set a pair of metal slides, on which the base of the theodolite could be moved back and forth a few inches in a direction perpendicular to that of the line of the instrument. This motion was necessary owing to the impossibility of having the axis of revolution of the theodolite coincident with that of the revolving mirror. To determine and eliminate any rotary motion in sliding, a fixed collimator was set on the theodolite pier at the height of the telescope of the theodolite. Both telescopes were on the same level with the receiving telescope of the phototachometer.

To make the determinations the telescope of the phototachometer was set near one of its extreme positions. The theodolite was then slid into its line of sight and alternate readings were made when the theodolite was placed upon the fixed micrometer wires of the phototachometer and upon the collimator. The difference of the circle readings thus obtained was the angle between the optical axes of the collimator

and of the phototachometer telescope. The latter was then changed to its other extreme reading, division 207 being taken for this purpose; the theodolite was again slid into position and the same determination was again made. The difference of the angles thus measured is the motion of the optical axis of the phototachometer when the reading of the microscope is changed by 206 divisions.

With a view of eliminating errors of division of the theodolite, seven sets of determinations of each angle were made, the position of the graduated circle of the theodolite being changed between each set. In each set the measures were repeated seven times in two positions of the divided circle differing by 180°. The instrument was read by three microscopes, so that each set depends on the mean of six divisions. In the following table the final results are arranged in chronological order. In the second column the numerals indicate the seven different positions of the divided circle of the theodolite. The next two columns give the readings of the microscopes of the phototachometer as explained in the observations. The sixth column gives the angles between the fixed collimator and the axis of the phototachometer telescope, measured from a micrometer wire in the focus. The number of readings in the next column indicates the number of complete sets of readings made. Applying the corrections for microscopes, we have the seconds of corrected readings given in the last column.

Date.	Position.	Division.	Microscopes— R.	Microscopes— L.	Arc between collimator and receiving telescope.	No. of readings.	Corr. for mic.	Seconds of corr. reading.
1880.					° ′ ″		″	″
Nov. 29	I	1	0.0	+ 0.3	86 29 29.01	7	— 0.36	28.65
29	I	207	0.0	— 0.4	92 53 14.96	7	+ 0.48	15.44
29	II	207	0.0	— 0.4	92 53 16.87	3	+ 0.48	17.35
29	II	1	0.0	+ 0.3	86 29 27.93	3	— 0.36	27.57
30	II	1	0.0	+ 0.3	86 29 28.95	4	— 0.36	28.59
30	II	207	0.0	— 0.3	92 53 15.95	4	+ 0.36	16.31
30	III	207	0.0	— 0.3	92 53 18.66	7	+ 0.36	19.02
30	III	1	0.0	+ 0.4	86 29 27.79	7	— 0.48	27.31
Dec. 1	IV	1	0.0	+ 0.4	86 29 38.44	7	— 0.48	37.96
1	IV	207	0.0	— 0.3	92 53 27.67	7	+ 0.36	28.03
1	V	207	0.0	— 0.3	92 53 23.61	7	+ 0.36	23.97
1	V	1	0.0	+ 0.4	86 29 36.13	7	— 0.48	35.65
2	VI	1	0.0	+ 0.4	86 29 34.97	7	— 0.48	34.49
2	VII	1	0.0	+ 0.4	86 29 33.66	7	— 0.48	33.18
2	VII	207	0.0	— 0.3	92 53 17.78	7	+ 0.36	18.14
2	VI	207	0.0	— 0.3	92 53 18.21	7	+ 0.36	18.57

The following are the separate results for the value in arc of 206 divisions and of one division, respectively:

Position of inst.	206 div.	One div.
	″	″
I	23026.79	111.781
II	23028.60	111.789
III	23031.71	111.804
IV	23030.07	111.796
V	23028.32	111.788
VI	23024.08	111.767
VII	23024.96	111.772
Mean	23027.79	111.785 ± 0″.004

The agreement between the final results obtained by the two methods must be regarded as purely accidental, and leads to the conclusion that the true value of one division is $111''.784$. As a matter of fact the value actually used in the reductions was $111''.783$, a result which arose from the fact that the corrections for the readings of the microscopes were accidentally neglected in getting the final value by the second method. This value, however, cannot be considered as having less weight than the other because among the possible sources of error to which measures of the distance between the revolving mirror and the meter scale, in using the first method, are subjected, must be included a possible constant error tending to make the measured distance come out too small. The source of this error is the liability of the scales to not being perfectly fitted together, owing to their deviations from a horizontal position. Although its probable magnitude is but a small fraction of a millimeter, we may consider it as sufficient to make a possible diminution of one or more units in the third place of decimals in the final results.

Effects of errors of division.—By the preceding methods measures were, on three occasions, made on a number of consecutive divisions with a view of determining their errors. The results of these measures from the 1st to the 20th division are shown in the following table. It does not appear from a general examination of the results that the differences are otherwise than purely accidental; whatever general errors of division exist are lost in the unavoidable errors of readings.

Further evidence on this subject is afforded by a comparison of the readings made by the two microscopes. It will be remembered that in all observations and determinations a mean of the results of two microscopes was taken. The distance apart of these microscopes is 206 divisions, hence whenever the one microscope was set on division N, the other would be set on division N + 206. We may therefore consider that every measured angle is a mean of two angles on different parts of the arc, and the difference of the errors of these angles is given by the comparison of the readings of the microscopes. Moreover, since it is in the highest degree improbable that there should be any relation by which divisions differing by the number 206 are affected by common errors, we may regard this discrepancy as affording an index to the accidental errors of division A comparison of the readings of the microscopes, which are given with every observation, shows that at neighboring parts of the arc, the discrepancies between the readings of the two microscopes, are not generally more than one or two tenths of a unit, which is as near as the readings could be made by the imperfect means afforded. Each unit corresponding to $2''.4$ of arc, the accidental errors thus indicated are less than $0''.5$. The case is, however, different when we measure long arcs. It will be noted that the second microscope L, generally showed a negative reading at that end of the arc corresponding to the greatest number of divisions, thus showing a systematic difference in the results of the two microscopes amounting to about $2''$ for the longest arc measured. This difference is about a ten-thousandth part of the whole, and would be important if it affected the final results in this proportion; but since both classes of determinations, those used to measure the velocity of light and those by which the value of one division was fixed, correspond

to the mean of the results from the two microscopes, any systematic difference of this sort is completely eliminated.

All errors of division would be completely eliminated from the result if the measures of arc upon the limb were made upon the same divisions and in the same numbers as those with which the velocity of light is observed. It will be seen by a comparison of the divisions used in the two cases that this condition was not rigorously fulfilled, at the same time a sufficient approximation to it was obtained to make it fairly certain that the systematic error thus arising can only be a very small fraction of a second for the entire length of the arc, and that it may therefore be left out of consideration.

Table of the measured distances in arc between consecutive divisions.

S.	(1)	(2)	(3)	Mean.
	″	″	″	″
0				
	110.0	111.8		110.9
1				
	113.7	110.8	110.9	111.8
2				
	110.2	112.5	112.6	111.8
3				
	112.8	112.4	111.8	112.3
4				
	111.0	110.9	111.6	111.2
5				
			111.3	111.3
6				
			111.8	111.8
7				
			112.0	112.0
8				
			112.2	112.2
9				
			112.3	112.3
10				
	110.5	111.7	112.5	111.6
11				
	112.3	112.8	111.2	112.1
12				
	112.2	110.8	112.3	111.8
13				
	110.4	111.8	112.0	111.4
14				
	112.1	112.2	112.3	112.2
15				
	113.0	111.8	111.6	112.1
16				
	110.7	111.7	111.8	111.4
17				
	111.6	111.8	111.9	111.8
18				
	112.4	112.8	112.5	112.6
19				
		112.4	111.5	112.0
20				

Chapter V.

RELATIVE POSITIONS OF THE STATIONS.

After a careful examination of a number of suggested positions in the neighborhood of Washington City the most eligible one for the revolving mirror was found to be Fort Myer, then Fort Whipple, the central post of the Army Signal Service, on the Virginia side of the Potomac, overlooking the city of Washington. There was no structure in the neighborhood of such permanence that the position of the revolving mirror could be safely referred to it. The position has therefore been marked by a stone rising a few inches above the ground. It is situated a few yards to the east of the road passing in front of the principal range of buildings.

As a station for the fixed mirror a place was chosen in the Observatory grounds, on the brow of the hill, at a distance of 272 feet to the south-southeast of the present foundation of the great equatorial. The large brick pier supporting the fixed mirror still stands, at the time of writing, and when removed it is intended that a permanent mark shall be placed showing the position of the face of the mirror.

It being found that observations could be made at a considerably greater distance than that of the Observatory, another point was selected at the base of the Washington Monument, a short distance northwest of its northwest corner.

The relative positions of these stations, and the system of triangulation by which their distances apart were measured, are shown in Plate I.*

The triangulation connecting the stations was executed, under the direction of the United States Coast and Geodetic Survey, by Assistant C. H. Sinclair, of that work, in conjunction with Ensign J. H. L. Holcombe, U. S. N. The details of this part of the work will be given in an Appendix, from which the data employed in the following discussion may be derived.

Owing to practical difficulties in placing the theodolite and getting clear lines of sight, the points at which the fixed mirrors were placed could not themselves be used as triangulation points. It was therefore necessary to choose points in their neighborhood for this purpose.

The following remarks pertain to the relations of the principal points of triangulation to the stations at which the mirrors were mounted:

Fort Myer.—The point at which the center of the theodolite was set was vertically above the revolving mirror, the theodolite was inside the station-house, and the signal on the roof above. No reduction is therefore necessary to the point occupied by the mirror.

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*The station, marked 6, at Fort Myer, where the revolving mirror was fixed, appears to be incorrectly laid down on the map. It is nearly in front of a point between the two northernmost houses, instead of being, as represented on the map, in front of a point south of the second one.

Observatory.—The Monument station not being visible from the pier supporting the mirrors, a point was chosen at a distance $39^{m}.3147$, on a horizontal projection, from the west face of the pier towards the fort. This distance is described as measured from the "southwest corner of the pier," but the corner represented on the diagram would rather be designated as the northwest corner. It is however sufficient for the purpose of reduction to know that the distance to some point of the nearest face of pier was $39^{m}.3147$ greater than that to the triangulation point.

Monument.—The triangulation point near the Monument is found by measuring AH, $2^{m}.588$ south from the southwest corner of the marble base of the Monument, and thence measuring HT, $5^{m}.130$ on the line towards Fort Myer. The point T thus reached was found to be $6^{m}.013$ from the corner A of the Monument. The preceding description fixes its position with such accuracy that no mark was deemed necessary.

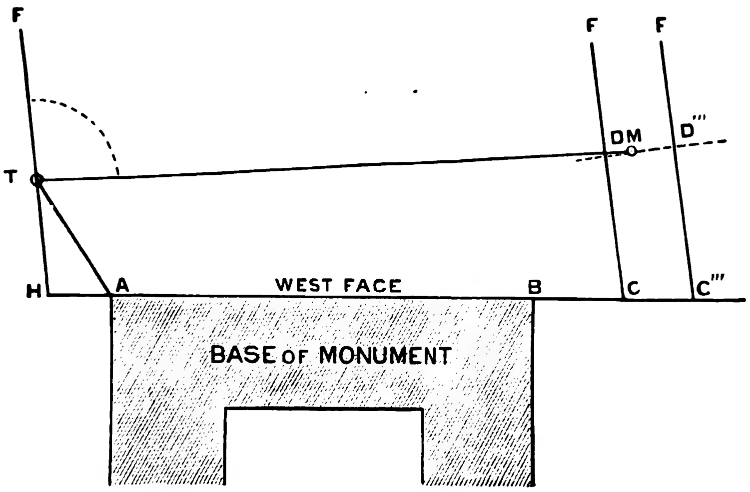


FIG. 5.

As a check upon the position of the pier which supported the fixed mirrors their position relative to the Monument was approximately determined as follows:

Four points, C C′ C″ C‴, of which only the first and last are laid down in the figure, being chosen on the ground in the prolongation of the west face of the Monument, distances CD, etc., were measured towards Fort Myer, each terminating in the vertical plane which contained the faces of the mirrors, with the following results:

	in.	m.		in.	m.
BC =	134.	= 3.404;	CD =	231.5	= 5.880
BC′ =	142.	= 3.607;	C′ D′ =	232.5	= 5.905
BC″ =	234.7	= 5.962;	C″ D″ =	244.6	= 6 213
BC‴ =	239.	= 6.071;	C‴ D‴ =	245.4	= 6.233

These measures would suffice to rediscover the position of the mirror, but do not suffice to fix its distance from Fort Myer, owing to the uncertainty of the angle between the face of the Monument and the line to Fort Myer.

Heights of mirrors above mean tide water.—For Fort Myer and the Observatory these heights were determined by the Coast and Geodetic Survey with the following results:

	m.
Foot of flagstaff at Fort Myer above mean tide-level of Potomac -	69.75
Revolving mirror above flagstaff - - - - - - - - - - - - -	2.69
Mirror above mean tide-level - - - - - - - - - - - -	72.44

Top of Observatory pier above tide-level	32.06
Center of fixed mirrors above pier	0.32
Mirror above tide-level	32.38

The first of the following data was supplied by Col. T. L. Casey, C. E., U. S Army, engineer in charge of the Monument:

	m.
Height of zero bolt above mean low tide, $41^{ft}.851$	= 12.7
Height of mirrors above base	3.0
Reduction to mean tide	— 0.4
Height of mirror above tide-level	15.3

The mirror station, near the Monument, was connected with the triangulation point (7) by an independent measure of the angles Monument–Myer–Mirror. The results were:

	°	′	″
Angle at Monument	94	13	6.6
Angle at Myer	0	22	19.5
Whence, angle at mirror	85	24	33.9

The horizontal distance Myer–Monument being $3718^{m}.89$, this triangle gives:

	m.	
Horizontal distance from Myer to mirror	3720.75	
Correction for mean height	+ .027	
Correction for difference of heights, $57^{m}.1$	+ .436	
Resulting distance between mirrors	3721.21	(A)
Double distance traveled by light	7442.42	

The connection between the triangulation point (3) and the west face of the pier supporting the mirrors in the grounds of the Naval Observatory was made in connection with the triangulation, and is reported in the papers from the Coast and Geodetic Survey found in the Appendix. The result is:

	m.	
Distance to face of pier	2550.72	
Mirror back of face of pier	0.23	
Total distance between mirrors	2550.95	(B)
Double distance traveled by light	5101.90	

A part of this line depends upon a measurement of $39^{m}.3$ from the triangulation point to the pier, up a steep hill. When it was desired to test this measure no mark in the ground indicating the triangulation point could be found. Feeling that the determination should be strengthened, the angles of the triangle, whose vertices were

in the actual planes containing the mirrors, were determined in the summer of 1884, by Lieut. C. H. AMSDEN, U. S. N., with the following results:

	°	′	″
Angle at Observatory	136	49	38.3
Angle at Monument	27	58	22.2
Angle at Fort Myer	15	12	3.8
Excess			4.3

The excess of 4″.3 being taken equally from the three angles will give, as the most probable result:

	°	′	″
Angle at Observatory	136	49	36.9
Angle at Monument (mirror pt.)	27	58	20.8

the sines of which angles should be proportioned to the horizontal projections of the distances A and B traveled by the ray of light.

Yet another triangle was measured, identical with the last, except that the triangulation point near the Monument was selected instead of the place occupied by the mirrors. The results were:

	°	′	″
Angle at Observatory	135	56	36.3
Angle at Monument (triangulation point 7)	28	29	6.6
Angle at Fort Myer	15	34	22.3
Excess			5.2

Dividing the excess as before, we have:

	°	′	″
Angle at Observatory	135	56	34.6
Angle at Monument	28	29	4.9

Using the distance, Myer–Monument $\Delta = 3718.89$ as a base, the first triangle gives as the distance B:

Meyer–Obs., mirror (horizontal)	2550.74
While the second, with the base 3720.75, gives	2550.61

The difference corresponds to a relative error of 5″ in the angles. Correcting by 0 .32 for difference of heights, the results for the distance B will be $2550^{m}.93$ and $2551^{m}.06$. The mean result differs by less than half a decimeter from the first determination, which seems to need no alteration.

Chapter VI.

OBSERVATIONS AND REDUCTIONS IN DETAIL.

Chronological.—The details of construction of the phototachometer were finally decided upon in the summer of 1879, and in September of that year a contract was made with the Messrs. Clark for the apparatus. The instrument was completed and ready for delivery in May, 1880. The problem of securing a location for it at a convenient and favorable point was, however, one of considerable difficulty, especially as a steam-engine was required to operate the revolving mirror, and it was essential that the whole apparatus should be under public protection. Careful and repeated inspections of the various hills around the city of Washington led, as already stated, to the selection of Fort Myer as the most available point for mounting the apparatus.

The first trial of the apparatus was made on June 22, 1880, but of course was little more than tentative, as it was not decided how the observations could best be made. A very few trials were, however, sufficient to settle upon the method of observation, which will be described presently. After a few days' trial it was found that the wheel-work to count the revolutions was destroyed by the rapid rotation to which it was subjected. Curiously enough, this simple part of the apparatus, the consideration of which had given rise to no solicitude, was the only part which gave serious trouble during the whole course of the experiments. New wheels were tried, which, however, wore out with such rapidity that it was scarcely practicable even to obtain a set of readings before they were gone. After losing three weeks in vain efforts of this kind, the revolving mirror was sent to the Messrs. Clark to make an improved wheel-work. They found, by trial, that the best metal wheels which could be made were rapidly worn away when subject to the peculiar conditions of the apparatus. They, therefore, hit upon the device of employing raw-hide as a material for the first wheel, a device which proved entirely successful. Experiments with the raw-hide wheel were commenced on August 9, and from that time forward were continued without interruption until September 20, after which the southern declination of the sun was not favorable to the reflection of the ray into the receiving telescope.

In the spring of 1881 the observations were resumed, using the same station for the fixed mirror. It appeared, however, that a longer distance could be used, and in order to gain this advantage the fixed mirror was transported to the Monument station, the position of which has already been described. The pier supporting the mirrors was built on made ground surrounding the base of the Monument, and was at first so unsteady that great difficulty was found in securing any permanent adjustment. It was not until August 8 that observations could be commenced.

The systematic differences which will hereafter be discussed were first noticed during this series of observations. They led to the conclusion that all results heretofore attained were affected by a torsional vibration of the mirror in the course of its revolution, and that to attain a reliable result this source of error must be eliminated. Portions of the apparatus were, therefore, sent to the makers for such alterations that the receiving and sending telescopes could be interchanged in position. Observations on the new plan were resumed on July 24, 1882, and continued until September 5. The number was sufficient to reduce the final accidental error to a very small amount. Indeed, throughout all the observations, the merely accidental differences were so small that the probable error of a complete determination was, under favorable circumstances, not much more than the ten-thousandth part of the whole.

Method of observation.—In devising the apparatus it was supposed that the best course would be to run the mirror for a few minutes with a velocity as nearly uniform as possible, and to make as many measures as possible at equidistant intervals of time upon the deflected image. For this purpose the eye-piece of the receiving telescope was supplied with a micrometer. It was soon found, however, that the rotation of the mirror could not be made sufficiently steady for the satisfactory application of this method. The plan, therefore, was adopted of leaving the micrometer wires in a fixed position, and adjusting the speed of the mirror so as to keep the image upon the wires This was done by means of the air-valves already described. The valves were so arranged that when the air went through one of them the mirror was driven in one direction, and when through the other in the opposite direction. Hence, by opening one valve and closing the other, the full blast could be applied to turn the mirror in either direction at pleasure. When the mirror was going at nearly full speed the latter could be restrained or diminished by letting a slight counterblast of air through the opposite valve. To enable the observer at the telescope to do this, an endless cord was passed around a groove in the wheel *T*, Plate 2, through pulleys upon the wall, around the side of the instrument, and along the pier *W*, in front of the observer. Thus the latter, by moving the cord back or forth, could adjust the counterblast with great delicacy to produce any desired change of speed. The method of making an observation was as follows:

It being ascertained that everything was in working order, the receiving telescope was set by the aid of its reading microscope upon some division near the zero end of its arc. The valve which would direct the blast so as give a negative motion to the mirror was then opened, the other one was closed, and the cord was placed around the wheel of the latter. The observer then took his place at the eye-piece, and gave a signal for the engineer to start the blower. As the speed of the mirror approached the proper limit, the image would be seen entering the field on the right and moving across it with greater or less rapidity. As it approached the parallel wires of the micrometer an assistant at the chronograph was directed to start it. Every 28th revolution of the mirror was then recorded on the chronograph-sheet, with the second beats of the chronometer, as already described. As the image approached the parallel wires, the observer began to move the cord to his right, thus slightly opening the closed valve, and admitting a minute counterblast of air. When the image just reached the middle

he rattled with his left hand a telegraph key, through which an electric circuit passed, and continued to do so until he found the image adjusted steadily and satisfactorily on the wires. The key was then let go, and the record on the chronograph proceeded without interruption. Whenever the slightest deviation of the image could be detected, which would, of course, generally happen at the end of every few seconds, a slight motion of the cord, in the right direction, would bring it back upon the wires. In general, the cord was held constantly in the hand, and the hand continually moved in such a way as to correct every deviation of the image. The object aimed at was that the deviations on the one side should as nearly as possible balance those on the other. Each "run," as it was called, generally continued about two minutes or more, unless something occurred to interrupt it. The engineer was then signaled to stop the air-blast, the key was pressed down to break the record on the chronograph, and the latter was stopped by the assistant. The windows for admitting light were then opened, the microscopes read, and the chronograph-sheet examined to see that the record was perfect.

The receiving telescope was then unclamped and set on some division near the other end of its arc. The air-valve which had been closed was opened and the other one closed, and the cord passed around it in a reverse direction. During this process the greatest care was of course taken not to touch or disarrange the receiving telescope. The shutters being all closed, the observer again took his seat at the eye-piece, signaled to the engineer to turn on the blast, and repeated the same process. The image would now be seen coming in on the left-hand side of the field. As it approached the parallel wires the counterblast was turned on by moving the cord towards the left, instead of towards the right, as in the first set. The result of the reversal of the cord was that in order to make the image move in any direction the observer had to make the cord move in that direction, and was thus relieved of all consideration of the direction in which the mirror was moving, and hence of any constant possible error depending on his idea of that direction. So far as his volitions were concerned, the relation of the deviation of the image to his efforts was the same in the two directions of revolution.

The run being completed, as before, the microscopes were again read. A comparison of the two sets sufficed for a complete determination of the time occupied by the light in going and coming.

The tabular exhibit which follows shows with sufficient fullness the different details of the observations.

In the third column the number of the set is simply the number in order of the observations made on each day and recorded for purposes of reference.

The next column gives the number of the division of the arc on which the left-hand microscope was set. The right-hand microscope was set on a division whose number was 206 greater, and the fine adjustment was always made by this microscope.

As already stated, these readings of the microscopes are estimates in measures whose unit is the distance between the individual wires of the system, four in number, set in the eye-piece of each microscope. This plan was adopted merely to save the time and expense of micrometers. Although the uncertainty of the readings is not such as to appreciably affect the results, great convenience would have been gained

had micrometers been used. The value of the unit was found to be $2''.4$, and the correction to the reading in seconds is found by multiplying the mean reading of the microscopes by this factor.

The concluded angular reading given in the next column is found by multiplying the division used by $111''.783$ and adding the correction for microscopes.

The next column gives the velocity of the mirror as determined from the readings of the chronograph sheet, expressed in chronometer seconds. It is considered negative or positive according to the direction of motion of the mirror.

The weights given in the next column refer almost entirely to the quality of the image and the generally satisfactory way in which the image was kept on the cross-wires. The maximum weight is 5, and indicates that the image was clear, sharp, and steady, and was kept on the cross-wires with entire accuracy. Weight 3 indicates an average observation, and weight 1 an observation which the observer considered very poor. When the weight is not recorded it may be considered as 3, indicating an average observation.

The next column gives the difference of the readings, and is supposed to express the angle through which the receiving telescope moved between the two sets.

The next paragraph gives the difference of the velocities, and does not require any explanation. The last column gives the mean time of passage of light in going and coming, expressed in millionths of a second. In order to deduce it, it is necessary to consider the chronometer rates, which will be done presently.

In the last column are given the weights used in combining the observations each day into a single mean result. Only a small range is assigned to the weights, because from the very nature of the case it is impossible to determine them with actual precision. Since each separate determination from a pair of sets, no matter how good it may be, is probably subject to systematic errors common to it, and to the poorer results, it was thought best not to assign so wide a range of weights as if the errors had depended only upon the quality of the image

The Chronometer rates and final formulæ.—In order to reduce the speed of the mirror to seconds of mean solar time, the chronometer time of the dropping of the noon-signal ball at the Observatory was occasionally noted.

During the season of 1880 a sidereal chronometer was used. The time-ball observations showed that, in the mean, it gained $4^m\ 0^s.3$ a day on mean solar time.

During 1881 the rate of the chronometer on sidereal time was quite small and rather irregular; it has therefore been assumed to run exactly on sidereal time.

During 1882 a mean-time chronometer was used, which, on the average, was found to lose $1^s.2$ per day on mean time.

We have, therefore, the following logarithms of the factors to reduce intervals of time expressed in chronometer seconds to mean solar seconds:

1880. log. factor = 0.001205
1881. " " = 0.001188
1882. " " = 9.999994

We have next to form an expression for the time in which the mirror, turning with an indicated speed, will move through half the arc indicated by the measures.

If we put h for the factors whose logarithms have just been given, and v for the number of turns in a chronometer second, the turns of the mirror in a mean solar second will be hv. Hence, in one such second, the mirror revolves through $1\,296\,000'' \, hv$ in arc. Hence, if we put Δ for the difference of angular readings, expressed in seconds of arc, the time in seconds occupied by the light in going and coming will be—

$$\frac{\Delta}{2\,592\,000\,hv}$$

Substituting for h its different values, and taking a millionth of a second as the unit, we have the following formulæ for the computation of the time of passage:

$$1880. \quad \tau = [9\ 585160]\frac{\Delta}{v}$$

$$1881. \quad \tau = [9.585177]\frac{\Delta}{v}$$

$$1882. \quad \tau = [9.586371]\frac{\Delta}{v}$$

From these formulæ the times of passage of the light, as given in the proper column of the table, are computed. In forming the times τ it is to be noted that since we do not measure from an absolute zero-point, but only take the algebraic differences of pairs of readings made when the mirror revolves in opposite directions, we are to take for Δ the difference of angular readings, and for v the algebraic difference or numerical sum of the velocities.

In one or two cases it was found necessary to use three sets in obtaining the result; one set being compared with the mean of the two others.

TABULAR EXHIBIT OF INDIVIDUAL OBSERVATIONS AND RESULTS.

FIRST SERIES.—*Measures from Observatory Station.*

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Reading of microscopes. Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
1880.						″			″		Millionths of second.	
June 28	NEWCOMB .	2	162	—0.2	+0.7	18109.5	+156.45					
		3	33	—0.1	+1.7	3690.8	—169.18	.	14418.7	325.63	17.036	.
June 29	NEWCOMB .	1	44	0.0	+0.5	4919.0	—140.79					
		2	170	+0.1	—0.1	19003.1	+177.36	.	14084.1	318.15	17.032	1
		3	23	—0.1	+0.1	2571.0	—194.19					
		4	173	0.0	—0.5	19337.9	+184.56	.	16766.9	378.75	17.031	1
		5	27	+0.1	+0.1	3018.4	—183.98					
		6	174	0.0	—0.5	19449.6	+187.20	.	16431.2	371.18	17.031	2
June 30	MICHELSON.	1	171	0.0	—0.5	19114.3	+176.93					
		2	23	0.0	+0.2	2571.2	—196.71	.	16878.5	381.22	17.034	3
		3	177	0.0	—0.4	19785.1	+192.10					
		4	177	0.0	—0.4	19785.1	+192.03					
		5	28	0.0	0.0	3129.9	—184.01	.	16655.2	376.04	17.040	2
		6	173	0.0	—0.5	19337.9	+182.02					
		7	28	0.0	0.0	3129.9	—183.95	.	16208.0	365.97	17.039	2
July 3	NEWCOMB .	1	39	—0.2	+0.2	4359.5	—152.10	1				
		2	145	0.0	—0.3	16208.2	+115.38	1	11848.6	267.48	17.043	1
		3	154	+0.1	+0.1	17214.8	+137.92	3				
		4	32	0.0	+0.3	3577.4	—170.24	2	13637.4	308.16	17.027	2
July 9	NEWCOMB .	1	34	0.0	+0.1	3800.7	—169.72	4				
		2	173	—0.2	—0.8	19337.2	+181.29	.	15536.5	351.01	17.029	2
Aug. 9	NEWCOMB .	1	19	0.0	+1.6	2185.8	—201.97	2				
		2	177	—0.9	—0.2	19784.3	+197.02	3	17658.5	398.99	17.028	1
		3	17	0.0	+0.4	1900.8	—206.98	3				
		4	183	0.0	—0.5	20455.7	+212.27	3	18554.9	419.25	17.028	2

June 28 to 30.—No weights were assigned to the observations made during the first three days. A few observations were made before June 28, but they were somewhat irregular, and of little value except to get the apparatus into working order.

June 29.—Set 5: beautiful bisections from 20 seconds after second rattle to 10 seconds before last one. Set 6: excellent bisection. During sets 1 and 2 the micrometer was set at 30.6 rev. Sets 3 and 4, 30.3 rev., and sets 5 and 6, 29.5 rev.

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
1880.						″			″		Millionths of second.	
Aug. 9	Newcomb .	5	15	0.0	+0.6	1677.5	−212.11	3				
		6	186	0.0	−0.6	20790.9	+219.90	3	19113.4	432.01	17.022	2
		7	13	0.0	+0.4	1453.6	−217.15	3				
		8	187	0.0	−0.5	20902.8	+222.36	4	19449.2	439.51	17.025	2
		9	9	0.0	+0.4	1006.5	−227.26	4				
		10	189	0.0	0.0	21127.0	+227.38	4	20120.5	454.64	17.027	2
		11	7	0.0	+0.5	783.1	−232.11	3				
		12	184	+0.1	−0.6	20567.5	+215.05	.	19784.4	447.16	17.023	2
Aug. 10	Newcomb .	1	50	+0.3	+0.8	5590.5	−122.33	1				
		2	151	0.0	−0.1	16879.1	+132.71	2	11288.6	255.04	17.029	1
Aug. 13	Newcomb .	1	32	0.0	+0.5	3577.7	−168.86	2				
		2	168	0.0	−0.2	18779.3	+174.51	3	15201.6	343.37	17.033	1
		3	30	0.0	+0.5	3354.1	−174.13	3				
		4	168	−0.2	−0.4	18778.8	+174.46	3	15424.7	348.59	17.024	2
		7	31	0.0	+0.0	3465.3	−171.58	4				
		8	167	0.0	−0.1	18667.6	+171.94	.	15202.3	343.52	17.026	2
		9	29	0.0	+0.2	3241.9	−176.82	4				
		10	175	0.0	−0.2	19561.8	+192.05	4	16319.9	368.87	17.022	2
		11	25	0.0	+0.5	2795.2	−186.80	4				
		12	177	0.0	−0.4	19785.1	+197.12	.	16989.9	383.92	17.026	2
		13	23	−0.1	+0.5	2571.5	−191.87	3				
		14	171	+0.1	−0.3	19114.7	+181.94	.	16543.2	373.81	17.027	2
Aug. 16	Michelson.	1	13	0.0	+0.3	1453.5	−217.10	4				
		2	181	0.0	−0.5	20232.1	+207.23	4	18778.6	424.33	17.027	2
		3	20	0.0	+0.4	2236.1	−199.38	3				
		4	180	0.0	−0.3	20120.6	+204.71	4	17884.5	404.09	17.028	2

June 30.—It was found that the teeth of the brass wheel which geared into the pinion of the revolving mirror in order to mark the number of revolutions were nearly worn away. A new wheel was hastily constructed by a local watch-maker.

July 3.—Trouble has been experienced in consequence of the pinion working loose on the shaft of the mirror.

July 9.—It has been found impossible to get a wheel that will last more than five minutes. Although there is no pressure upon the wheel (or, perhaps, for this very reason), the brass teeth are *chewed up* as it were by the steel teeth of the pinion. Friction upon the brass wheel, to prevent its shaking, does not help the matter. The mirror was, therefore, entirely dismounted and sent to Messrs. Alvan Clark & Sons for repair. After another vain trial the Messrs. Clark proposed a toothed wheel of raw hide.

August 9.—Apparatus works beautifully with the raw-hide wheel. Set 9: two runs were made at this setting, the sun having disappeared behind a cloud during the first run. The speeds were .227.23 and 227.29 respectively.

August 10.—Set 1 very poor. Worthless except from 15 seconds after first rattle to 30 seconds before last. Often no image visible.

Set 2. Fair bisection, but vibrating considerably, owing to faintness of images.

August 13.—Sets 5 and 6: chronograph stopped.

[Value of 1 div. microscopes = 2''.4. Value of 1 div. of limb = 111''.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes.		Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
				Right.	Left.							
						''			''		Millionths of second.	
1880. Aug. 16	MICHELSON.	5	25	0.0	+0.4	2795.0	—186.86	3				
		6	177	0.0	—0.3	19785.2	+197.12	4	16990.2	383.98	17.023	2
		7	24	0.0	+0.5	2683.4	—189.39	4				
		8	156	0.0	—0.2	17437.9	+144.23	3	14754.5	333.62	17.015	2
		9	20	0.0	+0.4	2236.1	—199.32	4				
		10	172	0.0	—0.4	19226.2	+184.46	4	16990.1	383.78	17.032	2
		11	18	0.0	+0.3	2012.4	—204.38	5				
		12	175	0.0	—0.1	19561.9	+192.17	4	17549.5	396.55	17.027	2
Aug. 17	NEWCOMB.	1	30	+0.2	+0.6	3354.5	—177.06	4				
		2	175	0.0	—0.2	19561.8	+189.12	4	16207.3	366.18	17.028	2
		3	15	0.0	+0.7	1677.6	—214.84	.				
		4	180	0.0	—0.4	20120.5	+202.09	3	18442.9	416.93	17.018	2
		5	13	0.0	+0.4	1453.7	—219.54	5				
		6	161	0.0	—0.3	17996.7	+154.19	4	16543.0	373.73	17.030	2
		7	14	—0.1	+0.3	1565.2	—217.08	5				
		8	182	0.0	—0.5	20343.9	+207.18	5	18778.7	424.26	17.030	2
		9	12	0.0	+0.4	1341.9	—221.98	5				
		10	178	+0.1	—0.2	19897.3	+197.19	.	18555.4	419.17	17.031	2
		11	13	0.0	+0.4	1453.7	—219.52	4				
		12	183	0.0	—0.5	20455.7	+209.84	4	19002.0	429.36	17.027	2
Aug. 21	MICHELSON.	1	10	0.0	+0.4	1118.3	—223.33	3				
		2	186	0.0	+0.2	20791.9	+219.98	4	19673.6	443.31	17.074	0
		3	11	0.0	0.0	1229.6	—221.81	3				
		4	185	0.0	—0.5	20679.3	+217.71	3	19449.6	439.52	17.025	1
		5	12	0.0	+0.4	1341.9	—218.81	5				
		6	184	0.0	—0.6	20567.4	+215.57	5	19225.5	434.38	17.028	1
		7	13	0.0	+0.3	1453.5	—216.20	4				
		8	183	0.0	—0.5	20455.7	+213.02	4	19002.2	429.22	17.033	2

August 17.—Set 4: two runs at this setting, the first not being entirely satisfactory. Results 201.79 and 202.09. Set 9: stopped by passing cloud.

August 21.—Comparison of speeds with settings shows an unexplained disturbance of instrument during first few sets.

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Reading of microscopes. Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
						″			″		Millionths of second.	
1880. Aug. 21	MICHELSON.	9	14	0.0	+0.3	1565.3	−213.79	3				
		10	182	0.0	−0.5	20343.9	+210.45	4	18778.6	424.24	17.030	2
		11	15	0.0	+0.5	1677.3	−211.39	4				
		12	181	0.0	−0.6	20232.0	+207.90	4	18554.7	419.29	17.026	2
Aug. 24	MICHELSON.	1	15	0.0	+0.5	1677.3	−211.50	3				
		2	185	0.0	−0.5	20679.2	+217.89	4	19001.9	429.39	17.026	2
		3	16	0.0	+0.4	1789.0	−208.90	3				
		4	184	0.0	−0.7	20567.2	+215.47	3	18778.2	424.37	17.020	2
		5	17	0.0	+0.3	1900.7	−206.47	2				
		6	183	0.0	−0.5	20455.7	+211.83	4	18555.0	418.30	17.027	2
		7	18	0.0	+0.3	2012.5	−209.41	3				
		8	182	0.0	−0.6	20343.8	+204.72	4	18331.3	414.13	17.030	2
		9	19	0.0	+0.4	2124.4	−206.34	3				
		10	181	0.0	−0.6	20232.0	+202.84	4	18107.6	409.18	17.026	2
		11	20	0.0	+0.4	2236.1	−203.77	4				
		12	180	0.0	−0.5	20120.3	+200.38	4	17884.2	404.15	17.025	2
Aug. 25	MICHELSON.	1	20	0.0	+0.5	2236.3	−203.02	2				
		2	180	0.0	−0.4	20120.5	+201.10	4	17884.2	404.12	17.026	1
		3	21	0.0	+0.3	2347.8	−200.21	3				
		4	179	0.0	−0.5	20008.6	+198.75	4	17660.8	398.96	17.031	2
		5	22	0.0	+0.4	2459.7	−197.89	4				
		6	178	0.0	−0.4	19896.9	+196.25	4	17437.2	394.14	17.021	2
		7	23	0.0	+0.4	2571.5	−195.22	3				
		8	177	0.0	−0.4	19785.1	+193.61	4	17213.6	388.83	17.032	2
		9	24	0.0	+0.5	2683.4	−192.65	4				
		10	176	0.0	−0.5	19673.2	+191.26	5	16989.8	383.91	17.026	2
		11	25	0.0	+0.5	2795.2	−190.23	5				
		12	175	0.0	−0.2	19561.8	+188.73	5	16766.6	378.96	17.022	2

August 2 to 25.—Mr. MICHELSON always set the division under the zero-point of the right microscope.
August 25.—Set 1 very faint. Cloudy.

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Reading of microscopes. Left.	Concluded angular reading.	Turn of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
						″			″		Millionths of second.	
1880. Sept. 3	MICHELSON.	1	10	0.0	+0.4	1118.3	−230.53	4				
		2	190	0.0	−0.3	21238.4	+224.16	5	20120.1	454.69	17.025	2
		3	11	0.0	+0.2	1229.9	−228.03	4				
		4	189	0.0	−0.1	21126.9	+221.60	4	19897.0	449.63	17.025	2
		5	12	0.0	+0.4	1341.9	−225.50	4				
		6	188	0.0	−0.5	21014.6	+219.01	4	19672.7	444.51	17.027	2
		7	13	0.0	+0.4	1453.7	−223.02	4				
		8	187	0.0	−0.5	20902.8	+216.43	4	19449.2	439.45	17.027	2
		9	14	0.0	+0.3	1565.3	−220.49	5				
		10	186	0.0	−0.7	20790.8	+213.99	5	19225.5	434.48	17.024	2
		11	15	0.0	+0.5	1677.3	−217.92	5				
		12	185	0.0	−0.4	20679.4	+211.33	5	19002.0	429.25	17.031	2
Sept. 4	MICHELSON.	1	15	0.6	+0.6	1677.5	−221.54	4				
		2	185	0.0	−0.5	20679.3	+207.96	5	19001.8	429.50	17.021	2
		3	16	0.0	+0.4	1789.0	−218.99	5				
		4	184	0.0	−0.6	20567.4	+205.34	5	18778.3	424.33	17.027	2
		5	15	0.0	+0.6	1677.5	−221.53	4				
		6	183	0.0	−0.5	20455.7	+202.78	5	18778.2	424.31	17.027	2
		7	18	0.0	+0.3	2012.5	−213.69	4				
		8	182	0.0	−0.5	20343.9	+200.69	4	18331.5	414.38	17.020	2
		9	19	0.0	+0.3	2124.2	−210.95	5				
		10	181	0.0	−0.6	20232.0	+198.10	5	18107.8	409.05	17.032	2
		11	20	0.0	+0.5	2236.3	−208.42	5				
		12	180	0.0	−0.5	20120.3	+195.60	5	17884.0	404.02	17.031	2
		13	21	0.0	+0.2	2347.7	−205.91	5				
		14	179	0.0	−0.5	20008.6	+193.07	5	17660.9	398.98	17.030	2
		15	22	0.0	+0.4	2459.7	−203.38	5				
		16	178	0.0	−0.4	19896.9	+190.68	5	17437.2	394.06	17.023	2
		17	23	0.0	+0.4	2571.5	−200.65	5				
		18	177	0.0	−0.4	19785.1	+188.20	5	17213.6	388.85	17.031	2
		19	24	0.0	+0.4	2683.3	−198.06	5				
		20	179	0.0	−0.5	20008.6	+193.39	5	17325.3	391.45	17.028	2

[Value of 1 div. microscopes = 2''.4. Value of 1 div. of limb = 111''.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes.		Concluded angular reading.	Turns of mirror per second.	Wt	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
				Right.	Left.							
						''			''		Millionths of second.	
1880. Sept. 10	MICHELSON.	1	25	0.0	+0.6	2795.3	−194.49	4				
		2	175	0.0	—0.2	19561.8	+184.36	3	16766.5	378.85	17.027	1
		3	26	0.0	+0.3	2906.7	−191.79	3				
		4	174	0.0	—0.3	19449.9	+182.01	2	16543.2	373.80	17.027	1
		5	27	0.0	+0.4	3018.6	−189.27	3				
		6	173	0.0	—0.3	19338.1	+179.47	3	16319.5	368.74	17.027	1
		7	28	0.0	+0.4	3130.4	−186.80	4				
		8	172	0.0	—0.5	19226.1	+177.01	3	16095.7	363.81	17.022	2
		9	28	0.0	+0.4	3130.4	−186.71	3				
		10	171	0.0	—0.4	19114.4	+174.43	3	15984.0	361.14	17.028	2
		11	30	0.0	+0.4	3354.0	−181.80	4				
		12	170	0.0	—0.1	19003.0	+171.76	4	15649.4	343.56	17.029	2
		13	31	0.0	+0.4	3465.8	−179.31	5				
		14	169	0.0	—0.1	18891.2	+169.24	5	15425.4	348.55	17.027	2
		15	32	0.0	+0.5	3577.7	−176.80	5				
		16	168	0.0	—0.3	18779.2	+166.61	5	15201.5	343.41	17.031	2
		17	33	0.0	+0.6	3689.6	−174.24	5				
		18	167	0.0	—0.2	18667.5	+164.12	5	14978.0	338.36	17.031	2
		19	34	0.0	+0.5	3801.2	−171.80	4				
		20	166	0.0	—0.4	18555.5	+161.65	5	14754.3	333.45	17.023	2
		21	35	0.0	+0.7	3913.2	−169.22	5				
		22	165	0.0	—0.1	18444.0	+159.11	5	14530.8	328.33	17.028	2

September 10.—Sets 2 and 4; images very faint. Sets 1, 3, 5, and 10; images faint, cloudy.

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of Microscopes.		Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
				Right.	Left.							
						″			″		Millionths of second.	
1880. Sept. 11	MICHELSON.	1	36	0.0	+0.2	4024.4	−166.36	3				
		2	164	0.0	−0.5	18331.8	+156.79	4	14307.4	323.15	17.034	1
		3	37	0.0	+0.5	4136.6	−163.87	5				
		4	163	0.0	−0.1	18220.5	+154.19	5	14083.9	318.06	17.036	2
		5	38	0.0	+0.4	4248.2	−161.43	4				
		6	162	0.0	−0.3	18108.5	+151.64	4	13860.3	313.07	17.033	2
		7	39	0.0	+0.5	4360.1	−158.99	5				
		8	161	0.0	−0.4	17996.6	+149.61	5	13636.5	308.60	17.000	0
		9	40	0.0	+0.5	4471.9	−156.02	5				
		10	160	0.0	−0.1	17885.2	+146.99	5	13413.2	303.01	17.031	2
		11	41	0.0	+0.3	4583.5	−153.53	5				
		12	149	0.0	−0.2	16655.4	+119.19	5	12072.0	272.72	17.030	2
		13	42	0.0	+0.3	4695.2	−151.03	5				
		14	158	0.0	0.0	17661.7	+141.91	4	12966.5	292.94	17.030	2
		15	43	0.0	+0.4	4807.1	−148.52	.				
		16	157	0.0	0.0	17549.9	+139.34	5	12742.8	287.86	17.031	2
		17	44	0.0	+0.5	4919.1	−145.95	5				
		18	156	0.0	−0.3	17437.8	+136.79	5	12518.7	282.74	17.035	2
		19	45	0.0	+0.5	5030.8	−143.59	5				
		20	155	0.0	+0.4	17326.8	+134.23	5	12296.0	277.82	17.028	2
		21	46	0.0	0.0	5142.0	−140.99	5				
		22	164	0.0	−0.4	18331.9	+157.00	5	13189.9	297.99	17.029	2
		23	47	0.0	+0.3	5254.2	−138.52	5				
		24	153	0.0	+0.3	17103.2	+129.14	4	11849.0	267.66	17.031	2
		25	48	0.0	+0.4	5366.1	−135.91	3				
		26	154	0.0	+0.3	17214.9	+131.72	5	11848.9	267.63	17.033	2
		27	50	0.0	+0.5	5589.7	−130.85	4				
		28	151	0.0	0.0	16879.2	+124.11	5	11289.5	254.96	17.036	2
		29	50	0.0	+0.5	5589.7	−130.85	5				
		30	150	0.0	0.0	16767.4	+121.60	5	11177.7	252.45	17.035	2

September 11.—Hazy sky. Set 1; image faint. Sets 7 and 8; readings show some change in the instrument between these sets.

[Value of 1 div. microscopes = 2''.4. Value of 1 div. of limb = 111''.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Reading of microscopes. Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
						''			''		Millionths of second.	
1880. Sept. 13	MICHELSON.	1	50	0.0	+0.5	5589.7	−130.94	3				
		2	150	0.0	−0.1	16767.3	+121.40	3	11177.6	252.34	17.042	1
		3	51	0.0	+0.4	5701.4	−128.61	5				
		4	149	0.0	−0.2	16655.4	+118.75	3	10954.0	247.36	17.038	2
		5	52	0.0	+0.7	5813.6	−126.08	5				
		6	148	0.0	−0.1	16543.8	+116.26	4	10730.2	242.34	17.035	2
		7	53	0.0	+0.6	5925.2	−123.61	4				
		8	147	0.0	−0.3	16431.7	+113.70	5	10506.5	237.31	17.032	2
Sept. 15	NEWCOMB.	1	30	−0.1	+0.5	3354.0	−180.79	.				
		2	170	+0.1	+0.3	19003.6	+172.69	.	15649.6	353.48	17.033	2
		3	32	0.0	+0.6	3577.8	−175.86	.				
		4	168	0.0	−0.2	18779.3	+167.59	.	15201.5	343.45	17.028	2
		5	25	(*)	(*)	2794.6	−193.51	.				
		6	171	0.0	−0.4	19114.4	+175.18	.	16319.8	368.69	17.030	2
		7	24	+0.1	+0.6	2683.6	−196.15	.				
		8	172	0.0	−0.4	19226.2	+177.64	.	16542.6	373.79	17.027	2
		9	27	0.0	+0.4	3018.6	−188.66	4				
		10	171	+0.1	−0.2	19114.8	+174.96	4	16096.2	363.62	17.031	2
Sept. 17	NEWCOMB.	1	37	0.0	+0.5	4136.6	−161.20	2				
		2	160	−0.1	−0.2	17884.9	+149.40	2	13748.3	310.60	17.030	1
		3	36	−0.1	+0.3	4024.4	−163.80	2				
		4	161	−0.1	−0.4	17996.5	+151.87	3	13972.0	315.67	17.029	1
		5	36	0.0	+0.4	4024.7	−163.83	3				
		6	160	0.0	+0.1	17885.4	+149.38	.	13860.7	313.21	17.026	1

September 13.—Set 1: Heliostat adjusted.

September 15.—Set 5: Readings of microscopes not recorded; uncertain by ± 1.0. After set 6 took out mirror and oiled its bearings.

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
1880.						″			″		Millionths of second.	
Sept. 17	Newcomb .	7	39	+0.1	+0.6	4360.4	−156.29	2				
		8	158	0.0	+0.3	17662.1	+144.26	2	13301.7	300.55	17.028	1
		9	40	0.0	+0.5	4471.9	−153.79	2				
		10	157	(*)	(*)	17550.4	+141.68	.	13078.5	295.47	17.030	1
		11	41	0.0	+0.4	4583.6	−151.32	3				
		12	159	−0.1	+0.3	17773.7	+146.70	3	13190.1	298.02	17.028	1
		13	42	0.0	+0.4	4695.4	−148.80	3				
		14	156	+0.1	+0.1	17438.4	+139.11	.	12743.0	287.91	17.028	1
Sept. 18	Newcomb .	1	15	0.0	+0.6	1677.5	−216.54	2				
		2	195	+0.1	−0.3	21797.4	+238.04	2	20120.0	454.58	17.029	1
		3	2	+0.1	+0.4	224.2	−249.43	2				
		4	198	−0.1	−0.6	22132.2	+245.58	3	21908.0	495.01	17.027	1
		5	0	0.0	+0.5	0.6	−254.50	3				
		6	200	+0.2	−0.4	22356.4	+250.58	3	22355.8	505.08	17.029	2
		7	1	0.0	+0.4	112.3	−252.03	.				
		8	199	0.0	−0.5	22244.2	+247.94	3	22131.9	499.97	17.031	2
		9	3	0.0	+0.3	335.7	−247.02	3				
		10	197	0.0	−0.4	22020.8	+242.87	4	21685.1	489.89	17.030	2
		11	4	0.0	+0.4	447.6	−244.61	3				
		12	196	+0.1	−0.4	21909.1	+240.31	4	21461.5	484.92	17.028	2
		13	5	0.0	+0.5	559.5	−241.97	4				
		14	193	+0.2	−0.3	21574.0	+232.78	4	21014.5	474.75	17.030	2
		15	4	+0.1	+0.5	447.9	−244.42	4				
		16	191	−0.2	−0.4	21349.8	+227.92	1	20902.0	472.34	17.025	1
		17	6	+0.1	+0.3	671.2	−239.41	.				
		18	194	−0.1	−0.5	21685.2	+235.40	.	21014.0	474.81	17.027	2

September 17.—Cloudy. Observations frequently stopped. Set 10: Setting not recorded, but probably microscopes read as usual. Reduced observations on this supposition.

September 18.—Set 1: Blast so strong found it difficult to keep image on wires. Set 16: Image very unsteady.

[Value of 1 div. microscopes = 2''.4. Value of 1 div. of limb = 111''.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
1880.						''			''		Millionths of second.	
Sept. 20	NEWCOMB .	1	15	+0.2	+0.6	1677.7	—216.33	2				
		2	185	0.0	—0.5	20679.3	+212.88	2	19001.6	429.21	17.032	1
		3	0	+0.2	+0.7	1.1	—254.31	.				
		4	200	0.0	—0.5	22356.0	+250.71	.	22354.9	505.02	17.030	1
		5	1	—0.1	—0.1	111.5	—251.92	4				
		6	199	0.0	—0.5	22244.2	+248.14	3	22132.7	500.06	17.028	2
		7	2	0.0	+0.3	223.9	—249.42	3				
		8	198	0.0	—0.6	22132.3	+245.61	3	21908.4	495.03	17.027	2
		9	3	+0.4	+0.5	336.4	—246.93	2				
		10	197	0.0	—0.3	22020.9	+242.96	2	21684.5	489.89	17.030	1
		11	4	0.0	+0.4	447.6	—244.08	4				
		12	196	+2.7	+2.8	21916.1	+240.83	5	21468.5	484.91	17.033	2
		13	5	+0.1	+0.5	559.6	—241.57	5				
		14	195	0.0	—0.3	21797.3	+238.30	4	21237.7	479.87	17.027	2
		15	6	0.0	+0.0	670.7	—239.06	4				
		16	194	+0.1	—0.3	21685.7	+235.79	4	21015.0	474.85	17.027	2
		17	7	0.0	+0.4	783.0	—236.56	5				
		18	193	0.0	—0.4	21573.6	+233.08	2	20790.7	469.64	17.031	1

September 20.—Set 4: After this set instrument disturbed by a slight knock.

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes.		Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
				Right.	Left.							
1881.						″			″		Millionths of second.	
Mar. 25	Newcomb .	1	31	0.0	0.0	3465.3	−170.42	3				
		2	163	−0.1	−0.4	18220.0	+163.01	2	14754.7	333.43	17.026	1
		3	22	−0.1	+0.3	2459.5	−193.22	3				
		4	176	−0.1	−0.5	19673.1	+195.77	3	17213.6	388.99	17.026	2
		5	6	0.0	−0.3	670.3	−233.71	2				
		6	182	0.0	−0.6	20343.8	+210.97	3	19673.4	444.68	17.022	1
		7	17	+0.1	+0.4	1900.9	−205.74	4				
		8	183	+0.1	−0.5	20455.8	+213.52	4	18554.9	419.26	17.028	2
Mar. 28	Newcomb .	1	15	+0.2	+0.6	1677.7	−212.27	3				
		2	188	+0.3	−0.4	21015.1	+223.14	.	19337.4	435.41	17.087	0
		3	15	+0.2	+0.6	1677.7	−212.83	2				
		4	189	0.0	−0.4	21126.5	+226.65	3	19448.8	439.48	17.027	2
		5	9	0.0	+0.2	1006.3	−227.26	4				
		6	190	−0.1	−0.4	21238.2	+229.93	4	20231.9	457.19	17.026	2
		7	7	0.0	+0.3	782.8	−232.41	4				
		8	190	0.0	−0.4	21238.2	+229.83	5	20455.4	462.24	17.026	2
Apr. 7	Newcomb .	1	9	+0.1	+0.2	1006.4	−226.60	3				
		2	186	−0.2	−1.1	20790.1	+220.33	3	19783.7	446.93	17.031	2
		3	17	−0.1	+0.2	1900.4	−207.16	2				
		4	176	+0.2	−0.4	19673.6	+194.55	.	17773.1	401.71	17.023	1
		5	4	+0.1	+0.3	447.6	−240.81	3				
		6	190	+0.3	−0.2	21238.9	+229.02	3	20791.3	469.83	17.026	2
		7	5	+0.1	+0.4	559.5	−238.27	.				
		8	189	+0.4	0.0	21127.5	+226.47	.	20568.0	464.74	17.028	2
		9	6	0.0	+0.3	671.1	−235.77	3				
		10	189	+0.1	−0.4	21014.8	+223.93	.	20343.8	459.70	17.027	2
		11	7	0.0	+0.4	783.0	−233.26	.				
		12	187	+0.4	−0.3	20903.5	+221.37	.	20120.6	454.63	17.028	2

March 28.—The result of sets 1 and 2 rejected on account of discordance. The apparatus seems to have been disarranged between the sets.

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes.		Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
				Right.	Left.							
1881.						″			″		Millionths of second.	
Apr. 15	NEWCOMB .	1	8	0.0	+0.3	894.6	−230.80	3				
		2	190	−0.1	−0.5	21238.0	+228.90	3	20343.4	459.70	17.027	2
		3	7	−0.2	0.0	782.2	−233.32	3				
		4	191	+0.3	−0.3	21350.6	+231.43	3	20568.3	464.75	17.028	2
		5	5	+0.1	+0.3	559.4	−238.38	3				
		6	192	+0.3	0.0	21462.7	+233.94	3	20903.3	472.32	17.028	2
		7	3	0.0	+0.1	335.5	−244.40	4				
		8	193	+0.2	−0.4	21573.9	+235.54	4	21238.4	479.94	17.026	2
		9	4	+0.1	+0.4	447.7	−241.83	.				
		10	189	+0.3	−0.2	21127.1	+225.46	.	20679.4	467.29	17.027	2
		11	1	−0.4	−0.6	110.6	−249.51	3				
		12	198	0.0	−0.7	22132.2	+248.09	3	22021.6	497.60	17.027	2
		13	2	0.0	+0.1	223.7	−246.93	.				
		14	197	+0.4	0.0	22021.7	+245.60	.	21798.0	492.53	17.029	2

During the spring and summer of 1881 the instrument was dismounted, and its direction changed so as to receive the return reflected ray from the station at the base of the Washington Monument. The pier supporting the revolving mirror was, however, left unaltered in position.

SECOND SERIES.—*Measures from Monument Station.*

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
1881.						″			″		Millionths of second.	
Aug. 8	NEWCOMB.	1	2	—0.2	+0.2	223.6	—220.40	1				
		2	247	—0.2	—0.1	27610.0	+203.89	2	27386.5	424.29	24.834	1
		3	13	0.0	+0.5	1453.8	—201.31	2				
		4	268	—0.1	+1.2	29959.2	+240.27	3	28505.4	441.58	24.836	1
		5	0	+0.2	+1.7	2.3	—223.75	.				
		6	268	—0.1	—0.3	29957.4	+240.23	3	29955.1	463.98	24.840	2
		7	3	—0.1	+0.4	335.7	—218.61	3				
		8	270	+0.2	+0.6	30182.4	+243.68	3	29846.7	462.29	24.840	2
		9	4	—0.1	+0.3	447.4	—216.91	3				
		10	269	+0.1	+0.3	30070.1	+242.06	3	29622.7	458.97	24.833	2
		11	13	—0.1	+0.5	1453.7	—201.36	.				
		12	267	—0.2	—0.4	29845.3	+238.52	3	28391.7	439.88	24.834	2
		13	5	0.0	+0.4	559.4	—215.16	2				
		14	266	0.0	+0.3	29734.6	+236.80	1	29175.2	451.96	24.837	1
Aug. 10	NEWCOMB.	1	5	—0.2	0.0	558.7	—215.87	3				
		2	265	0.0	—0.2	29622.3	+234.45	3	29063.6	450.32	24.831	2
		3	4	+0.1	+0.2	447.5	—217.53	3				
		4	263	0.0	—0.3	29398.6	+231.02	4	28951.1	448.55	24.834	2
		5	3	—0.1	—0.2	335.0	—219.23	4				
		6	267	+0.1	—0.4	29845.7	+238.08	4	29510.7	457.31	24.829	2
		7	2	0.0	+0.1	223.7	—220.88	3				
		8	268	0.0	—0.4	29957.4	+239.81	2	29733.7	460.69	24.831	1
		9	0	+0.1	+0.4	0.6	—224.29	2				
		10	266	0.0	—0.2	29734.0	+236.34	3	29733.4	460.63	24.835	1
		11	0	0.0	+0.1	0.1	—224.47	3				
		12	264	0.0	—0.4	29510.2	+232.81	3	29510.2	457.28	24.830	2
		13	— 1	0.0	+0.1	— 111.7	—226.01	4				
		14	263	0.0	—0.4	29398.4	+231.28	.	29510.1	457.29	24.829	2
		15	— 2	—0.1	+0.2	— 223.4	—227.70	3				
		16	262	0.0	—0.3	29286.8	+229.47	3	29510.2	457.17	24.835	2
Sept. 12	NEWCOMB.	1	21	0.0	—0.2	2347.2	—181.74	3				
		2	241	+0.1	—0.2	26939.6	+199.25	2	24592.4	380.99	24.835	.
		3	11	0.0	0.0	1229.6	—199.17	3				
		4	243	0.0	0.0	27163.3	+202.28	2	25933.7	401.45	24.854	.

September 12.—A comparison of sets 2 and 4 shows some change in the instrument before set 4; the only result used is, therefore, that found by combining the mean of 1 and 3 with 2. This gives 24.831.

[Value of 1 div. microscopes = 2''.4. Value of 1 div. of limb = 111''.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
1881.						''			''		Millionths of second.	
Sept. 13	NEWCOMB .	1	51	+0.1	+0.3	5701.4	—129.22	2				
		2	228	0.0	—0.5	25485.9	+177.16	1, 2	19784.5	306.38	24.845	1
		3	35	0.0	+0.4	3912.9	—157.02	2				
		4	240	—0.5	—0.7	26826.5	+197.97	1	22913.6	354.99	24.834	1
		5	34	+0.1	+0.3	3801.1	—158.38	2				
		6	235	+3.4	+4.0	26277.9	+189.63	2	22476.8	348.01	24.849	1
		7	36	—0.1	+0.1	4024.2	—155.02	3				
		8	228	0.0	—0.4	25486.0	+177.65	4	21461.9	332.67	24.821	2
		9	37	—0.4	+0.1	4135.6	—153.34	4				
		10	234	+0.2	—0.1	26157.3	+187.83	3	22021.7	341.17	24.835	2
		11	20	0.0	+0.3	2236.0	—182.34	1				
		12	233	0.0	—0.1	26045.3	+186.24	2	23809.3	368.58	24.853	1
Sept. 19	NEWCOMB .	1	19	0.0	—0.1	2123.8	—184.76	2				
		2	224	0.0	—0.5	25038.8	+170.27	2	22915.0	355.03	24.832	1
		3	6	+0.2	+0.3	671.3	—205.31	2				
		4	235	+0.2	—0.4	26268.8	+191.20	2	25597.5	396.51	24.839	1
		5	20	0.0	+0.1	2235.8	—182.70	3				
		6	236	0.0	—0.4	26380.3	+191.32	3	24144.5	374.02	24.838	2
		7	20	0.0	0.0	2235.7	—182.94	3				
		8	230	+0.4	+0.3	25710.9	+180.90	2	23475.2	363.84	24.824	1
		9	5	0.0	+0.1	559.0	—207.53	3				
		10	237	0.0	—0.4	26492.1	+194.39	3	25933.1	401.92	24.825	2
		11	6	0.0	+0.3	671.1	—205.60	3				
		12	235	+0.2	+0.4	26269.7	+190.89	3	25598.7	396.49	24.831	2
		13	4	0.0	+0.1	447.3	—209.07	3				
		14	238	0.0	—0.4	26603.9	+196.28	3	26156.6	405.35	24.827	2

For some time past I have been suspicious that the various parts of the image formed by the different faces of the mirror were not accurately in the same vertical line. Were this suspicion well founded, reflections from different faces of the mirror would give different results. Hitherto no reasons for any such differences had presented itself to my mind, but to-day, on starting set 4, the four images formed by the different faces of the mirror were found to be arranged thus: = =. The cause of this appearance, which at once suggested itself, was a torsional vibration of the mirror around its own axis of revolution, having a period of, probably, half the time of rotation. A source of possible systematic error in the results was thus presented, the discussion of which will be given subsequently. Repeated trials show that the vibration did not set in, or at least did not become sensible, until the mirror attained a certain speed, which limit of speed, however, was very variable. On the following days I endeavored to keep the speed within this limit, yet the results must be recorded as very doubtful in consequence of the liability of the vibration to set in.

September 13.—Set 1: Had to run at low speed, owing to vibration of mirror at high speed. Set 4: Images begin to separate by torsional vibration; tried to observe the mean position of the four images. Sets 5 and 6: Images all right, but very faint and intermitting.

On *September* 13 the mirror was unshipped and sent to the Messrs. CLARK to be balanced. They reported it as sensibly out of balance and the pivot as not being perfectly round.

September 19.—First trial after the mirror had been adjusted by the Messrs. CLARK. Set 1: Image fluctuating during firs half of run. Set 4: Image fluctuating 5 or 10 seconds. Set 8: After 6 runs readjusted and oiled the pivots of the mirror.

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Reading of microscopes. Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
1881.						″			″		Millionths of second.	
Sept. 24	NEWCOMB .	1	10	—0.2	—0.4	1117.1	—198.94	2				
		2	237	+0.5	+0.4	26493.7	+194.73	2	25376.5	393.67	24.800	1
		3	0	+0.1	+0.3	0.5	—215.15	2				
		4	240	0.0	—0.3	26827.6	+200.14	3	26827.1	415.29	24.854	1
		5	1	+0.1	—0.1	111.8	—213.65	3				
		6	239	0.0	—0.4	26715.7	+198.44	3	26603.9	412.09	24.838	2
		7	2	0.0	—0.3	223.2	—211.90	3				
		8	242	0.0	—0.2	27051.2	+203.62	3	26828.0	415.52	24.842	2
		9	3	—0.1	—0.4	334.7	—209.86	3				
		*10a	244	0.0	+0.4	27275.5	+207.45	3	26940.8	417.31	24.839	1
		9	3	—0.1	—0.4	334.7	—209.86	3				
		*10b	235	0.0	0.0	26269.0	+191.48	.	25934.3	401.34	24.862	1
		†11a	4	+0.2	+0.4	447.9	—208.17	2				
		12	236	0.0	—0.4	26380.3	+193.56	4	25932.4	401.73	24.836	1
		†11b	9	0.0	—0.2	1005.8	—199.52	4				
		12	236	0.0	—0.4	26380.3	+193.56	4	25374.5	393.08	24.837	2
		13	7	0.0	+0.2	782.7	—202.72	2				
		14	222	—0.1	+0.5	24816.3	+169.39	3	24033.6	372.11	24.850	2

* No reason is given why 10*b* was taken. Combined with 9, the result is discordant with that of 9 and 10*a*.

† No reason is given why 11*b* was taken.

September 24.—Image rather faint and unsteady. Set 9: During this set the torsional vibration again became apparent. There were, however, three intervals during which the image was good.

During some subsequent trials of the apparatus the pivot of the mirrors suddenly cohered to its conical cap, and the mirror was sent to the makers for another thorough overhauling of its pivots.

THIRD SERIES.—*Measures from Monument Station with new arrangement of Receiving Telescope.*

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer, and order of telescopes.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Reading of microscopes. Left.	Concluded angular reading.	Turns of mirror per secoud.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
1882.						″			″		Millionths of second.	
July 24	NEWCOMB .	1	12	—0.4	—0.3	1341.3	—196.12	3				
		2	241	0.0	+0.6	26940.4	+201.71	.	25599.1	397.83	24.828	2
	Reversed . .	3	5	+0.2	+0.3	559.5	—208.35	3				
		4	245	+0.5	+0.4	27387.9	+208.57	3	26828.4	416.92	24.826	2
		5	1	—0.2	—0.3	111.2	—215.18	2				
		6	241	+0.3	0.0	26940.1	+201.63	3	26828.9	416.81	24.833	1
July 26	NEWCOMB .	1	11	—0.1	—0.3	1229.1	—198.14	2				
		2	234	—0.1	—0.1	26157.0	+189.30	2	24927.8	387.44	24.824	1
	Reversed (?)	3	19	0.0	+0.2	2124.1	—184.10	3				
		4	247	+0.2	0.0	27610.6	+211.83	1	25486.5	395.93	24.834	1
		5	14	—0.1	+0.2	1565.1	—192.70	3				
		6	253	+0.1	+0.4	28281.7	+223.66	4	26716.6	416.36	24.756	0
		7	11	+0.3	+0.1	1230.1	—196.62	3				
		8	244	0.0	+0.1	27275.2	+208.12	3	26045.1	404.74	24.827	2
July 31	HOLCOMBE .	1	20	—0.1	—0.4	2235.1	—184.13	2				
		2	255	—0.3	+0.4	28504.8	+224.28	1	26269.7	408.41	24.816	2
	Direct . . .	3	14	0.0	+0.4	1565.4	—194.32	1				
		4	250	—0.1	—0.4	27945.1	+215.42	1	26379.7	409.74	24.840	1
		5	8	+0.2	+0.4	895.0	—204.93	1				
		6	257	0.0	+0.4	28728.7	+228.10	2	27833.7	433.03	24.798	2
		7	7	—0.5	—0.4	781.4	—206.37	2				
		8	254	—0.1	+0.3	28393.1	+222.61	2	27611.7	428.98	24.829	3
Aug. 9	HOLCOMBE .	1	10	0.0	+0.3	1118.2	—195.34	2				
		2	256	—0.4	—0.5	28615.4	+232.05	2	27497.2	427.39	24.822	1
	Reversed . .	3	4	+0.2	+0.5	448.0	—205.80	2				
		4	253	—0.4	—0.2	28280.4	+226.68	2	27832.4	432.48	24.824	1
		5	3	+0.1	0.0	335.5	—207.49	1				
		6	255	0.0	+0.6	28505.4	+230.38	3	28169.9	437.87	24.821	1

July 24 and 26 (?).—Telescopes interchanged; sending telescope below. Set 2: After set 2 adjusted mirror and microscopes.

July 26.—Set 2: Had to make three trials before image was all right. Set 4: The image distorted into an inclined ellipse. Sets 5 and 6: Some change in the zero-point.

July 31.—Sending telescope *above*, in the old direct position. The discordances do not seem to arise from any well-marked, definite changes in the zero-point, but solely from the badness of the conditions.

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer and order of telescopes.	No. of set.	Div. of arc used.	Reading of microscopes.		Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
				Right.	Left.							
1882.						″			″		Millionths of second.	
Aug. 10	Newcomb .	1	10	0.0	+0.3	1118.2	−195.52	3				
		2	236	0.0	0.0	26380.8	+197.09	3	25262.6	392.61	24.825	1
	Direct . . .	3	11	0.0	0.0	1229.6	−193.72	2				
		4	230	0.0	+0.2	25710.3	+186.65	3	24480.7	380.37	24.830	2
		5	8	−0.1	−0.3	894.5	−199.04	2				
		6	232	+0.2	+0.5	25934.5	+190.14	3	25040.0	389.18	24.823	1
		7	12	+0.1	+0.5	1342.1	−192.00	4				
		8	229	0.0	0.0	25598.3	+184.91	4	24256.2	376.91	24.829	2
		9	13	+0.2	+0.5	1454.0	−190.22	4				
		10	228	0.0	0.0	25486.5	+183.18	3	24032.5	373.40	24.831	2
	Reversed .	11	10	0.0	+0.4	1118.3	−194.37	3				
		12	230	−0.2	0.0	25709.8	+187.92	3	24591.5	382.29	24.819	2
		13	9	+0.1	+0.4	1006.6	−195.94	3				
		14	232	0.0	+0.4	25934.1	+191.46	4	24927.5	387.40	24.824	2
		15	8	−0.3	+0.1	894.0	−197.58	4				
		16	233	. .	. .	26045.4	+193.37	3	25151.4	390.95	24.820	2
		17	7	+0.3	+0.6	783.6	−196.50	4				
		18	231	−0.3	−0.4	25821.0	+192.43	2	25037.5	388.93	24.836	2
Aug. 11	Newcomb .	1	4	0.0	+0.4	447.6	−201.90	2				
		2	236	+0.2	+0.3	26381.4	+201.02	2	25933.8	402.92	24.832	1
	Reversed .	3	5	−0.8	−0.6	557.2	−200.22	3				
		4	234	+0.1	+0.3	26157.7	+197.44	2	25600.5	397.66	24.836	1
		3*b*	5	−0.8	−0.6	557.2	−200.22	3				
		4*b*	230	+0.2	+0.4	25710.8	+190.66	4	25153.6	390.88	24.828	2
		5	6	0.0	0.0	670.7	−199.01	3				
		6	231	+0.1	+0.2	25822.2	+191.87	4	25151.5	390.88	24.825	2
		7	17	0.0	+0.4	1900.8	−179.75	4				
		8	233	+0.3	+0.5	26046.4	+195.57	2	24145.6	375.32	24.821	1
		9	8	+0.1	+0.4	894.9	−195.34	4				
		10	229	−0.1	0.0	25598.2	+188.54	4	24703.3	383.88	24.828	2

August 10.—Sets 1, 2: Images slightly distorted. Sets 3, 7: Images all right, but in most of the sets to 9, a slight distortion was seen. After 9, no distortion was remarked. Set 16: Reading of microscopes not recorded.

[Value of 1 div. microscopes = 2''.4. Value of 1 div. of limb = 111''.783.]

Date.	Observer and order of telescopes.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
1882.						''			''		Millionths of second.	
Aug. 25	HOLCOMBE.	1	8	+0.3	0.0	894.6	− 191.80	2				
		2	238	−0.2	−0.2	26603.9	+ 267.67	2	25709.3	399.47	24.829	1
	Direct . . .	3	11	0.0	−0.7	1228.8	− 186.81	1				
		4	249	−0.3	−0.3	27833.2	+ 226.48	2	26604.5	413.29	24.837	1
		5	7	+0.2	0.0	782.7	− 193.95	3				
		6	248	−0.4	−0.3	27721.3	+ 224.72	3	26938.6	418.67	24.825	2
		7	6	−0.4	−0.8	669.3	− 195.68	3				
		8	245	−0.4	−0.6	27385.6	+ 219.47	3	26716.4	415.15	24.828	2
Aug. 29	HOLCOMBE.	1	3	+0.7	+0.8	337.1	− 208.12	2				
		2	242	−0.5	0.0	27050.9	+ 207.03	3	26713.8	415.15	24.826	1
	Reversed .	3	11	+0.2	+0.4	1230.3	− 190.43	3				
		4	230	+0.1	0.0	25710.2	+ 189.92	3	24479.9	380.35	24.830	2
		5	10	0.0	+0.7	1118.7	− 183.48	2				
		6	211	0.0	+0.1	23586.3	+ 165.58	3	22467.6	349.06	24.832	1
		7	1	0.0	+0.5	112.4	− 207.77	3				
		8	246	0.0	0.0	27498.6	+ 217.65	3	27386.2	425.42	24.836	2
		9	2	0.0	+0.4	224.0	− 205.97	3				
		10	247	0.0	0.0	27610.4	+ 219.62	1	27386.4	425.59	24.826	1
		11	3	0.0	+0.3	335.7	− 204.14	2				
		12	248	+0.2	+0.3	27722.8	+ 221.39	3	27387.1	425.53	24.830	1
Aug. 30	HOLCOMBE.	1	4	+0.1	+0.5	447.9	− 201.23	2				
		2	241	+0.2	+0.5	26940.5	+ 210.54	4	26492.7	411.77	24.822	2
	Reversed .	3	5	0.0	−0.1	558.8	− 197.82	3				
		4	247	−0.1	+0.4	27610.8	+ 222.41	4	27052.0	420.23	24.836	2
		5	1	+0.1	−0.3	111.5	− 204.84	.				
		6	239	0.0	−0.4	26715.7	+ 208.64	.	26604.1	413.48	24.823	2

August 29.—After set 2 struck sending telescope, and had to readjust it.

[Value of 1 div. microscopes = 2''.4. Value of 1 div. of limb = 111''.783.]

Date.	Observer and order of telescopes.	No. of set.	Div. of arc used.	Reading of microscopes.		Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
				Right.	Left.							
1882.						''			''		Millionths of second.	
Sept. 1	HOLCOMBE .	1	4	+0.1	+0.1	447.4	−199.00	2				
		2	230	+0.4	+0.4	25711.1	+193.60	4	25263.7	392.60	24.827	1
		3	9	0.0	−0.1	1005.9	−190.21	3				
		4	230	−0.3	−0.4	25709.2	+193.67	3	24703.3	383.88	24.827	2
		5	8	0.0	−0.3	893.9	−191.89	2				
		6	233	−0.3	−0.4	26044.6	+198.94	4	25150.7	390.83	24.828	1
		7	7	+0.4	+0.4	783.4	−193.70	3				
		8	234	0.0	0.0	26157.2	+200.60	3	25373.8	394.30	24.827	2
		9	12	+0.4	+0.4	1342.4	−184.99	3				
		10	238	−0.9	−0.8	26602.3	+207.47	4	25260.0	392.46	24.831	2
		11	9	+0.2	0.0	1006.3	−190.18	3				
		12	239	+0.3	0.0	26716.5	+209.36	4	25210.2	399.54	24.827	2
		13	17	0.0	0.0	1900.3	−176.18	3				
		14	212	−0.4	−0.2	23697.3	+162.48	3	21797.0	338.74	24.826	2
		15	16	0.0	0.0	1788.5	−177.87	1				
		16	233	−0.9	−1.2	26042.9	+198.94	3	24254.4	376.81	24.833	1
		17	8	−0.1	−0.3	893.8	−191.89	3				
		18	240	−1.0	−1.4	26825.0	+211.09	2	25931.3	402.98	24.826	1

September 1.—The order of the telescopes not recorded. Set 15: Very faint.

[Value of 1 div. microscopes = 2″.4. Value of 1 div. of limb = 111″.783.]

Date.	Observer and order of telescopes.	No. of set.	Div. of arc used.	Reading of microscopes. Right.	Reading of microscopes. Left.	Concluded angular reading.	Turns of mirror per second.	Wt.	Difference of angular readings.	Diff. of velocities of mirror.	Time of passage of light.	Wt.
1882.						″			″		Millionths of second.	
Sept. 2	Holcombe.	1	10	+0.7	0.0	1118.7	−189.23	3				
		2	242	0.0	−0.1	27051.4	+213.67	2	25932.7	402.90	24.832	1
		3	8	0.0	−0.2	894.0	−192.66	2				
		4	244	−1.9	−2.0	27270.4	+217.14	3	26376.3	409.80	24.832	1
		5	9	+0.4	+0.1	1006.6	−191.00	3				
		6	247	−0.3	−0.6	27609.3	+222.46	4	26602.7	413.46	24.824	2
		7	6	+0.5	+0.3	671.7	−196.12	4				
		8	254	0.0	0.0	28392.9	+234.45	2	27721.2	430.57	24.839	2
		9	7	+0.1	0.0	782.6	−194.48	3				
		10	255	−1.9	−1.9	28500.1	+236.24	3	27717.5	430.72	24.828	2
		11	2	+0.3	+0.2	224.2	−203.16	3				
		12	254	0.0	0.0	28392.9	+234.63	3	28168.7	437.79	24.824	2
		13	7	0.0	0.0	782.5	−194.44	4				
		14	245	−0.5	−0.8	27385.3	+219.00	4	26602.8	413.44	24.825	2
		15	6	+0.3	0.0	671.1	−195.99	4				
		16	240	+0.4	0.0	26828.4	+210.41	3	26157.3	406.40	24.832	2
		17	7	−0.3	−0.1	782.0	−194.48	4				
		18	222	+0.1	−0.1	24815.8	+179.02	4	24033.8	373.50	24.825	2
Sept. 5	Holcombe.	1	11	+0.2	−0.2	1229.6	−187.38	3				
		2	238	−1.5	−1.6	26600.6	+206.86	3	25371.0	394.24	24.829	2
	Direct . . .	3	8	0.0	0.0	894.3	−192.58	3				
		4	237	−0.5	−0.7	26491.1	+205.20	4	25596.9	397.78	24.827	2
		5	5	0.0	−0.1	559.0	−198.03	4				
		6	240	+0.3	+0.5	26828.9	+210.21	3	26269.8	408.24	24.828	2
		7	6	0.0	−0.1	670.6	−196.33	4				
		8	242	0.0	0.0	27051.5	+213.60	2	26380.9	409.93	24.829	2
		9	5	−0.4	−0.4	558.0	−196.64	3				
		10	241	+0.6	+0.4	26940.9	+213.52	3	26382.9	410.16	24.816	2
		11	3	+0.4	+0.4	336.3	−199.96	3				
		12	236	−0.4	−0.5	26379.7	+204.81	4	26043.4	404.77	24.823	2

September 2.—Order of telescopes not recorded.
September 5.—Set 10: Image broken as by torsional vibration.

CHAPTER VII.

DISCUSSION OF RESULTS.

It was originally intended to determine the zero-point of the scale after every set of observations, if possible, with a view of having a comparison of measures in the two directions of rotation. Although a knowledge of the zero-point would not have changed the final result, since it would have been eliminated in any case, it would have facilitated the detection of slight changes in its position, and would have afforded evidence of any difference in the behavior of the mirror while rotating in the two directions. It was also intended to use a much wider range of velocities than were actually employed. It was not, however, found convenient to introduce either of these features. The zero-point could not be determined with the mirror absolutely at rest, on account of the appreciable breadth of the image of the slit formed on and near the distant mirror. The image was, in fact, wider than the mirror itself. The measures would, therefore, have been doubtful by half the angular diameter of the image, which was often nearly a minute of arc. To get a correct zero the image should be flashed back and forth across the field by a slight movement of the mirror. But it was found impracticable, without more elaborate and special devices, to produce a movement which would be sufficiently slow not to cause a very sensible motion of the image while the light was going and coming, and would repeat the image rapidly enough to admit of a satisfactory pointing upon it. Again, it was found that the higher the velocity of the mirror the more steadily the image could be kept upon the wires, and that the steadiness deteriorated very rapidly when the velocity fell far below 200 turns per second. A repugnance to making uncertain determinations at a low speed when good ones could be made at a high speed prevented any serious attempt to secure a wide range in this respect.

The mean result of each day's work is shown in the following exhibit, and is derived without change from the times already given. The general causes influencing all the observations of each series are so much more important than the individual peculiarities of the several determinations that no comments upon the latter are added. Owing to the evidence of constant differences between the results of the work on different days, the final weights are not proportional to the sums of the separate weights on each day, but approach more nearly to equality.

It may be remarked that the three series should be regarded as quite distinct, and that the pivots of the revolving mirror were examined and reground by the makers before each series, so that the torsional vibration might, *a priori*, be supposed different in each series. It is also supposed that the form of the pivots was more perfect in the last series.

Times of passage of light (in fractions of a mean solar second) as concluded from observations of each day.

I.—TO AND FROM OBSERVATORY.

1880.	June 28.	$0^s.000\ 017.036$	Wt. =	1
	29.	17.031		3
	30.	17.037		3
	July 3.	17.032		2
	9.	17.029		1
	Aug. 9.	17.025		5
	10.	17.029		0
	13.	17.026		5
	16.	17.025		5
	17.	17.027		5
	21.	17.029		3
	24.	17.026		5
	25.	17.026		5
	Sept. 3.	17.026		5
	4.	17.027		7
	10.	17.027		7
	11.	17.032		7
	13.	17.036		3
	15.	17.030		4
	17.	17.028		3
	18.	17.029		6
	20.	17.029		5
1881.	Mar. 25	17.026		2
	28.	17.026		2
	Apr. 7.	17.028		4
	15.	17.027		6
Mean - -		0.000 017.0282		

II.—TO AND FROM MONUMENT IN 1881.

1881.	Aug. 8.	$0^s.000\ 024.836$	Wt. =	4
	10.	24.832		4
	Sept. 12.	24.831		1
	13.	24.837		3
	19.	24.831		4
	24.	024.840		2
Mean for 1881 - -		0 000 024.8344		

III.—TO AND FROM MONUMENT AFTER CHANGE OF APPARATUS TO ELIMINATE TORSIONAL VIBRATION.

1882.	July	24	$0^s.000\ 024.828$;	Pos. R.;	Wt. = 4
		26	24.828	R.	3
		31	24.819	D.	2
	Aug.	9	24.822	R.	2
		10	24.828	D.	5
		10	24.825	R.	5
		11	24.828	R.	6
		25	24.829	D.	4
		29	24.831	R.	6
		30	24.827	R.	4
	Sept.	1	24.828	(?)	8
		2	24.829	(?)	9
		5	24.826	D.	6

Mean of 1882.	Direct pos.,	24.8265	Wt. = 17
	Reversed pos.,	24.8276	30
	Not recorded,	24.8285	17
	Both pos.,	24.8275	64

The differences between the two classes of results is too small for taking account of. The general mean is therefore adopted. Combining the separate means with the distances traveled, we have the following results for the velocity of light in air expressed in kilometers per second:

Observatory, 1880–81.	Dist. = 5101.90 m.;	Time = .000 017 0282 s.;	V = 299615
Monument, 1881.	" 7442.42	" .000 024 8344	V = 299682
Monument, 1882.	" 7442.42	" .000 024 8275	V = 299766

The differences of these results far exceed the probable errors arising from the accidental differences between the separate daily means. Perhaps the strongest way of showing this is to give the greatest and least daily results of each series of observations. They are as follows:

Observatory, 1880.	Least	299460
	Greatest	299671
Monument, 1881.	Least	299662
	Greatest	299723
Monument, 1882.	Least	299736
	Greatest	299878

In order to obtain a definitive result, it is necessary to investigate the possible causes of these systematic discrepancies. Let us then enumerate the hypotheses which lie at the basis of the result, and which may possibly need modification. They are:

I. That the motion of the revolving mirror is uniform in running.

II. That the figure of the mirror remains invariable.

If both these conditions are fulfilled, the angular motion of every part of the mirror during the interval between the two reflections will be correctly given by multiplying the velocity of the mirror, deduced from the chronographic record, by the elapsed time.

III. That the angle of reflection is always equal to the angle of incidence.

IV. That the changes in the direction of the ray thus reflected are correctly measured by the angular motion of the receiving telescope around the axis of the revolving mirror.

We shall consider these hypotheses in order, omitting, however, the third, which does not seem to need discussion. Notwithstanding the rapid motion of the mirror, that motion is too small between the impacts of two successive waves of light to suppose it capable of affecting the angle. Even if there were any such effect it would change all the measures in the same degree, and would therefore not explain discrepancies among them.

First hypothesis: Uniform revolution of the mirror around its axis.—The air-blast moves the mirror by striking twelve fan-wings. The pressure upon the mirror should therefore be considered as not constant, but as subject to a periodic inequality repeating itself twelve times in each revolution. From the general action and apparent continuity of the air-blast, I do not think this inequality to be sensible; but to make the case as extreme as possible, let us suppose the mirror to have been moved by a series of impacts, 12 in each revolution. Although, after the air-blast was removed, the mirror would generally run two minutes before coming to rest, I think it would have come to rest in 30 seconds were the resistance as strong as during the motion. Its rate of motion averaging something over 200 turns per second, the resistance of the air and other obstacles to its motion would have brought it to rest in about 3,000 turns. Therefore, when going at this average speed, the resistance would suffice to make it lose about one 3,000th part of its speed during each revolution, and therefore about $\frac{1}{36,000}$ part during the interval between two impacts. This seems to me the extreme limit of the error that could have been introduced from this cause. The corresponding maximum change in the velocity of light deduced on the hypothesis of uniform motion would be less than four kilometers, and need not be further considered.

Second hypothesis: Invariability of form of the mirror.—Let us recapitulate the process by which the velocity is measured, in order to see how it would be affected by a change in the figure of the mirror. The latter is a square prism, of which the height is double the breadth of each face: the outgoing ray strikes one half of each face and the returning ray is reflected from the other half. Let us designate that half of the face which the light strikes first as face A, and the half which receives and reflects it on its return as face B. Now, considering the distant reflector as a point, which we may do, because its angular magnitude does not affect the result, the face A of the mirror reflects light to this point only when in a certain definite position. We may define this position by the angle which the normal to face A makes with some arbitrary fixed line. Let us put—

ω, this angle of position of face A, at the moment of reflecting the light.

When the light returns it is reflected by face B. Let us put—

ω', the angular position of face B, when the return ray is reflected. There will be two values of the angle ω', the one corresponding to negative and the other to positive rotation of the mirror. The angle ω has the same value in the two directions of rotation. And the fundamental hypothesis is that the difference between the two values of ω' is equal to the rotation of the mirror during twice the period required for the light to go and come. This hypothesis is justified by the following considerations: Suppose the two faces A and B to differ by a minute constant, β; and suppose, also, that the motion of light were absolutely instantaneous; the value of ω' would be equal to $\omega + \beta$ in either direction of rotation. But if v and v' be the velocities, and τ the time of passage of light, then, when the light strikes the mirror on its return, the respective value of ω and ω' corresponding to positive and negative rotations will be—

$$\omega + \beta + v\tau \text{ and } \omega + \beta - v'\tau$$

The difference of these is $(v + v')\ \tau$ as stated.

But this result presupposes that the angle β between the faces A and B is invariable. Now it is possible that β, which represents the twist of the mirror, may have a separate value for the two rotations arising from torsion, and may also be subject to a periodic inequality.

It follows that all observations made by having the sending telescope throw the light upon one invariable part of the mirror, and the receiving telescope receive it from another invariable part, may be affected by systematic errors arising from torsion and torsional vibration of the mirror. That torsion of the mirror produces such a systematic error was foreseen before commencing the construction of the apparatus, and to avoid this effect the fan-wheels were attached both to the top and bottom of the mirror, in order that there might be no torsional force during its motion. But the possibility of torsional vibration of the mirror was not foreseen until its effect was noticed during the observations of 1881, by the formation of distinct images from the different faces appreciably separated in the direction of motion. That this result could have no other cause than such a vibration seemed to me quite clear. I therefore assumed it to be due to this cause. Between the seasons of 1881 and 1882, changes were made in the apparatus by which the sending telescope could be lowered, so as to throw the light upon the lower part of the face of the mirror and the receiving telescope raised, so as to take it from the upper part of the face. It will now be shown that the effect of any probable torsional vibration is thus completely eliminated from the mean of two complete series of observations.

The only hypothesis that I shall make respecting this vibration is that it is uniform throughout the whole length of the mirror; that is, that it may be represented by a uniform twist of the entire mirror, this twist not remaining constant, but varying harmonically through a regular period. With reference to this hypothesis it may be remarked that no such vibration would produce an evil effect unless its period were an aliquot part of the time of rotation. Now, while it seems to me possible that the figure of the mirror might, during its revolution, be subject to minute changes of

almost any sort, it does not seem probable that these changes would have a period of the kind above mentioned except they were uniform from top to bottom, as supposed in the hypothesis. In accordance, then, with the hypothesis we shall assume that whenever one half of the face is thrown forward by the effect of vibration the other half will be thrown backward by an equal amount. Introducing only this hypothesis, and making no supposition whatever respecting any other relation between the different parts of the face of the mirror, it follows that the respective angles of position ω and ω' of the two halves of the face when the mirror is in rotation, may be represented by equations of the following form:

Positive Rotation.

$$\begin{aligned} &\text{Part A; } \omega = a + bt + h \sin(C + nt) + k \cos(C + nt) \\ &\text{Part B; } \omega' = a + bt - h \sin(C + nt) - k \cos(C + nt) + \beta \end{aligned} \tag{1}$$

Negative Rotation.

$$\begin{aligned} &\text{Part A; } \omega = a' - b't + h' \sin(H + mt) + k' \cos(H + mt) \\ &\text{Part B; } \omega' = a' - b't - h' \sin(H + mt) - k' \cos(H + mt) + \beta' \end{aligned} \tag{2}$$

Here a and a' represent the angles of position of part A of the face counted from an arbitrary direction at an arbitrary zero of time. b and b' are the mean speeds of rotation, which, in accordance with what has already been shown, we consider as constant in each case.

β and β' represent the twist of the face during rotation, defined as the angle between A and B, which may or may not be the same as when the mirror is at rest.

The terms in h and k are the periodic terms, in which we take for the epoch an entirely arbitrary moment. The only supposition we make is the one already stated, that the two halves of the face are equally and oppositely affected by it.

Since it is conceivable that the form of the mirror may be permanently different in the negative direction of rotation, we assign an independent twist β' to it, and entirely independent periodic terms.

What we shall now do is to find the mean result when the ray is received upon part A of the face, and when received on part B of the face, the velocities of rotation being supposed to be the same. Then in what we shall designate as the *direct* position of the two telescopes, the sending telescope will throw its light upon the half A of the face of the mirror. As already shown, at the moment when the flash is sent to the distant mirror, this face must have a definite angle of position, ω_0. Counting the time τ from the epoch of reflection, we shall have the equation—

$$\omega_0 = a + h \sin C + k \cos C = a' + h' \sin H + k' \cos H \tag{3}$$

The time of passage being τ, the position angles ω' will be those measured by the receiving telescope. We shall put—

ω'_1, the angle at which the return ray was received during positive rotation.

ω'_2, the same angle in the case of negative rotation.

We shall then have—

$$\omega_1' = a + \beta + b\tau - h \sin(C + n\tau) - k \cos(C + n\tau)$$
$$\omega_2' = a' + \beta' - b'\tau - h' \sin(H + m\tau) - k' \cos(H + m\tau)$$

and for the difference of the two angles—

$$\omega_1' - \omega_2' = a - a' + \beta - \beta' + (b + b')\,\tau - h \sin C' - k \cos C' + h' \sin H' + k' \cos H' \qquad (4)$$

where we have put for brevity—

$$C' = C + n\tau; \quad H' = H + m\tau$$

It appears that in this case, which was that of all the measures made during 1880 and 1881, the measured angle $\omega_1' - \omega_2'$ depends not merely upon τ and the speeds of rotation, but also upon the periodic terms in the vibration. But in 1882 the instrument was so arranged that the sending telescope could throw its light upon the part B of the face, and the receiving telescope take it from A. Counting the time as before, from the moment when the mirror was in position to throw the light towards the distant mirror, we have, when the telescopes are interchanged—

$$\omega_0' = a + \beta - h \sin C - k \cos C = a' + \beta' - h' \sin H - k' \cos H \qquad (5)$$

then for the angles at which the light was received in positive and negative rotation respectively—

$$\omega_1 = a + b\tau + h \sin C' + k \cos C'$$
$$\omega_2 = a' - b'\tau + h' \sin H' + k' \cos H'$$

The difference of these results gives

$$\omega_1 - \omega_2 = a - a' + (b + b')\,\tau + h \sin C' + k \cos C' - h' \sin H' - k' \cos H'$$

The sum of this equation and that obtained in the direct position of the two telescopes (4) is

$$\omega_1' - \omega_2' + \omega_1 - \omega_2 = 2a - 2a' + \beta - \beta' + 2\,(b + b')\,\tau \qquad (6)$$

the equations (3) and (5) may be put into the respective forms

$$0 = a - a' + h \sin C + k \cos C - h' \sin H - k' \cos H$$
$$0 = a - a' - h \sin C - k \cos C + h' \sin H + k' \cos H + \beta - \beta'$$

the sum of which gives

$$0 = 2a - 2a' + \beta - \beta'$$

the sum (6) of the deviations thus becomes

$$\omega_1' - \omega_2' + \omega_1 - \omega_2 = 2\,(b + b')\,\tau \qquad (7)$$

which is independent of all torsional vibration and of differences between the twists of the mirror.

In the table of results for 1882, already given, the separate results in the direct and reverse positions of the telescope are shown. It will be seen that there is no systematic difference in the two cases. We conclude, therefore, that at this time it

chanced that the effect of the torsional vibration was insensible. There is, therefore, no occasion for making any distinction between the two sets of results, and the general mean is taken.

We have now to consider all possible causes which might have resulted in the angular deviation of the receiving telescope not correctly representing twice the motion of the reflecting surface. If the face of the mirror were a perfect plane and the receiving telescope perfectly adjusted to focus, it does not appear that the measured deviation could be anything different from the double motion of the reflecting surface. Let us, therefore, consider the possible result in case the telescope is not accurately focused and in case the face of the mirror is not a perfect plane.

In the figure (5) let C be the common center of motion of the mirror and of the receiving telescope; MA a section of the face of the mirror in the position in which it receives a ray of light, MO, from the distant mirror while rotating positively;

$M'A'$, the position of the same face when it receives the return ray while rotating negatively;

α, the angle between these two positions of the face of the mirror.

L, the position of the objective of the receiving telescope when the light reflected from the center of the distant mirror would be seen on the cross wires at the focus F;

L', the position of the same lens when the receiving telescope is moved to the second position through the angle 2α.

M' R' the ray reflected from the point M (now M'), when the mirror is in negative motion. The distant reflector being several kilometers distant, while the distance $M'M'$ is only a fraction of a millimeter, we may regard the lines M O and M' O' as parallel.

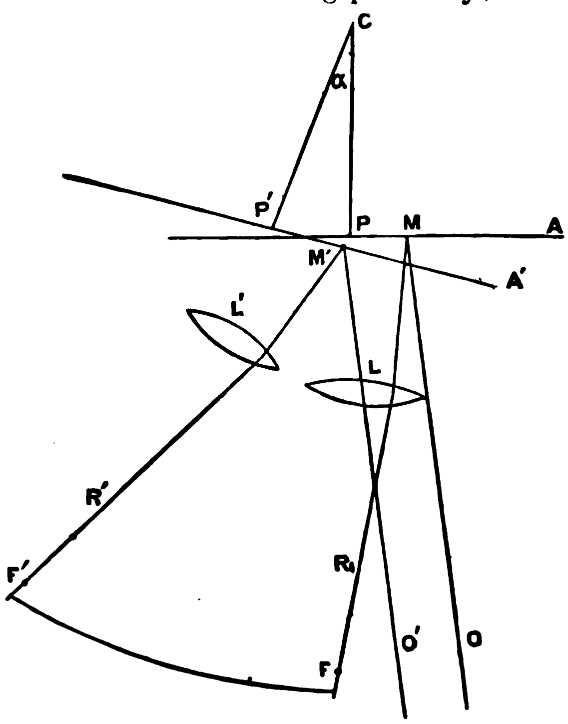


FIG. 5.

Then assuming the angles of incidence and reflection to be equal, it follows that the angle between the reflected rays ML and $M'L'$ will be rigorously equal to 2α. By hypothesis, we have moved the receiving telescope through the angle 2α, and so the question reduces itself to the inquiry whether the reflected ray will be received upon the cross wire at F' exactly as the ray L F was so received. In order that this may be the case it is necessary and sufficient that F shall be in the astronomical focus of the telescope, or, to be more precise, that it shall be in the conjugate focus of the distant mirror. We therefore reach the conclusion:

If the cross wires be set in the astronomical focus of the receiving telescope, and if we compare rays striking the same point of the reflector in the two directions of motion, then the motion of the image will rigorously represent twice the motion of the revolving mirror.

Since this will be true for every point of the face M A, taken separately, it will

be true for the whole reflected image, provided that every reflected ray is received by the objective; in other words, provided that that portion of the face which reflects the rays is smaller than the objective of the telescope by an amount equal to the displacement of any point of the mirror relative to the objective.

We cannot, however, regard this condition as absolutely fulfilled, because owing to the displacement just referred to, some minute portions of the mirror may be thrown out of the line of sight from the objective at the right-hand edge in the case of left-handed rotation, and vice versa. If, however, we suppose the focus to be so adjusted that all the rays reflected from the revolving mirror are brought to the same foci, F and F', there will be no change in the position of the image arising from new portions of the reflected surface entering into the line of rays and other portions going out. If the mirror is perfectly plain the focus thus found will be the true astronomical focus, and since, in actual observations, the instrument was set as near as possible at this combined focus, we conclude that if the face of the revolving mirror were perfectly plane the movement of the receiving telescope would accurately measure twice the distance of the mirror. But, as a matter of fact, the faces were slightly convex. To inquire into the amount of error thus produced let us return to the figure. Let us put

$90^\circ - \beta$ for the angle AMO by which the ray was reflected from the face of the mirror in positive rotation. The angle β was then not far from 12°. The maximum value of the angle α was about 4°.

If we put r for the radius CP of the revolving mirror, and ρ for the distance CL, we shall have with sufficient approximation

$$MM' = r \sin \alpha$$

and the length of this line projected upon the line LL' will be approximately

$$r \sin \alpha \cos 14^\circ.$$

For our present purpose we may neglect the difference between the angles α and 2α and their sines and may suppose the cosine and secant of 14° unity. Since the rays ML and $M'L'$ diverge by the angle 2α, the distance between the points L and L', at which they meet the lens will be, with sufficient approximation

$$r \sin \alpha + (\rho - r) \sin 2\alpha$$

while the actual motion of the lens will be $\rho \sin 2\alpha$, for which we may put $2\rho\alpha$.

The displacement of the ray relative to the objective will then be, approximately

$$2\,\rho\alpha - 2\,(\rho - r)\,\alpha - r\alpha = \alpha r$$

We conclude that since $r = 18^{mm}$, corresponding rays entered the receiving telescope at points different by one millimeter in opposite directions of rotation.

If the micrometer wire on which the image is received be pushed in by a quantity whose ratio to the focal length FL of the receiving telescope is k, the difference between the linear displacements of the image from the wire arising from the displacement αr of the ray will be $\alpha k r$. The angular displacement will be equal to this amount divided by the focal length of the telescope; that is, we shall have

$$\text{Error of measured angle} = \alpha k \frac{r}{F}$$

F being the focal length of the telescope.

The ratio $r:F$ is approximately 0.016; whence we have

$$\text{Error of measured angle} = .016\ \alpha k$$

The actual velocity of light, as determined by the measures, is proportional to 2α. It follows that the velocity of light, as determined from the apparatus, will be multiplied by the factor 1.0080 k.

It is evident from the figure that the algebraic sign of the symbol k is such that when the focus actually used is longer than the astronomical focus the observed angle will be too great. Hence, the velocity of light will be too small and will require a positive correction.

A comparison of the focal points of the receiving telescope for rays received by reflection from the different faces of the mirror with the focus for direct vision, showed that one face was sensibly plane, while the other three were convex to a sensibly equal degree, each lengthening the focus about 15 millimeters. As the focus was set by trial, so as to get the best image in making the actual observations, 11 millimeters may be taken as the mean amount by which the focus was lengthened. The actual focal length of the receiving telescope was 235 centimeters. The value of the coefficient k in the preceding formula is therefore .005, and the factor by which the velocity is to be increased is 0.000040, or 12 kilometers per second.

Reduction to a vacuum.—The index of refraction of air at 0° C. and 760^{mm} pressure is supposed to be 1.000294. The measures were actually made at a mean temperature not far from 25° C., and a mean pressure not far from 767^{mm}. The corresponding factor to reduce the density of the air is 0.93, whence the corresponding index of refraction is 1.000273.

The suggestion of Professor Lord RAYLEIGH that the group velocity of waves of light in air, which is the quantity actually measured, may be less than the true wave velocity, has received a certain amount of confirmation from the researches of Professor MICHELSON on the velocity of light in bisulphide of carbon. This result shows that the actual reduction to a vacuum should be greater than that deduced from the index of refraction. But MICHELSON'S measures show the correction to be too small to be taken into account. We therefore adopt 1.000273 as the factor of correction.

Concluded velocity in vacuo.—The preceding investigations and discussions seem to show that our results should depend entirely on the measures of 1882. Adopting this course we shall have

Immediate result of measurement - - - -	V = 299766 k. m.
Correction for curvature of mirror - - - - -	+ 12
Reduction to vacuo - - - - - - - - - - - -	82
Concluded velocity in vacuo - - - - - - -	299860

If we estimated the probable error of this result from the discordance of the separate measures, it would be less than 10 kilometers. But we can have no ground for assigning any definite numerical value to the probable error, owing to the possibility of constant errors. Indeed, judges may not be wanting to maintain that the results of all three series of observations should have been taken into account, on the

ground that we cannot be sure of having eliminated all systematic errors from any of them. On this hypothesis we might fairly assign the respective weights 2, 3, and 6 to the three series. This would give:

Velocity in air - - - - - - - - - - - - - -	299728
Velocity in vacuo - - - - - - - - - - - - -	299810

The probable error might then be estimated at 40 or 50 kilometers.

Comparison of results.—As a matter of convenience, the velocities of light in vacuo, which have been published in these papers, are here collected. Professor MICHELSON'S result at the Naval Academy was given in Vol. I, p. 141, as 299944. But he reports (see post) that two errors were committed, which together made the result too great by 34 kilometers, and that it should have been 299910. We therefore have:

MICHELSON, at Naval Academy, in 1879 - - - -	299910
MICHELSON, at Cleveland, 1882. (See *Post*) - - -	299853
NEWCOMB, at Washington, 1882, using only results supposed to be nearly free from constant errors -	299860
NEWCOMB, including all determinations - - - -	299810

To these may be added, for reference—

FOUCAULT, at Paris, in 1862 - - - - - - - -	298000
CORNU, at Paris, in 1874 - - - - - - - - -	298500
CORNU, at Paris, second determination, made in 1878 -	300400
This last result, as discussed by LISTING, - - - -	299990
YOUNG and FORBES, 1880–'81 - - - - - - - -	301382

Making a liberal allowance for probable error, I think we may conclude as the most probable result—

Velocity of light in vacuo = 299860 ± 30 k. m.

The solar parallax.—According to BESSEL (*Tabulæ Regiomontana*, pp. xviii—xx), if we put—

V, the velocity of light;
$\varkappa$, the constant of aberration;
n, the earth's mean motion;
e, the eccentricity of its orbit;

then the time required for light to traverse the semi-major axis of the earth's orbit will be

$$\varkappa \frac{\sqrt{1-e^2}}{n}$$

and the mean distance of the sun will be

$$\varkappa V \frac{\sqrt{1-e^2}}{n}$$

Assuming the velocity of light to be 299860 km., and the earth's equatorial radius to be 6378.2 km. (CLARK), the following table shows the solar parallax and distance corresponding to different values of the constant of aberration:

κ	Dist. in millions of kilometers.	Solar parallax.
$''$		$''$
20.42	149.08	8.825
.44	149.23	8.816
.46	149.37	8.808
.48	149.52	8.799
.50	149.66	8.790
.52	149.81	8.782
.54	149.96	8.773

NYRÉN'S value of the constant of aberration from Pulkowa observations is 20''.492. To this corresponds—

$$\Pi = 8''.794$$

Is there any difference between the velocities of rays of different colors?—In making the experiments no special arrangements were made for detecting differences between the velocities of differently colored rays, for the reason that the phenomena of variable stars seem to be conclusive against the hypothesis of any such difference, a conclusion pointed out by Arago many years ago. Were there a difference of one hour in the times of the blue and red rays reaching us from Algol, this star would show a well-marked coloration in its phases of increase and decrease. No trace of coloration having ever been noticed, the difference of times cannot exceed a fraction of an hour. It is not at all probable that the parallax of this star amounts to one-tenth of a second, so that its distance, probably, exceeds two million radii of the earth's orbit, and the time which is required for its light to reach us probably exceeds 30 years, or a quarter of a million of hours. It is therefore difficult to see how there can be a difference as great as four parts in a million between the velocities of light coming from near the two ends of the bright part of the spectrum. While the experiments were in progress, however, it was announced that Messrs. FORBES and YOUNG had detected a difference of two per cent. This announcement led to occasional careful examination of the return image of the slit formed by reflection from the revolving mirror. Had there been a difference of one-thousandth, the resulting spectrum would have been 15'' in breadth, and the image would, therefore, have shown a well-marked iridescence on its edges. No trace of such iridescence could ever be seen.

Chapter VIII.

SUGGESTIONS RESPECTING IMPROVEMENTS IN THE METHOD.

The writer is of opinion that the velocity of light admits of being measured with sufficient precision to serve as a test of the invariability of our standards of length, although he does not feel qualified to judge whether this constant or the wave length of a special ray would be best adapted for this special purpose. A summary of the considerations which should guide the physicist in making a more precise determination are therefore given. They are based upon, and are principally a summary of, the considerations given at length in Chapter II, with additions and modifications suggested by experience with the apparatus.

I. As between the methods of the revolving tooth-wheel and the revolving mirror there can, in the writer's mind, be no question that the latter is alone to be considered. The fundamental principle of the Arago-Foucault method is that the time of passage of light is determined by the motion of a revolving reflecting surface while a ray of light is going to a distant reflector and returning. We assume that the velocity must be determined by an application of this principle.

II. A fundamental question in the arrangement of the apparatus is whether the lens of the receiving telescope should be between the eye-piece and the revolving mirror or between the revolving mirror and the distant mirror. The advantage of the latter arrangement is a much greater quantity of light, and therefore, it may be supposed, a greater distance of the fixed mirror. The disadvantage is that, with an eye-piece of given focal length, held at a fixed distance from the axis of revolution of the mirror, the image will be injured in the ratio of the whole focal length of the lens to the distance between the mirror and the eye-piece.

III. Should the return ray be received upon and reflected from the same point of the revolving mirror from which it was sent? An undoubted advantage is gained by doing so in lessening the danger of systematic errors arising from torsional vibration. At the same time it should be remembered that this vibration might take place between the mirror and the turbine or other attachment by which it is run, as well as between different parts of the face of the mirror itself. That is, if the axis on which the mirror turns has attached to it, either above or below the mirror, any apparatus whose moment of inertia is appreciable in comparison with that of the mirror, there must always be a possibility of such vibration. It is therefore extremely desirable that in any case the mirror itself should be many times heavier than any other object carried on its axis. In attempting to use a great distance we also meet with the difficulty that an ordinarily polished surface always reflects a considerable amount of diffused light at small angles around the mean deflected ray, owing to the existence of minute scratches produced by the polishing tool This difficulty may be probably avoided

by the use of some form of silvering which does not require the application of the polisher.

IV. A very desirable, if not essential condition is that the angle measured should be that between the two directions of the reflected ray in opposite directions of rotation of the mirror. The advantage of this is that we not only double the angle to be measured, but eliminate any possible error in the determination of the zero-point.

We have now to consider whether the two conditions of measures in opposite directions, and the reflection of both the outgoing and returning rays from the whole face of the mirror can be combined. Two difficulties are met in the combination. One is that with small velocities of rotation the return ray will be reflected from the mirror nearly on its original path, so that the slit sending the light, and the spider line receiving it, will be side by side. For the same reason, with slow velocities of rotation, it will be necessary to use the same objective for sending and receiving, which would require us to adopt the form of apparatus in which the revolving mirror is between the objective and the focus at which the observation is made. It is true, that if high velocities of rotation be employed all difficulty from this source can be avoided. If it is necessary to use the system in which the telescopes look into the revolving mirror, it would be advisable to place the sending telescope so far from the mirror that the eye-piece of the receiving telescope shall pass in front of the objective of the sending telescope. It would then be impossible to observe with low velocities, and all observations would have to be made with deflections of the returning ray sufficiently great to throw the object end of the receiving telescope entirely out of the pencil of rays from the objective of the sending telescope. This would be so troublesome that we may consider it absolutely necessary to abandon the use of sending and receiving telescopes, and to adopt MICHELSON's method of placing the mirror between the objective and its focus, thus using but a single objective for both the outgoing and returning ray.

We then meet with the difficulty, already pointed out, that a perfectly dark field cannot be obtained. To see any image at all, with the mirror in motion, it is necessary to incline either the mirror or the telescope; but with minute inclinations it would still be difficult to avoid a considerable amount of reflected light. In order that the deviation of the reflected ray may correctly measure the motion of the mirror between the two reflections, it is necessary that the measurement of the deviation be made in a plane at right angles to the axis of rotation. But, if the ray is sent out in this plane, the ray impinging on the mirror will be flashed into the observer's field of view by direct reflection from the revolving mirror once in each revolution. To avoid this, it is necessary to incline the mirror, and thus lose the advantages in accuarcy which arise from a vertical axis of rotation and a horizontal plane of measurement. There is, however, a system so well fitted to avoid all these difficulties that I regard it as superior to any yet proposed or employed. It consists in employing a prism for the revolving mirror, as in the preceding determinations, but receiving each returning flash on a face adjoining that from which it was reflected. The best form of prism for this purpose would be one of pentagonal section; the arrangement of the apparatus, which is very simple, is shown in Fig. 6.

M is a section of the revolving mirror, *J* is the objective of the fixed sending telescope, which receives light from the slit *S*, and throws it in the direction *P* to the distant reflector. The ray is reflected back on the path *P'*, and then reflected from another face of the revolving mirror into the receiving telescope *K*. The most advantageous size for the face of the revolving mirror would, probably, not be far from 40 millimeters in height and 50 in breadth. It might be impracticable to make the angles of the prism accurately equal. The result of inequalities would be, that, instead of the five images formed by reflection from the successive faces being superposed, they would be separated horizontally by different small intervals. This, however, need cause no serious difficulty. The images would be arranged in the same way in opposite directions of rotation, and all that would be necessary to avoid systematic error would be that the pointings on the images should be made in the same way in the two directions of rotation

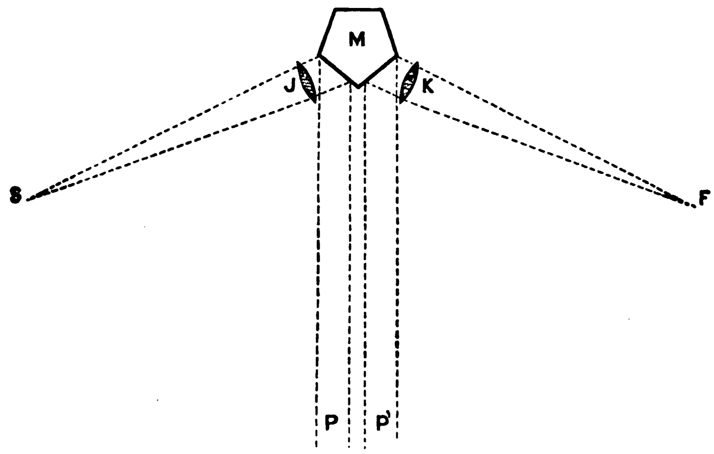


FIG. 6.

A still further perfection of the method, which would lead to a result of which the precision would be limited only by our means of linear measurement is, I conceive, within the power of art. It consists in placing the fixed mirror at so great a distance that the pentagonal revolving mirror could move through an arc of nearly 36 degrees while the ray is going and returning. If a speed of 500 turns per second could be attained the required distance would be 30 kilometers. Then, in opposite directions of rotation, the return ray would be reflected at phases of the mirror differing by the angle between two consecutive faces. The result would be that the receiving telescope would need to have but a small motion, and all the observer would have to measure would be the small angle by which the difference of postions of the mirror when the flash was received in opposite directions of rotation differed from 72°.

In the Rocky Mountains or the Sierra Nevada no difficulty would be found in finding stations at which a return ray could be received from a distance of 30, 40, or even 50 kilometers, with little more dispersion and loss than at a distance of 4 kilometers through the air of less favored regions. It is true that the surface of the distant reflector would have to be increased in proportion to the distance, but it would not be necessary to make a single reflector of great size. A row of ten reflectors, each 6 or 8 decimeters in diameter, might be sufficient to insure the visibility of the return ray.

APPENDIX I.

TRIANGULATION OF THE STATIONS.

The following papers give the details of the triangulation and computation of results as made in the Coast Survey Office:

U. S. Coast and Geodetic Survey Office,
Washington, April 29, 1881.

Dear Sir: In compliance with request contained in your letter of March 24, I forward herewith a report from Assistant C. A. Schott, Chief of the Computing Division of this Office, giving an account of the triangulation and summary of the computations, in connection with the experiments for determining the velocity of light.

Yours respectfully,

C. P. PATTERSON,
Superintendent.

Prof. Simon Newcomb,
Superintendent of the Nautical Almanac, Navy Department.

Computing Division, Coast and Geodetic Survey,
April 29, 1881.

A base-line was measured on Analostan Island, D. C., by C. H. Sinclair, Subassistant Coast and Geodetic Survey, in February, 1881, for the purpose of a trigonometrical determination of the distance between the revolving prism at Fort Myer, Va., and the reflector at the U. S. Naval Observatory, D. C., this distance being required in the observations, for the determination of the velocity of light conducted by Prof. S. Newcomb.

The base is nearly a quarter of a mile in length, and was measured twice by means of two steel contact slide-rods, agate capped, and each 4 meters long. By comparison with a 4-meter standard rod, No. 9 was found = 4 meters at 36°.66 Fahr., and rod No. 10, of this length, at 42°.55 Fahr., the coefficient of expansion being 0.0000061. The length of the base corrected for inclination and for change of temperature of rods was found as follows:

By first measure, 374.6432; by second measure, 374.6394; mean, 374.6413 ± 0.0037.

(The reduction to the sea level is insignificant, and not needed for this especial purpose.)

The horizontal angles between the stations, as shown on the accompanying sketch of triangles, were measured by Mr. Sinclair by means of a 15 c. m. (6-inch) Gambey theodolite (No. 56) with the following resulting directions at each station.

	Observed directions.	Corrected directions.
NORTH BASE.	° ′ ″	″
Naval Observatory .	0 00 00.0	[59.8]
South base	61 18 20.8	21.7
Canal	85 54 03.6	04.3
Graveyard	109 04 51.6	50.1
NAVAL OBSERVATORY.		
Monument	0 00 00.0	[59.1]
Canal	117 40 19.2	20.5
Fort Myer	136 59 09.4	07.4
South base	148 45 59.6	59.8
North base	170 39 05.5	06.0
GRAVEYARD.		
North base	0 00 00.0	00.9
South base	25 23 42.3	41.9
Canal	120 12 30.1	32.3
Fort Myer	214 35 53.0	50.3
SOUTH BASE.		
North base	0 00 00.0	[58.5]
Naval Observatory .	96 48 30.3	30.3
SOUTH BASE—Cont'd.	° ′ ″	″
Monument	116 23 54.8	55.8
Canal	214 44 39.9	39.8
Graveyard	253 10 06.7	08.9
Fort Myer	258 47 35.6	36.0
CANAL.		
Fort Myer	0 00 00.0	02.7
Graveyard	58 08 22.2	19.9
North base	94 45 01.0	02.7
South base	104 54 03.5	01.5
Naval Observatory .	135 52 12.9	12.8
Monument	166 27 56.9	56.0
FORT MYER.		
South base	0 00 00.0	00.2
Graveyard	3 34 37.0	40.4
Naval Observatory .	6 14 03.3	02.1
Monument	21 49 14.3	15.6
Canal	31 03 07.6	05.1

These angles give rise to the following 12 conditional equations between the points forming the figure of the triangulation in order to satisfy the geometrical requirements. The equations 1 to 8 are so-called "angle equations," the numerical term being expressed in seconds. Equations 9 to 12 are so-called "side-equations," with their numerical terms expressed in units of the 5th place of decimals in the logarithms. The numbers in parenthesis indicate corrections to directions; thus $\binom{5}{2}$ is the correction at (2 or) South base to (5 or) Graveyard. Every angle is thus composed of two corrected directions.

(1) $0 = +6.4 - \binom{5}{2} + \binom{1}{2} - \binom{2}{1} + \binom{5}{1} - \binom{1}{5} + \binom{2}{5}$

(2) $0 = -4.1 - \binom{4}{2} + \binom{5}{2} - \binom{2}{5} + \binom{4}{5} - \binom{5}{4} + \binom{2}{4}$

(3) $0 = -3.1 - \binom{5}{4} + \binom{1}{4} - \binom{4}{1} + \binom{5}{1} - \binom{1}{5} + \binom{4}{5}$

(4) $0 = -3.0 - \binom{3}{1} + \binom{2}{1} - \binom{1}{2} + \binom{3}{2} - \binom{2}{3} + \binom{1}{3}$

(5) $0 = -0.6 - \binom{3}{2} + \binom{4}{2} - \binom{2}{4} + \binom{3}{4} - \binom{4}{3} + \binom{2}{3}$

(6) $0 = +6.8 - \binom{6}{4} + \binom{2}{4} - \binom{4}{2} + \binom{6}{2} - \binom{2}{6} + \binom{4}{6}$

(7) $0 = +15.7 - \binom{6}{4} + \binom{5}{4} - \binom{4}{5} + \binom{6}{5} - \binom{5}{6} + \binom{4}{6}$

(8) $0 = +7.4 - \binom{6}{4} + \binom{3}{4} - \binom{4}{3} + \binom{6}{3} - \binom{3}{6} + \binom{4}{6}$

(9) $0 = +5.5 + .58\binom{2}{1} - .12\binom{3}{1} - .46\binom{4}{1} - .52\binom{1}{3} + .87\binom{2}{3} - .35\binom{4}{3}$
$- 1.18\binom{1}{4} + 1.53\binom{2}{4} - .35\binom{3}{4}$

(10) $0 = +1.1 + .31\binom{2}{1} - .12\binom{3}{1} - .19\binom{5}{1} - .52\binom{1}{3} + .87\binom{2}{3} - .35\binom{4}{3}$
$+ .55\binom{2}{4} - .35\binom{3}{4} - .20\binom{5}{4} - .44\binom{1}{5} + .42\binom{2}{5} + .02\binom{4}{5}$

(11) $0 = +4.9 - .66\binom{2}{3} - .35\binom{4}{3} + 1.01\binom{6}{3} + .29\binom{2}{4} - .35\binom{3}{4} + .06\binom{6}{4}$
$- 1.58\binom{2}{6} + 1.93\binom{3}{6} - .35\binom{4}{6}$

(12) $0 = -14.5 - .27\binom{4}{2} + 2.41\binom{5}{2} - 2.14\binom{6}{2} - .20\binom{2}{4} + .33\binom{5}{4} - .13\binom{6}{4}$
$- 3.37\binom{2}{6} + 3.77\binom{5}{6} - .40\binom{4}{6}$

From these equations and the ordinary treatment by the method of least squares the corrections to the angles were obtained.

North Base to South Base						m. 374.6413	2.573616
	°	′	″	″	″		
Graveyard	25	23	42.3	− 1.3	41.0		0.367693
North Base	47	46	30.8	− 2.4	28.4		9.869529
South Base	106	49	53.3	− 2.7	50.6		9.980987
Graveyard to South Base							2.810838
" North Base							2.922296

North Base to South Base						2.573616
	°	′	″	″	″	
Canal	10	08	62.5	− 3.8	58.7	0.753946
North Base	24	35	42.8	− 0.2	42.6	9.619306
South Base	145	15	20.1	− 1.4	18.7	9.755816
Canal to South Base						3.946868
Canal to North Base						4.083378

Graveyard to South Base						2.810838
	°	′	″	″	″	
Canal	46	45	41.3	+ 0.2	41.5	0.137565
Graveyard	94	48	47.8	+ 2.5	50.3	9.998465
South Base	38	25	26.8	+ 1.4	28.2	9.793429
Canal to South Base						2.946868
" Graveyard						2.741832

North Base to Canal						3.083378
	°	′	″	″	″	
Graveyard	120	12	30.1	+ 1.3	31.4	0.063387
North Base	23	10	48.0	− 2.2	45.8	9.395067
Canal	36	36	38.8	+ 4.0	42.8	9.775531
Graveyard to Canal						2.741832
" North Base						2.922296

South Base to North Base						2.573616
	°	′	″	″	″	
Naval Observatory	21	53	05.9	+ 0.4	06.3	0.428587
South Base	96	48	30.3	+ 1.5	31.8	9.996926
North Base	61	18	20.8	+ 1.1	21.9	9.943097
Naval Observatory to North Base						2.999129
" " South Base						2.945300

Canal to South Base						2.946868
	°	′	″	″	″	
Naval Observatory	31	05	40.4	− 1.1	39.3	0.286974
Canal	30	58	09.4	+ 1.9	11.3	9.711458
South Base	117	56	09.6	− 0.2	09.4	9.946193
Naval Observatory to South Base						2.945300
" " Canal						3.180035

Canal to North Base						3 083378
	°	′	″	″	″	
Naval Observatory	52	58	46.3	— 0.8	45.5	0.097770
Canal	41	07	11.9	— 1.9	10.0	9.817982
North Base	85	54	03.6	+ 0.9	04.5	9.998888
Naval Observatory to North Base						2.999130
" " Canal						3.180036

South Base to Canal						2.946868
	°	′	″	″	″	
Fort Myer	31	03	07.6	— 2.7	04.9	0.287513
South Base	44	02	55.7	+ 0.6	56.3	9.842155
Canal	104	53	63.5	— 4.7	58.8	9.985147
Fort Myer to Canal						3.076536
" South Base						3.219528

South Base to Graveyard						2.810838
	°	′	″	″	″	
Fort Myer	3	34	37.0	+ 3.2	40.2	1.204785
South Base	5	37	28.9	— 0.8	28.1	8.991260
Graveyard	170	47	49.3	+ 2.4	51.7	9.203905
Fort Myer to Graveyard						3.006883
" South Base						3.219528

Graveyard to Canal						2.741832
	°	′	″	″	″	
Fort Myer	27	28	30.6	— 5.9	24.7	0.335980
Graveyard	94	23	22.9	— 4.9	18.0	9.998725
Canal	58	08	22.2	— 4.9	17.3	9.929073
Fort Myer to Canal						3.076537
" Graveyard						3.006885

Fort Myer to South Base						3.219528
	°	′	″	″	″	
Naval Observatory	11	46	50.2	+ 2.2	52.4	0.689997
Fort Myer	6	14	03.3	— 1.4	01.9	9.035777
South Base	161	59	05.3	+ 0.4	05.7	9.490334
Naval Observatory to South Base						2.945302
" Fort Myer						3.399859

Naval Observatory to Canal						3.180035
	°	′	″	″	″	
Fort Myer	24	49	04.3	— 1.2	03.1	0.377030
Naval Observatory	19	18	50.2	— 3.4	46.8	9.519472
Canal	135	52	12.9	— 2.8	10.1	9.842793
Fort Myer to Canal						3.076537
" Naval Observatory						3.399858

At the Naval Observatory the distance from the trigonometrical station to the southwest corner of the pier supporting the reflector was measured by the two base rods, corrected for temperature and inclination. This distance is as follows:

	m.	
5 applications of rod No. 9 - - - - - -	19.9864	Total, $39^m.3147$
4 " " No. 10 - - - - - -	15.9753	
Measure by steel tape - - - - - - -	3.3530	

Supposing the prism at Fort Myer, over which the trigometrical signal was erected, the trigonometrica station at the Naval Observatory, and the southwest corner of the pier to be in a right line, we would have

	m.
Distance (in the horizon of the base) prism to N. O. △	2511.068
" " N. O. △ to SW. corner of pier	39.315
" " prism to SW. corner of pier	2550.383

In order to show the influence of the elevation of the two terminal objects on the distance between, I made the following rough estimate: Ground foot of flagstaff Fort Myer, above sea, 70^m; prism above this place, 5^m; height of prism above sea, 75^m; top of pier Naval Observatory above sea, 34^m; difference of height of prism and mirror 41^m (or 134½ feet nearly); hence, taking $3\frac{1}{2}^m$ for the height of rods during the base measure,

Correction to distance for 53^m, average height above base - - - -	$+ 0^m.021$
Correction to distance for difference of height of 41^m - - - - - - -	$+ 0.329$
Corrected distance, prism to SW. corner of pier - - - - - - - - -	2550.733

The distance and direction from this corner to the mirror are unknown.

We have also the following estimate of the probable error of the distance:

That arising from the base measure $\pm 4^{mm} \times \frac{2551}{375} = \pm 27^{mm}$

That arising from the elevation $\pm 2^{mm}$

That arising from the difference of height $\pm 85^{mm}$

$$\text{Total } \sqrt{27^2 + 2^2 + 85^2} = \pm 89^{mm}, \text{ or about } \frac{1}{29\,000} \text{ of the whole distance.}$$

This probable error may be greatly reduced by the measure of the heights.

The distances, Fort Myer to Washington Monument, and Naval Observatory to the same, were like wise determined, but they are supposed not to be needed now.

The path of the light may be taken as a straight line, the tortuosity of the ray being insignificant.

CHARLES A. SCHOTT,
Assistant Coast and Geodetic Survey.

COAST AND GEODETIC SURVEY OFFICE,
May 31, 1881.

DEAR SIR: The heights of the mirrors above the base-line on Analostan Island having been ascertained, the previous results communicated to you for distance Fort Myer to Naval Observatory, viz, 2550.733 meters, can now be corrected as follows:

LENGTH OF LINE BY TRIANGULATION.

	m
Fort Myer *prism* to SW. *corner* of pier near Naval Observatory	2550.383
Correction for average height of objects above base	+ 0.017
Correction for difference of heights of objects ($40^m.38$ or 132½ ft.)	+ 0.320
Corrected distance	2550.720

with an estimated probable error of $\pm 31^{mm}$, or about $\frac{1}{82\,000}$ of the length.

Yours respectfully,

C. P. PATTERSON,
Superintendent United States Coast and Geodetic Survey.

Prof. SIMON NEWCOMB, U. S. N.

Addition to report of April 29, 1881, *giving the connection of the Washington Monument station with the U. S. Naval Observatory station and Fort Myer.*

COMPUTING DIVISION, COAST AND GEODETIC SURVEY,
December 17, 1883.

I herewith submit in a supplementary report the geodetic connection in detail of the Washington Monument station with that at Fort Myer, as requested by Professor NEWCOMB. When submitting my first report the line had not been specially asked for; it was, however, computed and the position adjusted. As shown in the accompanying figure, the Monument station connects by means of 4 (black) lines with the previously adjusted figure, and gives rise to five additional equations, all showing very small residuals or errors to be dispersed.

[The figure described by Mr. SCHOTT is omitted. It can be replaced by Plate I. The four lines in question are those from the Monument to Observatory, the two bases, and Fort Myer.]

RESULTING DIRECTIONS OBSERVED AT MONUMENT STATION, WASHINGTON MONUMENT.

		Corrected.
	° ′ ″	″
Canal	0 00 00.0	[59.8]
Fort Myer	4 18 16.9	16.9
South Base	20 05 21.8	21.4
Naval Observatory	31 43 54.6	55.2

Conditional equations connecting station Washington Monument with previous quadrilateral taken as unchangeable.

$$0 = -2.9 - \binom{5}{1} + \binom{5}{2} - \binom{2}{5} + \binom{1}{5}$$
$$0 = +0.2 - \binom{5}{2} + \binom{5}{3} - \binom{3}{5} + \binom{2}{5}$$
$$0 = -0.8 - \binom{5}{1} + \binom{5}{4} - \binom{4}{5} + \binom{1}{5}$$
$$0 = -2.5 + .03\binom{5}{2} + 1.30\binom{5}{3} - 0.58\binom{2}{5} + 2.79\binom{3}{5} - 2.21\binom{4}{5}$$
$$0 = -0.5 - .11\binom{5}{1} + 0.03\binom{5}{2} + 0.34\binom{1}{5} - 0.58\binom{2}{5} + 0.24\binom{4}{5}$$

Forming equations of correlatives and normal equations there result the small corrections to the angles, as shown in the triangle side computation, herewith added.

The resulting length of side, Fort Myer to Washington Monument, in the horizontal plane of the base-line, or projected thereto, is 3718.89 meters [3.570413], with an estimated probable error between $\pm$ $0^{m}.0$ and $0^{m}.04$, but the slant line, or actual distance between the two stations, cannot be given for want of information about the height of the Monument station above the base-line.

C. A. S.

South Base to Naval Observatory							2.945300
	°	′	″	″	″		
Monument	11	38	32.8	+ 1.0	33.8		0.695060
South Base	19	35	24.5	+ 1.0	25.5		9.525426
Naval Observatory	148	45	59.8	+ 0.9	60.7		9.714767
Monument to Naval Observatory						1464.83	3.165786
" South Base							3.355127

Fort Myer to Naval Observatory 3.399858

	°	′	″	″	″	
Monument	27	25	37.7	+ 0.5	38.2	0.336655
Fort Myer	15	35	12.2	+ 1.3	13.5	9.429272
Naval Observatory	136	59	07.4	+ 0.9	08.3	9.833900

Monument to Naval Observatory	1464.82	3.165785
" " Fort Myer		3.570413

Fort Myer to South Base 3.219528

	°	′	″	″	″	
Monument	15	47	04.9	— 0.4	04.5	0.565397
Fort Myer	21	49	14.1	+ 1.3	15.4	9.570201
South Base	142	23	41.2	— 1.1	40.1	9.785488

Monument to South Base	3.355126
" " Fort Myer	3.570413

Canal to Fort Myer 3.076536

	°	′	″	″	″	
Monument	4	18	16.9	+ 0.3	17.2	1.124581
Canal	160	27	54.2	— 0.9	53.3	9.369295
Fort Myer	9	13	50.8	— 1.3	49.5	9.205218

Monument to Fort Myer	3.570412
" " Canal	3.406335

Canal to South Base 2.946868

	°	′	″	″	″	
Monument	20	05	21.8	— 0.2	21.6	0.464092
Canal	61	33	55.4	— 0.9	54.5	9.944166
South Base	98	20	45.0	— 1.1	43.9	9.995377

Monument to South Base	3.355126
" " Canal	3.406337

Canal to Naval Observatory 3.180036

	°	′	″	″	″	
Monument	31	43	54.6	+ 0.8	55.4	0.279058
Canal	30	35	44.1	— 0.9	43.2	9.706693
Naval Observatory	117	40	20.5	+ 0.9	21.4	9.947245

Monument to Naval Observatory	1464.83	3.165787
" " Canal		3.406339

UNITED STATES COAST AND GEODETIC SURVEY OFFICE,
Washington, D. C., December 22, 1883.

DEAR SIR: In reply to yours of December 20, I herewith inclose the desired information relating to height of Fort Myer. The figures are supposed to be sufficiently accurate for your purpose. Also a copy of a letter just received from Mr. SINCLAIR.

I am yours, respectfully,

CHAS. A. SCHOTT,
Acting Assistant in charge of Office.

Prof. SIMON NEWCOMB,
Superintendent Nautical Almanac, Washington, D. C.

DATA OF HEIGHTS TAKEN FROM THE RECORD.

	m.
Foot of flag-staff at Fort Myer above average tide-level of the Potomac	69.75
Mirror of apparatus above base of flag-staff	2.69
Hence mirror (the prism was not in position) above the Potomac mean level	72.44

The base line is above the same level ½ (8.7 + 12.7) + 0.5 metres, or $11^m.2$.

The average low water is half a meter below the average or mean tide level of the river.

The top of the pier in Naval Observatory grounds where the mirror was mounted is $32^m.06$ above the mean tide level of the river.

C. A. S.

DECEMBER 22, 1883.

UNITED STATES COAST AND GEODETIC SURVEY,
Covington, Va., December 20, 1883.

DEAR SIR: In reply to your inclosure from Professor NEWCOMB, I make the following statement:

The theodolite used in connecting Fort Myer with the pier at the Naval Observatory was a six-inch Gambey, graduated to five seconds. The tripod was centered over the revolving mirror in the usual manner by means of a plumb-bob. The legs of the tripod were of unusual length, and, with the assistance of some two-inch boards firmly nailed together and placed on the floor, no difficulty was found in making the tripod stride the mirror and at the same time rendering it very stable.

An aperture in the side of the building and a window enabled me to see the signals used in the triangulation. For the purpose of observing on the revolving mirror, a board was nailed across the rafters of the building and a point fixed in this exactly over the mirror by a plumb-bob; then a hole was cut in the roof, and a small pole, resting centrally on the board over the point above the mirror and carefully plumbed, projected through the roof. See sketch showing elevation.

Yours, respectfully,

C. H. SINCLAIR.

Mr. C. A. SCHOTT,
Acting Assistant in charge of Office, Washington, D. C.

APPENDIX II.

THE CHRONOGRAPH RECORD FOR 1882.

The following tables show the chronograph readings by which the velocities were determined in 1882. They are given only for that year because it is not supposed that they can be the subject of serious discussion unless to form some general conception of the character of the observations. Those for 1882 will completely suffice to determine the degree of precision with which the mirror could be kept at a uniform speed. To understand the numbers it is to be remarked that the cylinder of the chronograph revolved once in about ten seconds; and that every second-beat of the chronometer, and every twenty-eighth revolution of the mirror, were recorded on the sheet by wedge-shaped projections on a continuous line, which were formed by lateral jerks of the pen which made the line. These projections are called "breaks." Since at the usual rate of speed of the mirror twenty-eight revolutions were completed seven or eight times in a second, the more numerous breaks on the line are those made by the mirror, and their distance apart is generally about three-fourths of a centimeter. The chronograph second-breaks are longer than the others, being about one-fourth of a centimeter, and their distance apart is about six centimeters. The result of this was that the position of the break made by the mirror could not be determined when it fell during the break of the chronometer.

The method of reading the sheets was as follows: The signal given by the observer for the commencement of a set being found on the sheet, the first second-break which did not contain within it any mirror-break was chosen as an arbitrary zero, and the following second-breaks were numbered continuously from it, until a signal was reached showing that an accurate setting of the deflected image of the slit upon the wires of the receiving telescope was about to cease. The mirror-break next preceding the zero second-break was also taken as an arbitrary zero, and the following ones were numbered continuously from it. It was only necessary to make the actual count of the mirror-breaks which occurred during the first ten seconds, since there never was any such uncertainty in the speed as to leave a doubt of the entire number of any break following it. The space between two consecutive mirror-breaks was then taken as the unit of measure, so that position along the broken line was expressed in terms of the number of mirror-breaks which had transpired from the zero point. The second-breaks were then measured by their position among the mirror-breaks, the point taken for this purpose being the commencement of the break. It was found quite sufficient to measure every tenth second in order to avoid all possibility of the count being an integer in error. The measures were made to hundredths of a unit, but it may be assumed that the mean accidental error of the measurement is several hundredths, possibly five or more.

In the tables the column "Rev." shows the points of the broken line at which the second-breaks whose numbers are given in the first column commenced, expressed in terms of the number of mirror-breaks. Subtracting any two consecutive numbers in the second column we have the number of mirror-breaks which occurred during ten chronometer seconds. From what has been said it will be seen that the beginning of the second-break could not be accurately fixed when it occurred during a mirror-break. In the early observations it was first the habit to take the first following second which could be accurately read; but it was soon found that the irregularity of intervals thus arising was entirely unnecessary, for it was sufficient to obtain a rough approximation to the position of the doubtful break, and to go on to the next one in order. Since the velocities depend only upon the accurate numbers, any errors in this approximate class are eliminated from the final result.

The column Δ gives the number of mirror-break intervals during each ten-second interval. Where the readings in column Rev. are accurate, Δ is always the difference between two consecutive readings; when uncertain, only the *sums* of the Δ's for the interval between two accurate readings are to be relied on.

In deducing the mean velocity it was supposed that the mean of all the velocities during each ten seconds of the run would be the most probable result. The sum of the numbers in the column Δ, being the

difference between the first and last numbers in the column preceding, the product of this difference by 28 gives the whole number of revolutions of the mirror. Thus, in strictness, the numbers of the second column, intermediate between the first and last, serve only the purpose of detecting any errors that may have been made in reading off, and to test the uniformity of speed of the mirror. The doubtful quantities are marked by a colon. In such cases the sum of the doubtful numbers in the last column is always necessarily correct, one being as much too large as another is too small.

CHRONOGRAPHIC RECORD.

1882. July 24.

Set 1.

s.	Rev.	Δ
0	0.21	. .
10	70.28	70.07
20	140.41	70.13
30	210.43	70.02
40	280.43	70.00
50	350.55	70.12
60	420.50	69.95
70	490.57	70.07
80	560.54	69.97
90	630.61	70.07
100	700.68	70.07
110	770.71	70.03

V = 196.12

Set 2.

s.	Rev.	Δ
0	0.41	. .
10	72.43	72.02
20	144.50	72.07
30	216.57	72.07
40	288.61	72.04
50	360.64	72.03
60	432.64	72.00

V = 201.71

Set 3.

s.	Rev.	Δ
0	0.37	. .
10	74.77	74.40
20	149.04:	74.27:
30	223.33	74.29:
40	298. :	74.67:
50	372.45	74.45:
60	446.88	74.43
70	521. :	74.12:
80	595.73	74.73:
90	679. :	74.27:
100	744.46	74.46:
110	818.90	74.44

V = 208.35

1882. July 24.

Set 4.

s.	Rev.	Δ
0	0.46	. .
10	74.89	74.43
20	149.30	74.41
30	223.88	74.58
40	298.29	74.41
50	372.91	74.62
60	447. :	74.09:
70	521.82	74.82:
80	596.2:	74.38:
90	670.85	74.65:
100	745.1:	74.25:
110	819.77	74.67:
120	894.1:	74.33:
130	968.81	74.71:

V = 208.58

Set 5.

s.	Rev.	Δ
0	0.58	. .
10	77.46	76.88
20	154. :	76.81:
30	231. :	76.76:
40	308.00	76.97:
50	384.85	76.85
0	0.73	. .
10	77.65	76.92
20	154.58	76.93
30	231.42	76.84

V = 215.18

1882. July 24.

Set 6.

s.	Rev.	Δ
0	0.44	. .
10	72.48	72.04
20	144.50	72.02
30	216.50	72.00
40	288.53	72.03
50	360.50	71.97
60	432.50	72.00
70	504.52	72.02
80	576.57	72.05
90	648.55	72.98
100	720.57	72.02
110	792.57	72.00

V = 201.63

1882. July 26.

Set 1.

s.	Rev.	Δ
0	0.29	. .
10	71.1:	70.82:
20	141.85	70.75:
30	212.64	70.79
40	283.40	70.76
50	354. :	70.60:
60	424.93	70.93:
70	495.71	70.78
80	566.40	70.69

V = 198.14

Set 2.

s.	Rev.	Δ
0	0.53	. .
10	68.2:	67.67:
20	135.79	67.59:
30	203.36	67.57
40	271.0:	67.64:
50	338.63	67.63:
60	406.2:	67.57:
70	473.83	67.63:
80	541.38	67.55

V = 189.30

CHRONOGRAPHIC RECORD—Continued.

1882. July 26.

Set 3.

s.	Rev.	Δ
0	0.60	. .
10	66.40	65.80
20	132. :	65.76 :
30	197.93	65.77 :
40	263.67	65.74
50	329.40	65.73
60	395. :	65.74 :
70	460.87	65.73 :
80	526.63	65.76
90	592.36	65.73

V = 184.11

Set 4.

s.	Rev.	Δ
0	0.50	. .
10	75.98	75.48
20	151.68	75.70
30	227.34	75.66
40	303.0 :	75.66 :
50	378.69	75.69 :
60	454.38	75.69
70	530. :	75.62 :
80	605.65	75.65 :
90	681.2 :	75.65 :
100	756.92	75.62 :
110	832.54	75.62
120	908.2 :	75.66 :
130	983.92	75.72 :
140	1059.65	75.73

V = 211.83

Set 5.

s.	Rev.	Δ
0	0.71	. .
10	69.55	68.84
20	138.34	68.79
30	207.1 :	68.86 :
40	216. :	68.85 :
50	244.93	68.88 :
60	413.80	68.87
70	482.55	68.75
80	551.28	68.73
90	620.1 :	68.82 :
100	689.0 :	68.90 :
110	757.89	68.89 :
120	826.58	68.69

V = 192.71

1882. July 26.

Set 6.

s.	Rev.	Δ
0	0.25	. .
10	80. :	79.75 :
20	160. :	80.00 :
30	240. :	80.00 :
40	319.91	79.91 :
50	399.80	79.89
60	479.64	79.84
70	559.56	79.92
80	639.28	79.72

V = 223.66

Set 7.

s.	Rev.	Δ
0	0.36	. .
10	70.55	70.19
20	140.86	70.31
30	211.1 :	70.24 :
40	281.2 :	70.10 :
50	351.50	70.30 :
60	421.67	70.17
70	491.89	70.22

V = 196.62

Set 8.

s.	Rev.	Δ
0	0.26	. .
10	74.67	74.41
20	148.96	74.29
30	223.26	74.30
40	297.70	74.44
50	372.0 :	74.30 :
60	446.2 :	74.20 :
70	520.59	74.39 :
80	594.90	74.31

V = 208.12

1882. July 31.

Set 1.

s.	Rev.	Δ
0	0.40	. .
10	66.20	65.80
20	131.95	65.75
30	197.67	65.72
40	263.50	65.83
50	329.20	65.70

V = 184.15

1882. July 31.

Set 2.

s.	Rev.	Δ
0	0.48	. .
10	80.25 :	79.77 :
20	160.36	80.11 :
30	240.64	80.28
40	320.84	80.20
52	416.92	96.08
0	0.80	. .
10	80.75	79.95
20	160.88	80.13
36	289.00	128.12

V = 224.28

Set 3.

s.	Rev.	Δ
0	0.50	. .
10	70.00	69.50
20	139.32	69.32
30	208.78	69.46
40	278.0 :	69.22 :
50	247.50	69.50 :
60	416.96	69.46
70	486.30	69.34

V = 194.32

Set 4.

s.	Rev.	Δ
0	0.85	. .
10	77.70	76.85
20	154.54	76.84
30	231.54	77.00
40	308.50	76.96
50	385.42	76.92
60	462.25	76.83
0	0.54	. .
10	77.46	76.92
20	154.48	77.02
30	231.46	76.98

V = 215.42

CHRONOGRAPHIC RECORD—Continued.

1882. July 31.

Set 5.

s.	Rev.	Δ
2	. . .	. .
12	. . .	. .
20	147.30	. .
30	220.52	73.22
40	293.70	73.18
50	366.92	73.22
62	454.75	87.83
70	513.18	58.43
80	586.44	73.26

V = 204.93

Set 6.

s.	Rev.	Δ
0	0.42	. .
10	81.90	81.48
20	163.50	81.60
30	244.84	81.34
40	326.3:	81.46:
50	407.75	81.45:

V = 228.08

Set 7.

s.	Rev.	Δ
0	0.20	. .
10	74.1:	73.90:
20	147.74	73.64:
30	221.54	73.80
40	295.20	73.66
50	368.92	73.72
60	442.44	73.52
68	501.37	58.93
80	589.82	88.45
90	663.37	73.55

V = 206.32

Set 8.

s.	Rev.	Δ
I. 6	48.80	. .
10	80.56	31.76
20	160.2:	79.64:
30	239.36	79.16:
40	318.88	79.52
II. 0	0.44	. .
10	80.0:	79.56
20	159.48	79.48:
30	239.1:	79.62:
40	318.48	79.38:
52	413.85	95.37

V = 222.51

1882. August 9.

Set 1.

s.	Rev.	Δ
0	0.54	. .
10	70.2:	69.66:
20	140.1:	69.90:
30	209.80	69.70:
40	279.50	69.70
48	335.40	55.90

V = 195.34

Set 2.

s.	Rev.	Δ
2	17.65	. .
10	84.1:	66.45:
22	183.50	99.40:
30	249.80	66.30
40	332.75	82.95
50	415.58	82.83
0	0.60	. .
10	83.50	82.90
20	166.35	82.85
30	249.1:	82.75:
40	332.0:	82.90:
50	414.83	82.83:

V = 232.05

Set 3.

s.	Rev.	Δ
0	0.40	. .
10	74.00	73.60
20	147.46	73.46
30	220.9:	73.44:
40	294.50	73.60:
50	367.90	73.40
60	441.40	73.50
70	514.85	73.45
80	588.36	73.51
90	661.90	73.54
100	735.37	73.47
110	808.90	73.53

V = 205.80

1882. August 9.

Set 4.

s.	Rev.	Δ
0	0.40	. .
10	81.42	81.02
20	162.40	80.98
30	243.40	81.00
40	324.44	81.04
50	405.32	80.88
60	486.2:	81.0:
70	567.32	81.0:
80	648.28	80.96
90	729.1:	80.88:
100	810.1:	80.88:
110	890.92	80.88:

V = 226.68

Set 5.

s.	Rev.	Δ
0	0.52	. .
10	74.74	74.22
20	148.82	74.08
30	222.88	74.06
40	296.92	74.04
50	371.0:	74.08:
60	445.1:	74.10:
70	519.15	74.05:
80	593.37	74.22
90	667.44	74.07

V = 207.49

Set 6.

s.	Rev.	Δ
0	0.60	. .
10	82.96	82.36
20	165.0:	82.04:
30	247.50	82.50:
40	329.75	82.25
50	412.0:	82.25:
60	494.1:	82.10:
70	576.60	82.50:
80	658.92	82.32
90	741.0:	82.08:
100	823.45	82.45:
110	905.66	82.21

V = 230.36

CHRONOGRAPHIC RECORD—Continued.

1882. August 10.

Set 1.

s.	Rev.	Δ
0	0.83	. .
10	70.62	69.79
20	140.42	69.80
30	210.2:	69.80:
40	280.0:	69.80:
50	349.90	69.88:
60	419.75	69.85
70	489.58	69.83
80	559.45	69.87
90	629.3:	69.85:
100	698.9:	69.80:
110	768.9:	69.80:
120	838.83	69.93:
130	908.58	69.75
140	978.45	69.87
150	1048.0:	69.55:
160	1118.0:	70.0:
170	1187.9:	69.9:
180	1257.72	69.82:

V = 195.52

Set 2.

s.	Rev.	Δ
0	0.42	. .
10	70.86	70.44
20	141.0:	70.14:
30	211.57	70.57:
40	282.0:	70.43:
50	352.50	70.50:
60	422.85	70.35
70	493.0:	70.15:
80	563.66	70.66:
90	634.0:	70.34:
100	704.46	70.46:
110	774.72	70.26

V = 197.09

1882. August 10.

Set 3.

s.	Rev.	Δ
0	0.48	. .
10	69.69	69.21
20	138.83	69.14
30	208.1:	69.27:
40	277.2:	69.10:
50	346.35	69.15:
60	415.55	69.20
70	484.83	69.28
80	554.0:	69.17:
90	623.2:	69.20:
100	692.35	69.15:

V = 193.72

Set 4.

s.	Rev.	Δ
0	0.53	. .
10	67.2:	66.67:
20	133.94	66.74:
30	200.54	66.60
40	267.1:	66.56:
50	333.87	66.77:
60	400.54	66.67
70	467.2:	66.66:
80	533.80	66.60:
90	600.48	66.68

V = 186.65

Set 5.

s.	Rev.	Δ
0	0.29	. .
10	71.40	71.11
20	142.50	71.10
30	213.50	71.00
40	284.57	71.07
50	355.78:	71.21:
60	426.80	71.02:

V = 199.03

1882. August 10.

Set 6.

s.	Rev.	Δ
0	0.50	. .
10	68.48	67.98
20	136.30	67.82
30	204.2:	67.90:
40	272.1:	67.90:
48	326.50	54.40:
58	394.42	67.92
70	475.86	81.44

V = 190.15

Set 7.

s.	Rev.	Δ
0	0.48	. .
10	69.1:	68.62:
20	137.69	68.59:
30	206.2:	68.51:
40	274.82	68.62:
50	343.35	68.53
60	412.0:	68.65:
70	480.36	68.56:
80	549.1:	68.54:
90	617.76	68.66:
98	672.55	54.79
110	754.80	82.25
120	823.35	68.55

V = 192.00

Set 8.

s.	Rev.	Δ
0	0.46	. .
10	66.50	66.04
20	132.56	66.06
30	198.56	66.00
40	264.60	66.04
50	330.68	66.08
60	396.66	65.98
70	462.72	66.06
80	528.80	66.08
90	594.84	66.04
100	660.87	66.03

V = 184.91

CHRONOGRAPHIC RECORD—Continued.

1882. August 10.

Set 9.

s.	Rev.	Δ
0	0.35	. .
10	68.2:	67.85:
20	136.1:	67.90:
28	190.48	54.38:
38	258.44	67.96
48	326.48	68.04
58	394.35	67.87
70	475.90	81.55
80	543.84	67.94
90	611.76	67.92
100	679.70	67.94
110	747.68	67.98
120	815.56	67.88
130	883.48	67.92
140	951.44	67.96

V = 190.22

Set 10.

s.	Rev.	Δ
0	0.62	. .
10	66.0:	65.38:
20	131.37	65.37:
30	196.87	65.50
40	262.2:	65.33:
50	327.73	65.53:
60	393.2:	65.47:
70	458.48	65.28
80	524.0:	65.52:
90	589.40	65.40:

V = 183.18

Set 11.

s.	Rev.	Δ
0	0.32	. .
10	69.84	69.52
20	139.25	69.41
30	208.70	69.45
40	278.1:	69.40:
50	347.50	69.40:
60	417.1:	69.60:
70	486.21	69.11:
80	555.72	69.51
90	625.2:	69.48:
100	694.58	69.38:
110	764.0:	69.42:
120	833.32	69.32:

V = 194.37

1882. August 10.

Set 12.

s.	Rev.	Δ
0	0.74	. .
10	67.80	67.06
20	134.94	67.14
30	202.0:	67.06:
40	269.2:	67.20:
50	336.27	67.07:
60	403.40	67.13
70	470.56	67.16
80	537.70	67.14
90	604.80	67.10
100	671.96	67.16
110	739.0:	67.04:
120	806.1:	67.10:
130	873.20	67.10:
140	940.36	67.16

V = 187.93

Set 13.

s.	Rev.	Δ
0	0.78	. .
10	70.70	69.92
20	140.70	70.00
30	210.64	69.94
40	280.56	69.92
50	350.54	69.98

V = 195.87

Set 14.

s.	Rev.	Δ
0	0.41	. .
10	68.69	68.28
20	137.00:	68.31:
30	205.55	68.55:
40	274.0:	68.45:
50	342.20	68.20:
60	410.76	68.56
70	479.0:	68.24:
80	547.44	68.44:

V = 191.47

1882. August 10.

Set 15.

s.	Rev.	Δ
0	0.43	. .
10	71.0:	70.57:
20	141.64	70.64:
30	212.2:	70.56:
40	282.71	70.51:
50	353.21:	70.50:
60	423.90	70.69:
70	494.36	70.46
80	564.93	70.57
90	635.44	70.51
100	706.0:	70.56:
110	776.64	70.64:

V = 197.58

Set 16.

s.	Rev.	Δ
0	0.41	. .
10	69.48	69.07
20	138.48	69.00
30	207.52	69.04
40	276.62	69.10
50	345.62	69.00
60	414.70	69.08
70	483.83	69.13

V = 193.36

Set 17.

s.	Rev.	Δ
0	0.20	. .
10	70.50	70.30
20	140.64	70.14
30	210.80	70.16
40	280.94	70.14
50	351.0:	70.06:
60	421.2:	70.20:
70	491.40	70.20:
80	561.64	70.24

V = 196.48

CHRONOGRAPHIC RECORD—Continued.

1882. August 10. Set 18.

s.	Rev.	Δ
0	0.38	. .
10	69.2:	68.62:
20	138.1:	68.90:
30	206.76	68.66:
40	275.40	68.64
50	344.2:	68.80:
60	412.90	68.70:
70	481.70	68.80
80	550.24	68.54
90	619.0:	68.76:
100	687.72	68.72:
110	756.35	68.63

V = 192.42

1882. August 11. Set 1.

s.	Rev.	Δ
0	0.76	. .
10	72.80	72.04
20	144.86	72.06
30	217.0:	72.14:
40	288.96	71.96:
50	361.0:	72.04:
60	433.29	72.29:
70	505.43	72.14
80	577.50	72.07
90	649.64	72.14
100	721.71	72.07
110	793.93	72.22

V = 201.90

Set 2.

s.	Rev.	Δ
0	0.64	. .
10	72.29	71.65
20	144.1:	71.81:
30	216.0:	71.90:
40	287.86	71.86:
50	359.64	71.78
60	431.36	71.72
70	503.0:	71.64:
80	594.9:	71.90:
90	646.80	71.90:
100	718.64	71.84
110	790.36	71.72

V = 201.02

1882. August 11. Set 3.

s.	Rev.	Δ
0	0.46	. .
10	72.0:	71.54:
20	143.50	71.50:
30	215.1:	71.60:
40	286.50	71.40:
50	358.1:	71.60:
60	429.57	71.47:
70	501.0:	71.43:
80	572.60	71.60:
88	629.86	57.26
100	715.57	85.71
110	787.0:	71.43:
120	858.56	71.56:

V = 200.22

Set 4.

s.	Rev.	Δ
0	0.54	. .
10	71.0:	70.46:
20	141.57	70.57:

V = 197.44

Set 4 (*bis*).

s.	Rev.	Δ
0	0.62	. .
10	68.75	68.13
20	136.86	68.11
30	204.90	68.04

V = 190.68

Set 5.

s.	Rev.	Δ
0	0.50	. .
10	71.57	71.07
20	142.60	71.03
30	213.71	71.11
40	284.86	71.15
50	355.90	71.04
60	426.96	71.06
70	498.0:	71.04:
80	568.96	70.96:
90	640.0:	71.04:
100	711.1:	71.10:
110	782.2:	71.10:
120	853.43	71.23:
130	924.50	71.07
140	995.57	71.07
150	1066.67	71.10
160	1137.71	71.04

V = 199.01

1882. August 11. Set 6.

s.	Rev.	Δ
0	0.55	. .
10	69.1:	68.55:
20	137.55	68.45:
32	219.82	82.27
40	274.70	54.88
50	343.2:	68.50:
60	411.72	68.52:
70	480.2:	68.48:
80	548.84	68.64:
90	617.3:	68.46:
100	685.86	68.56:
110	754.2:	68.34:
120	822.86	68.66:

V = 191.87

Set 7.

s.	Rev.	Δ
0	0.48	. .
10	64.70	64.22
20	128.84	64.14
30	193.0:	64.16:
40	257.2:	64.20:
50	321.48	64.28:
60	385.70	64.22
70	449.90	64.20
80	514.1:	64.20:
90	578.2:	64.10:
100	642.48	64.28:
110	706.64	64.16
120	770.84	64.20

V = 179.75

Set 8.

s.	Rev.	Δ
0	0.53	. .
10	70.3:	69.77:
20	140.2:	69.90:
30	210.1:	69.90:
40	279.90	69.80:
50	349.71	69.81
60	419.57	69.86
70	489.30	69.73
80	559.1:	69.80:
90	629.14	70.04:

V = 195.56

CHRONOGRAPHIC RECORD—*Continued.*

1882. August 11.

Set 9.

s.	Rev.	Δ
0	0.52	. .
10	70.3:	69.78:
20	140.1:	69.80:
30	209.90	69.80:
40	279.64	69.74
50	349.36	69.72
60	419.2:	69.84:
70	489.0:	69.80:
80	558.71	69.71:
90	628.50	69.79
100	698.2:	69.70:
110	768.0:	69.80:
120	837.86	69.86:
130	907.57	69.71
140	977.21	69.64

V = 195.34

Set 10.

s.	Rev.	Δ
0	0.55	. .
10	67.90	67.35
20	135.2:	67.30:
30	202.65	67.45:
40	270.0:	67.35:
50	337.3:	67.30:
60	404.61	67.31:
70	472.1:	67.49:
82	552.76	80.66:
90	606.60	53.84
100	673.95	67.35
110	741.3:	67.35:
112	754.69	13.39:

V = 188.54

1882. August 25.

Set 1.

s.	Rev.	Δ
0	0.41	. .
12	82.69	82.28
20	137.55	54.86
30	206.1:	68.55:
38	260.69	54.59:
50	342.90	82.21
60	411.34	68.44
70	479.90	68.56

V = 191.80

1882. August 25.

Set 2.

s.	Rev.	Δ
— 2	— 13.56	. .
0	1.0	14.56:
10	75.47	74.47:
20	149.54	74.07
30	224.2:	74.66:
36	268.52	44.32:
50	372. :	103.48:
58	431.44	59.44:
70	520.44	89.00

V = 207.67

Set 3.

s.	Rev.	Δ
0	0.60	. .
10	67.40	66.80
20	134.2:	66.80:
30	200.84	66.64:
40	267.60	66.76
50	334.2:	66.60:
58	387.53	53.33:

V = 186.79

Set 3 (*bis*).

s.	Rev.	Δ
0	0.33	. .
10	67.1:	66.77:
20	133.94	66.84:
30	200.53	66.59
40	267.2:	66.67:
50	333.93	66.73:

V = 186.82

Set 4.

s.	Rev.	Δ
0	0.92	. .
10	81.75	80.83
20	162.75	81.00
30	243.58	80.83
40	324.3:	80.72:
50	405.2:	80.90:
64	518.58	113.38:
70	567.4:	48.82:
78	631.83	64.43:
90	728.86	97.03
100	809.83	80.97
110	890.67	80.84
120	971.54	80.87

V = 226.48

1882. August 25.

Set 5.

s.	Rev.	Δ
0	0.60	. .
10	69.79	69.19
20	139.0:	69.21:
30	208.1:	69.10:
40	277.50	69.40:
50	346.79	69.29
60	416.0:	69.21:
70	485.1:	69.10:
80	554.57	69.47:
92	637.86	83.29

V = 193.95

Set 6.

s.	Rev.	Δ
0	0.52	. .
10	80.80	80.28
20	161.1:	80.30:
30	241.2:	80.10:
40	321.64	80.44:
50	401.88	80.24
60	482.1:	80.22:
74	594.40	112.30:
80	642.64	48.24
90	722.90	80.26
100	803.1:	80.20:
112	899.32	96.22:
120	963.60	64.28

V = 224.72

Set 7.

s.	Rev.	Δ
0	0.64	. .
10	70.57	69.93
20	140.36	69.79
30	210.1:	69.74:
40	280.0:	69.90:
50	350.0:	70.00:
60	420. :	70.00:
70	489.86	69.86:
80	559.71	69.85
90	629.57	69.86
100	699.50	69.93

V = 195.68

CHRONOGRAPHIC RECORD—Continued.

1882. August 25. Set 8.

s.	Rev.	Δ
0	0.48	. .
10	78.96	78.48
18	141.64	62.68
30	235.56	93.92
40	314.1:	78.54:
50	392.32	78.22:
60	470.80	78.48
68	533.48	62.68

V = 219.47

1882. August 29. Set 1.

s.	Rev.	Δ
0	0.52	. .
10	74.74	74.22
20	149.0:	74.26:
30	223.44	74.44:
40	297.89	74.45
50	372.1:	74.21:
60	446.59	74.49:

V = 208.17

Set 1 (*bis*).

s.	Rev.	Δ
0	0.52	. .
10	74.74	74.22
20	149.0:	74.26:
30	223.52	74.52:
40	297.74	74.22
50	372.1:	74.36:
60	446.48	74.38:
70	520.74	74.26
80	595.1:	74.36:
90	669.37	74.27:
100	743.74	74.37

V = 208.10

1882. August 29. Set 2.

s.	Rev.	Δ
0	0.56	. .
10	74.38	73.82
20	148.40	74.02
30	222.38	73.98
40	296.38	74.00
50	370.27	73.89
60	444.2:	73.93:
68	503.59	59.39:
80	592.0:	88.41:
90	666.0:	74.0 :
96	710.37	44.37:

V = 207.03

Set 3.

s.	Rev.	Δ
0	0.30	. .
10	68.34	68.04
20	136.30	67.96
30	204.34	68.04
40	272.34	68.00
50	340.34	68.00
60	408.34	68.00
70	476.40	68.06
80	544.40	68.00
90	612.48	68.08
100	680.41	67.93

V = 190.43

Set 4.

s.	Rev.	Δ
0	0.55	. .
10	68.41	67.86
20	136.3:	67.89:
30	204.1:	67.80:
40	271.90	67.80:
50	339.70	67.80
60	407.55	67.85
68	461.83	54.28
78	529.61	67.78
88	597.41	67.80
100	678.85	81.44

V = 189.92

1882. August 29. Set 5.

s.	Rev.	Δ
0	0.52	. .
10	66.2:	65.68:
20	131.48	65.28:
30	197.1:	65.62:
40	262.61	65.51:
50	328.0:	65.39:
60	393.69	65.69:

V = 183.48

Set 6.

s.	Rev.	Δ
0	0.51	. .
10	59.75	59.24
20	118.88	59.13
30	178.2:	59.32:
44	260.79	82.59:
50	296.3:	35.51:
60	355.36	59.06:
70	414.48	59.12
80	473.61	59.13
90	532.73	59.12

V = 165.58

Set 7.

s.	Rev.	Δ
0	0.89	. .
10	75.2:	74.31:
20	149.4:	74.20:
30	223.59	74.19:
40	297.77	74.18
50	372.0:	74.23:
60	446.3:	74.30:
70	520.4:	74.10:
80	594.67	74.27:
90	668.74	74.07
100	743.0:	74.26:
102	757.82	14.82:
110	817.2:	59.38:
120	891.34	74.14:

V = 207.77

CHRONOGRAPHIC RECORD—Continued.

1882. August 29.

Set 8.

s.	Rev.	Δ
0	0.80	. .
10	78.48	77.68
20	156.2:	77.72:
32	249.50	93.30:
40	311.72	62.22
50	389.40	77.68
60	467.1:	77.70:
70	545.0:	77.90:
80	622.76	77.76:
90	700.40	77.64

V = 217.65

Set 9.

s.	Rev.	Δ
0	0.70	. .
10	74.2:	73.50:
20	147.82	73.62:
28	206.67	58.85
40	295.0:	88.3[illegible]:
50	368.59	73.59:
60	442.1:	73.51:
70	515.74	73.64:
78	574.52	58.78
90	662.82	88.30
100	736.32	73.50

V = 205.97

Set 10.

s.	Rev.	Δ
0	0.68	. .
12	94.75	94.07
20	157.50	62.75
30	235.92	78.42
40	314.3:	78.38:
50	392.80	78.50:
60	471.3:	78.50
70	549.72	78.42
80	628.2:	78.48:
90	706.64	78.44:
100	785.1:	78.46:
102	800.72	15.62:

V = 219.62

1882. August 29.

Set 11.

s.	Rev.	Δ
0	0.64	. .
10	73.64	73.00
20	146.4:	72.76:
28	204.88	58.48:
38	277.67	72.79
48	350.59	72.92
58	423.55	72.96
68	496.37	72.82
80	583.86	87.49
90	656.82	72.96
100	729.70	72.88

V = 204.14

Set 12.

s.	Rev.	Δ
0	0.56	. .
10	79.88	79.32
20	158.88	79.00
30	237.88	79.00
40	316.96	79.08
50	396.0:	79.04:
62	490.88	94.88:
72	569.92	79.04
82	648.96	79.04
94	743.80	94.84
104	822.88	79.08
116	917.72	94.84
126	996.80	79.08

V = 221.39

1882. August 30.

Set 1.

s.	Rev.	Δ
0	0.60	. .
10	72.53	71.93
20	144.29	71.76
30	216.0:	71.71:
38	273.79	57.79:
48	345.71	71.92
60	431.93	86.22
70	503.71	71.78
80	575.50	71.79
90	647.40	71.90

V = 201.23

1882. August 30.

Set 2.

s.	Rev.	Δ
0	0.69	. .
10	75.77	75.08
20	151.0:	75.23:
30	226.2:	75.20:
40	301.31	75.11:
50	376.62	75.31
60	451.85	75.23

V = 210.54

Set 3.

s.	Rev.	Δ
0	0.43	. .
10	71.0:	70.57:
20	141.79	70.79:
30	212.50	70.71
40	283.2:	70.70:
50	353.71	70.51:
60	424.29	70.58
70	495.0:	70.71:
80	565.64	70.64:

V = 197.82

Set 4.

s.	Rev.	Δ
0	0.65	. .
10	80.1:	79.45:
20	159.48	79.38:
32	254.93	95.45
40	318.48	63.55
50	397.90	79.42
56	445.47	47.57
72	572.65	127.18
80	636.0:	63.35:
90	715.53	79.53:
100	794.90	79.37
110	874.3:	79.40:
120	953.82	79.52:
130	1032.92	79.10
140	1112.70	79.78

V = 222.41

CHRONOGRAPHIC RECORD—Continued.

1882. August 30. Set 5.

s.	Rev.	Δ
0	0.71	. .
10	73.93	73.22
20	147.1:	73.17:
30	220.3:	73.20:
40	293.36	73.06:
50	366.50	73.14
60	439.74	73.24
70	512.86	73.12
80	586.0:	73.14:
90	659.2:	73.20:
98	717.70	58.50:
110	805.44	87.74
120	878.5:	73.06:
130	951.77	73.27:
140	1024.90	73.13

V = 204.84

Set 6.

s.	Rev.	Δ
0	0.44	. .
10	74.92	74.48
20	149.44	74.52
30	223.96	74.52
40	298.52	74.56
50	372.96	74.44
60	443.52	74.56
70	522.0:	74.48:
80	596.52	74.52:
90	671.0:	74.48:
100	745.59	74.59:
110	820.0:	74.41:
120	894.59	74.59:
130	969.0:	74.41:
140	1043.62	74.62:

V = 208.64

1882. September 1. Set 1.

s.	Rev.	Δ
0	0.43	. .
10	71.50	71.07
20	142.71	71.21
30	213.71	71.00
40	284.79	71.08
50	355.86	71.07
60	427.0:	71.14:
70	497.90	70.90:
80	569.0:	71.10:
90	640.1:	71.10:
100	711.14	71.04:
110	782.21	71.07

V = 199.00

Set 2.

s.	Rev.	Δ
0	0.55	. .
10	69.69	69.14
20	138.83	69.14
30	208.0:	69.17:
40	277.1:	69.10:
50	346.2:	69.10:
60	415.41	69.21:
70	484.55	69.14

V = 193.60

Set 3.

s.	Rev.	Δ
0	0.87	. .
10	68.69	67.82
20	136.61	67.92
30	204.58	67.97
40	272.55	67.97
50	340.50	67.95
60	408.41	67.91
70	476.41	68.00
80	544.34	67.93

V = 190.21

1882. September 1. Set 4.

s.	Rev.	Δ
0	0.48	. .
10	69.55	69.07
20	138.69	69.14
30	207.90	69.21
40	277.0:	69.10:
50	346.2:	69.20:
60	415.34	69.14:
70	484.61	69.27
80	553.83	69.22

V = 193.67

Set 5.

s.	Rev.	Δ
0	0.83	. .
10	69.21	68.38
20	137.83	68.62
30	206.34	68.51
40	274.90	68.56
50	343.41	68.51
60	412.0:	68.59:
70	480.55	68.55:

V = 191.89

Set 6.

s.	Rev.	Δ
0	0.50	. .
10	71.57	71.07
20	142.57	71.00
30	213.60	71.03
40	284.79	71.19
50	355.80	71.01
60	426.86	71.06
70	497.86	71.00

V = 198.94

Set 7.

s.	Rev.	Δ
0	0.32	. .
10	69.50	69.18
20	138.64	69.14
30	207.86	69.22

V = 193.70

CHRONOGRAPHIC RECORD—Continued.

1882. September 1.

Set 8.

s.	Rev.	△
0	0. 57	. .
10	72. 2:	71. 63:
20	143. 86	71. 66:
30	215. 50	71. 64
40	287. 1:	71. 60:
50	358. 90	71. 80:
60	430. 43	71. 53

V = 200. 60

Set 9.

s.	Rev.	△
0	0. 53	
10	66. 57	66. 04
20	132. 67	66. 10
30	198. 80	66. 13
40	264. 87	66. 07
48	317. 67	52. 80
58	383. 83	66. 16
68	449. 87	66. 04
78	515. 87	66. 00
88	581. 93	66. 06
98	647. 93	66. 00
108	714. 0:	66. 07:
120	793. 27	79. 27:
130	859. 30	66. 03
140	925. 47	66. 17

V = 184. 99

Set 10.

s.	Rev.	△
0	0. 82	. .
10	74. 96	74. 14
20	149. 1:	74. 14:
30	223. 3:	74. 20:
38	282. 37	59. 07:
48	356. 52	74. 15
58	430. 60	74. 08
60	445. 40	14. 80

V = 207. 47

1881. September 1.

Set 11.

s.	Rev.	△
0	0. 83	. .
10	68. 65	67. 82
20	136. 61	67. 96
30	204. 55	67. 94
40	272. 50	67. 95
50	340. 41	67. 91
60	408. 41	68. 00
70	476. 28	67. 87

V = 190. 18

Set 12.

s.	Rev.	△
0	0. 38	. .
10	75. 0:	74. 62:
20	149. 92	74. 92:
30	224. 72	74. 80
40	299. 46	74. 74
50	374. 2:	74. 74:
60	449. 0:	74. 80:
70	523. 85	74. 85:
80	598. 54	74. 69

V = 209. 36

Set 13.

s.	Rev.	△
0	0. 50	. .
10	63. 45	62. 95
20	126. 32	62. 87
30	189. 36	63. 04
40	252. 18	62. 82
50	315. 0:	62. 82:
60	378. 20	63. 20:

V = 176. 26

Set 14.

s.	Rev.	△
0	0. 30	. .
10	58. 42	58. 12
20	116. 42	58. 00
30	174. 45	58. 03
40	232. 48	58. 03
50	290. 45	57. 97
60	348. 54	58. 09
70	406. 48	57. 94
80	464. 45	57. 97
90	522. 54	58. 09

V = 162. 48

1882. September 1.

Set 15.

s.	Rev.	△
0	0. 46	. .
10	64. 0:	63. 54:
20	127. 56	63. 56:
30	191. 1:	63. 54:
40	254. 65	63. 55:
50	318. 1:	63. 45:
60	381. 58	63. 48:
68	432. 39	50. 81
80	508. 65	76. 26
88	559. 39	50. 74
100	635. 70	76. 31

V = 177. 87

Set 16.

s.	Rev.	△
0	0. 43	. .
10	71. 50	71. 07
20	142. 57	71. 07
30	213. 64	71. 07
40	284. 64	71. 00
50	355. 64	71. 00
60	426. 71	71. 07
70	497. 79	71. 08

V = 198. 94

Set 17.

s.	Rev.	△
0	0. 55	. .
10	69. 0:	68. 45:
20	137. 61	68. 61:
28	192. 55	54. 94
40	274. 76	82. 21
50	343. 28	68. 52
60	411. 88	68. 60
70	480. 28	68. 40

V = 191. 89

CHRONOGRAPHIC RECORD—Continued.

1882. September 1.

Set 18.

s.	Rev.	△
0	0. 65	. .
10	76. 0 :	75. 35 :
20	151. 2 :	75. 20 :
30	226. 69	75. 49 :
40	302. :	75. 31 :
50	377. 38	75. 38 :
60	452. 92	75. 54
74	558. 38	105. 46
80	603. 69	45. 31
90	679. 1 :	75. 41 :
100	754. 54	75. 44 :

V = 211. 09

1882. September 2.

Set 1.

s.	Rev.	△
0	0. 60	. .
10	68. 14	67. 54
20	135. 83	67. 69
30	203. 34	67. 51
40	271. 0 :	67. 66 :
50	338. 55	67. 55 :
58	392. 55	54. 00
70	473. 69	81. 14
80	541. 0 :	67. 31 :
90	608. 76	67. 76 :
100	676. 41	67. 65

V = 189. 23.

Set 2.

s.	Rev.	△
0	0. 65	. .
10	76. 96	76. 31
22	168. 62	91. 66
30	229. 65	61. 03
40	306. 0 :	76. 35 :
52	397. 46	91. 46 :
60	458. 58	61. 12
70	534. 85	76. 27
80	611. 2 :	76. 35 :
90	687. 46	76. 26 :
100	763. 77	76. 31

V = 213. 67.

1882. September 2.

Set 3.

s.	Rev.	△
0	0. 48	. .
10	69. 28	68. 80
20	138. 1 :	68. 82 :
30	207. 0 :	68. 90 :
40	275. 80	68. 80 :
50	344. 61	68. 81
60	413. 48	68. 87
70	482. 2 :	68. 72 :
80	551. 0 :	68. 80 :
90	619. 76	68. 76 :
100	688. 55	68. 79
110	757. 34	68. 79

V = 192. 66.

Set 4.

s.	Rev.	△
0	0. 54	. .
10	78. 1 :	77. 56 :
20	155. 62	77. 52 :
28	217. 62	62. 00
40	310. 70	93. 08
48	372. 62	61. 92
60	465. 85	93. 23
68	527. 85	62. 00
80	620. 92	93. 07
90	698. 34	77. 42
100	776. 0 :	77. 66 :
110	853. 62	77. 62 .

V = 217. 14.

Set 5.

s.	Rev.	△
0	0. 48	. .
10	68. 69	68. 21
20	136. 83	68. 14
30	205. 0 :	68. 17 :
40	273. 2 :	68. 20 :
50	341. 50	68. 30 :
60	409. 69	68. 19
70	477. 97	68. 28

V = 191. 00.

1882. September 2.

Set 6.

s.	Rev.	△
0	0. 84	. .
10	80. 3 :	79. 46 :
20	159. 72	79. 42 :
30	239. 2 :	79. 48 :
40	318. 56	79. 36 :
50	398. 0 :	79. 44 :
60	477. 40	79. 40 :
70	557. 0 :	79. 60 :
80	636. 40	79. 40 :
90	715. 96	79. 56
98	779. 40	63. 44
110	874. 80	95. 40

V = 222. 46.

Set 7.

s.	Rev.	△
0	0. 53	. .
10	70. 57	70. 04
20	140. 60	70. 03
30	210. 71	70. 11
40	280. 75	70. 04
50	350. 86	70. 11
60	420. 93	70. 07
70	491. 0 :	70. 07 :
80	560. 93	69. 93 :
90	630. 93	70. 00

V = 196. 12.

Set 8.

s.	Rev.	△
0	0. 96	. .
10	84. 62	83. 66
18	151. 58	66. 96
32	268. 75	117. 17
40	335. 75	67. 00
50	419. 42	83. 67
60	503. 2 :	83. 78 :
72	603. 83	100. 63 :

V = 234. 45.

CHRONOGRAPHIC RECORD—Continued.

1882. September 2.

Set 9.

s.	Rev.	△
0	0.31	. .
10	69.76	69.45
20	139.1:	69.34:
30	208.69	69.59:
40	278.1:	69.41:
50	347.61	69.51:
60	417.0:	69.39:
70	486.48	69.48:
80	556.0:	69.52:
90	625.41	69.41:

V = 194.48.

Set 10.

s.	Rev.	△
0	0.42	. .
10	84.75	84.33
20	169.1:	84.35:
30	253.58	84.48:
40	337.92	84.34
50	421.92	84.00
60	506.58	84.66
70	591.0:	84.42:
78	658.42	67.42:
90	759.75	101.33

V = 236.24.

Set 11.

s.	Rev.	△
0	0.64	. .
10	73.1:	72.46:
20	145.79	72.69:
30	218.29	72.50
40	290.93	72.64
50	363.36	72.43
60	436.0:	72.64:
70	508.53	72.53:
80	581.1:	72.57:
90	653.64	72.54:

V = 203.16.

1882. September 2.

Set 12.

s.	Rev.	△
0	0.83	. .
10	84.58	83.75
20	168.42	83.84
30	251.96:	83.54:
40	336.0:	84.04:
50	419.75	83.75:
60	503.58	83.83
68	570.62	67.04
80	671.0:	100.38:
90	755.0:	84.0 :
100	838.83	83.83:
110	922.58	83.75

V = 234.63.

Set 13.

s.	Rev.	△
0	0.48	. .
10	69.90	69.42
20	139.34	69.44
30	208.83	69.49
40	278.1:	69.27:
50	347.65	69.55:
60	417.1:	69.45:
70	486.48	69.38:
80	556.0:	69.52:
90	625.48	69.48:

V = 194.44.

Set 14.

s.	Rev.	△
0	0.48	. .
10	78.88	78.40
20	157.0:	78.12:
28	219.64	62.64:
40	313.40	93.76
50	391.68	78.28
60	469.92	78.24
70	548.2:	78.28:
78	610.64	62.44:
90	704.56	93.92
100	782.64	78.08

V = 219.00.

1882. September 2.

Set 15.

s.	Rev.	△
0	0.50	. .
10	70.50	70.00
20	140.50	70.00
30	210.57	70.07
40	280.53	69.96
50	350.50	69.97
60	420.53	70.03
70	490.50	69.97
80	560.48	69.98

V = 195.99.

Set 16.

s.	Rev.	△
0	0.72	. .
10	75.88	75.16
20	151.0:	75.12:
30	226.2:	75.20:
40	301.3:	75.10:
50	376.40	75.10:
60	451.48	75.08
70	526.72	75.24
80	601.88	75.16

V = 210.41.

Set 17.

s.	Rev.	△
0	0.14	. .
10	69.76	69.62
20	139.3:	69.54:
30	208.61	69.31:
40	278.1:	69.49:
50	347.50	69.40:
60	416.90	69.40
70	486.34	69.44

V = 194.48.

CHRONOGRAPHIC RECORD—Continued.

1882. September 2.

Set 18.

s.	Rev.	Δ
0	0. 46	. .
10	64. 46	64. 00
20	128. 31	63. 85
28	179. 45	51. 14
38	243. 42	63. 97
48	307. 44	64. 02
58	371. 38	63. 94
68	435. 42	64. 04
78	499. 30	63. 88
88	563. 25	63. 95
100	639. 97	76. 72
110	703. 93	63. 96
120	767. 70	63. 77

V = 179. 02

1882. September 5.

Set 1.

s.	Rev.	Δ
0	0. 80	. .
10	67. 73	66. 93
20	134. 65	66. 92
30	201. 53	66. 88
40	268. 40	66. 87
50	335. 40	67. 00

V = 187. 38

Set 2.

s.	Rev.	Δ
0	0. 82	. .
10	74. 74	73. 92
20	148. 63	73. 89
30	222. 52	73. 89
40	296. 44	73. 92
50	370. 3 :	73. 86 :
60	444. 2 :	73. 90 :
70	518. 0 :	73. 80 :
80	591. 89	73. 89 :
90	665. 74	73. 85
100	739. 59	73. 85

V = 206. 86

1882. September 5.

Set 3.

s.	Rev.	Δ
0	0. 61	. .
10	69. 41	68. 80
20	138. 2 :	68. 79 :
30	207. 0 :	68. 80 :
40	275. 80	68. 80 :
50	344. 55	68. 75
60	413. 3 :	68. 75 :
70	482. 1 :	68. 80 :
80	550. 76	68. 66 :
90	619. 58	68. 82
100	688. 37	68. 79

V = 192. 57

Set 4.

s.	Rev.	Δ
0	0. 59	. .
10	73. 89	73. 30
20	147. 0 :	73. 11 :
28	205. 67	58. 67 :
40	293. 67	88. 00
50	367. 0 :	73. 33 :
58	425. 67	58. 67 :
70	513. 59	87. 92
80	586. 89	73. 30
88	645. 44	58. 55
98	718. 74	73. 30
110	806. 74	88. 00

V = 205. 20

1882. September 5.

Set 5.

s.	Rev.	Δ
0	0. 93	. .
10	71. 64	70. 71
20	142. 3 :	70. 66 :
30	213. 0 :	70. 70 :
40	283. 86	70. 86 :
50	354. 50	70. 64
60	425. 2 :	70. 70 :
70	496. 0 :	70. 80 :
80	566. 71	70. 71 :
90	637. 43	70. 72
100	708. 1 :	70. 67 :
110	778. 90	70. 80 :
120	849. 64	70. 74

V = 198. 03

Set 6.

s.	Rev.	Δ
0	0. 37	. .
10	75. 52	75. 15
20	150. 59	75. 07
30	225. 67	75. 08
40	300. 70	75. 03
50	375. 74	75. 04
60	450. 86	75. 12
70	525. 89	75. 03

V = 210. 21

Set 7.

s.	Rev.	Δ
0	0. 29	. .
10	70. 48	70. 19
20	140. 64	70. 16
30	210. 75	70. 11
40	280. 86	70. 11
50	351. 0 :	70. 14 :
60	421. 0 :	70. 00 :
70	491. 1 :	70. 10 :
80	561. 2 :	70. 10 :
90	631. 36	70. 16 :
100	701. 50	70. 14
110	771. 57	70. 07

V = 196. 33

CHRONOGRAPHIC RECORD—Continued.

1882. September 5. Set 8.

s.	Rev.	Δ
0	0.77	. .
10	77.0:	76.23:
22	168.62	91.62:
30	229.57	60.95
38	290.69	61.12
52	397.62	106.93
60	458.62	61.00
68	519.62	61.00
78	595.92	76.30
92	702.69	106.77
100	763.69	61.00
108	824.77	61.08
122	931.46	106.69
130	992.62	61.16
140	1068.85	76.23
150	1145.1:	76.25:
162	1236.62	91.52:
170	1297.62	61.00

V = 213.60

1882. September 5. Set 9.

s.	Rev.	Δ
0	0.50	. .
10	70.71	70.21
20	140.93	70.22
30	211.1:	70.17:
40	281.4:	70.30:
50	351.57	70.17:
60	421.86	70.29

V = 196.64

Set 10.

s.	Rev.	Δ
0	0.46	. .
10	76.69	76.23
20	153.0:	76.31:
26	198.77	45.77:
40	305.62	106.85
50	381.85	76.23
56	427.62	45.77
66	503.92	76.30
80	610.62	106.70
90	686.77	76.15
98	747.77	61.00

V = 213.52

1882. September 5. Set 11.

s.	Rev.	Δ
0	0.50	. .
10	72.0:	71.50:
20	143.3:	71.30:
30	214.79	71.49:
42	300.43	85.64
50	357.57	57.14
58	414.64	57.07
70	500.30	85.66
80	571.80	71.50

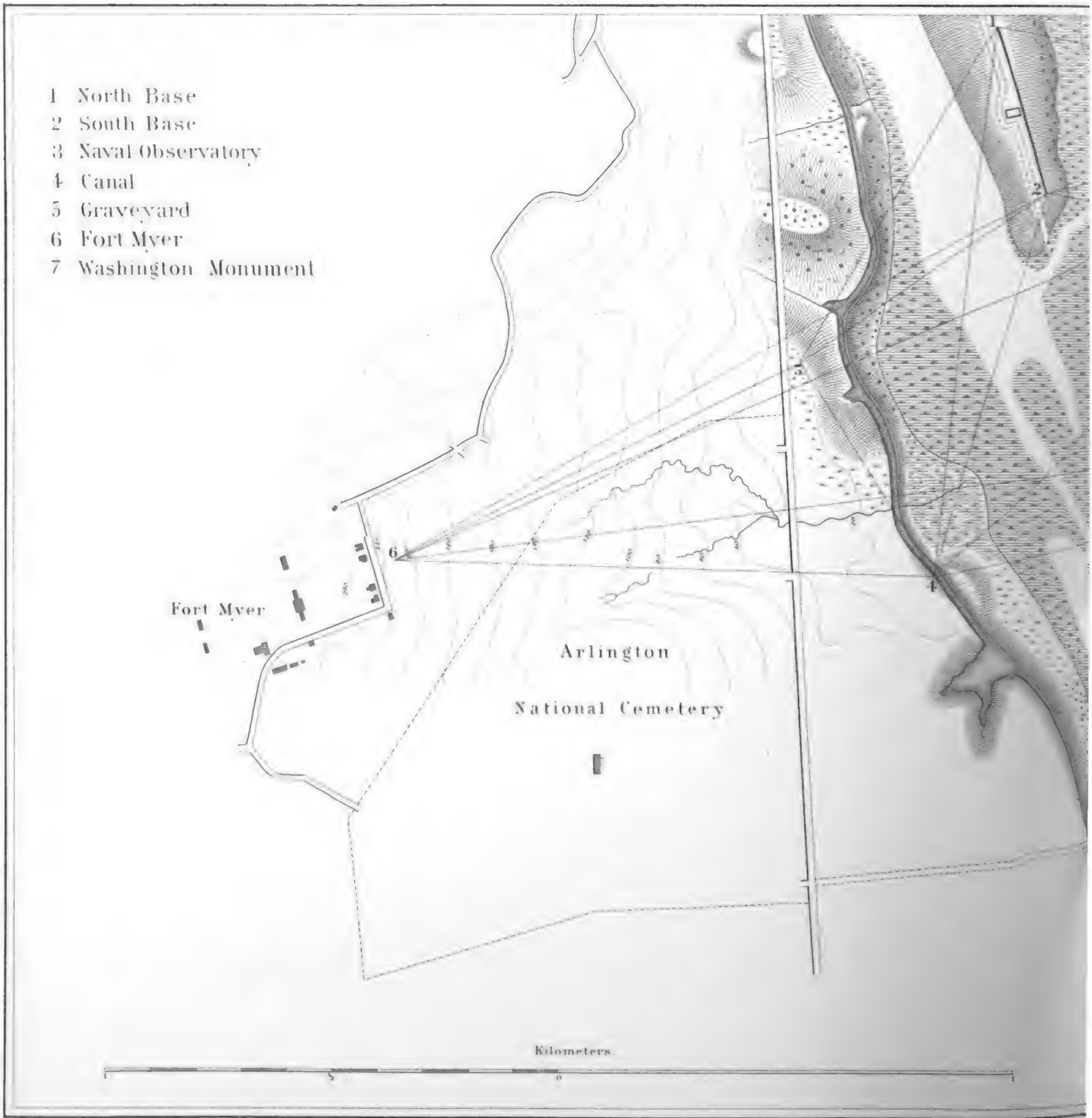
V = 199.96

Set 12.

s.	Rev.	Δ
0	0.52	. .
10	73.74	73.22
20	146.90	73.16
30	220.1:	73.20:
40	293.3:	73.20:
48	351.60	58.30:
58	424.74	73.14
70	512.44	87.70
80	585.60	73.16
90	658.82	73.22
102	746.60	87.78

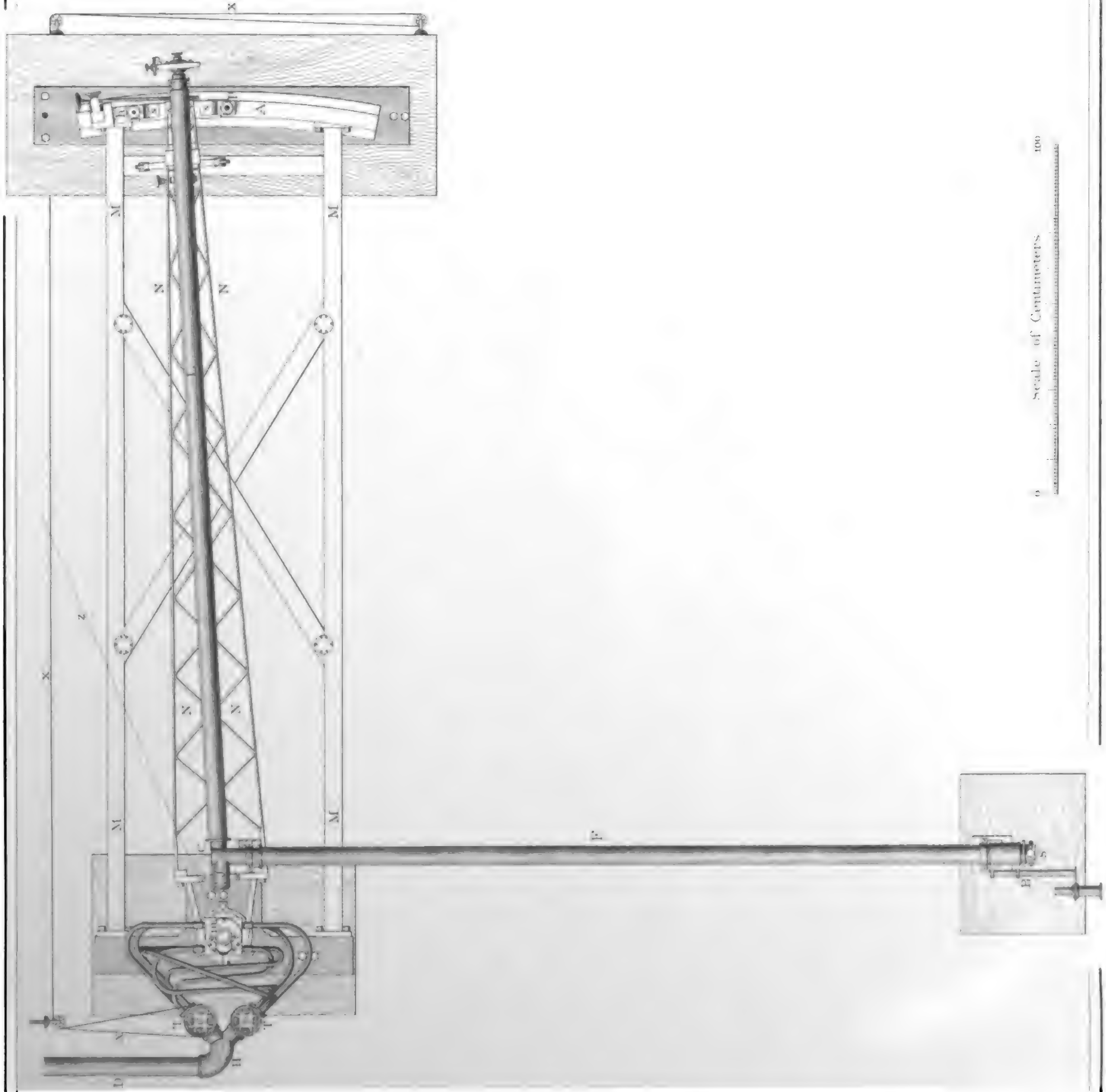
V = 204.81

MAP AND TRIANGULATION OF I



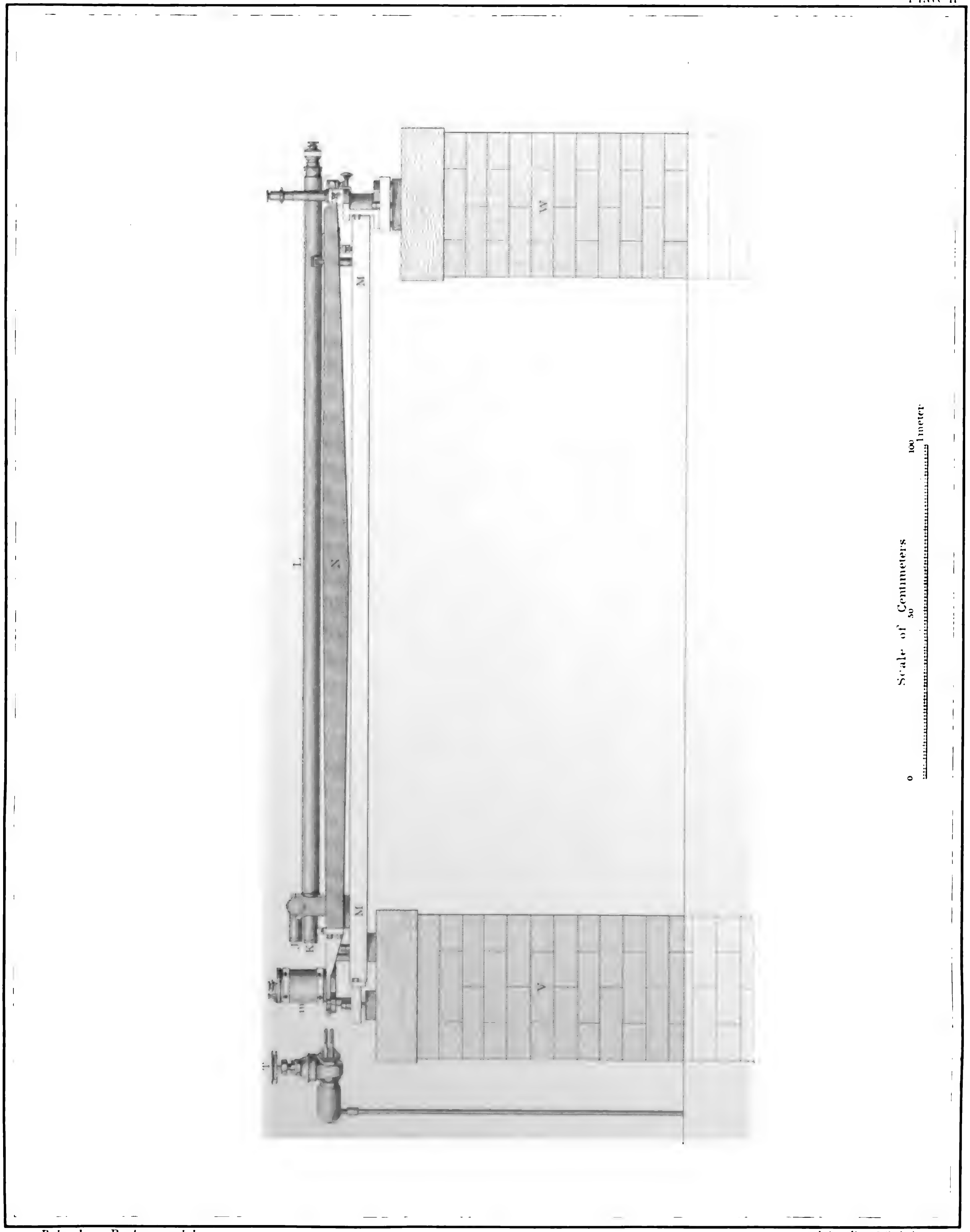
Lieut. C. E. Vreeland U. S. N. del.

EGION INCLUDING STATIONS.



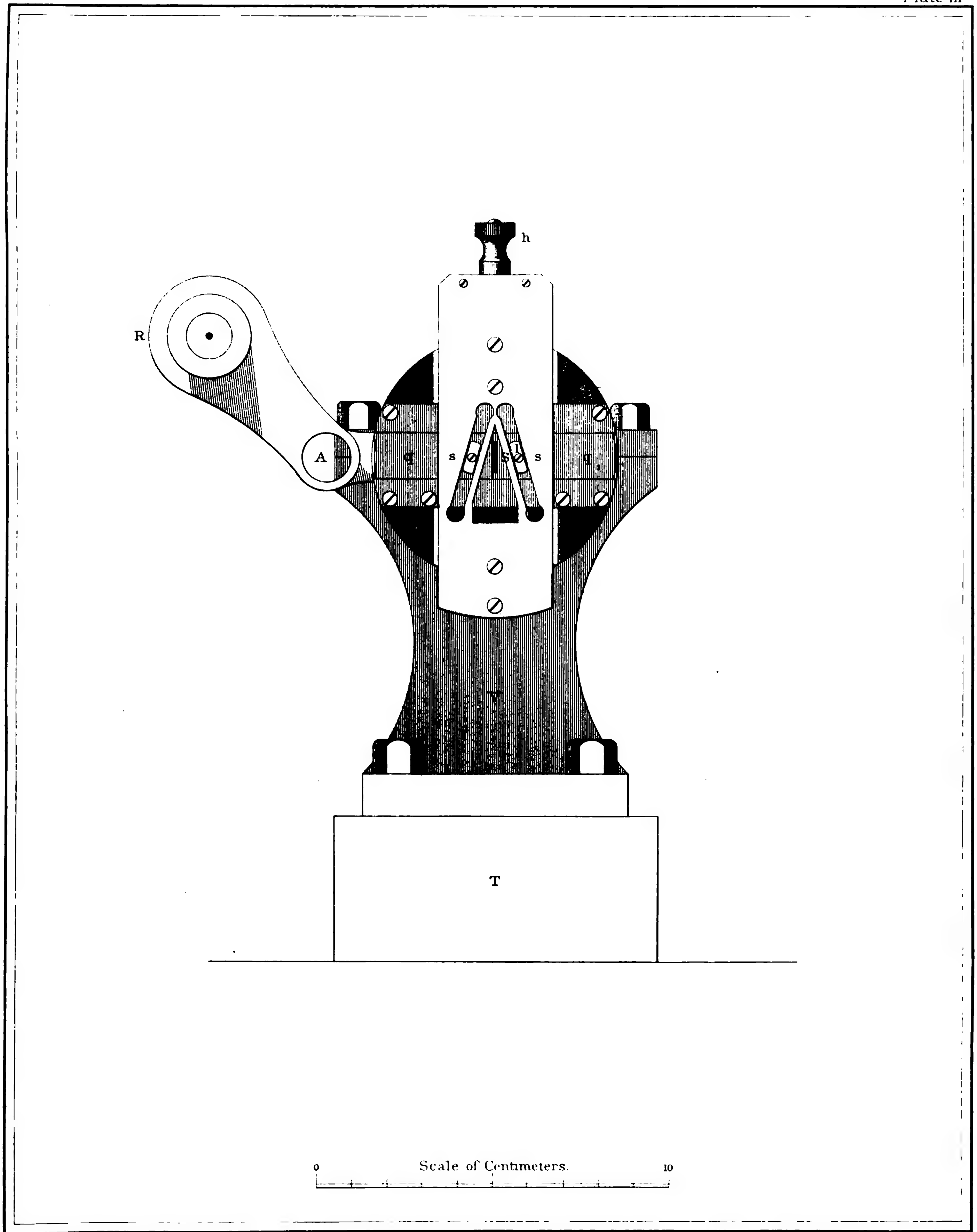
Roberdeau Buchanan del

Julius Bien & Co. Lith.



Roberdeau Buchanan del.

Julius Bien & Co. Lith.



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Julius Bien & Co. Lith.

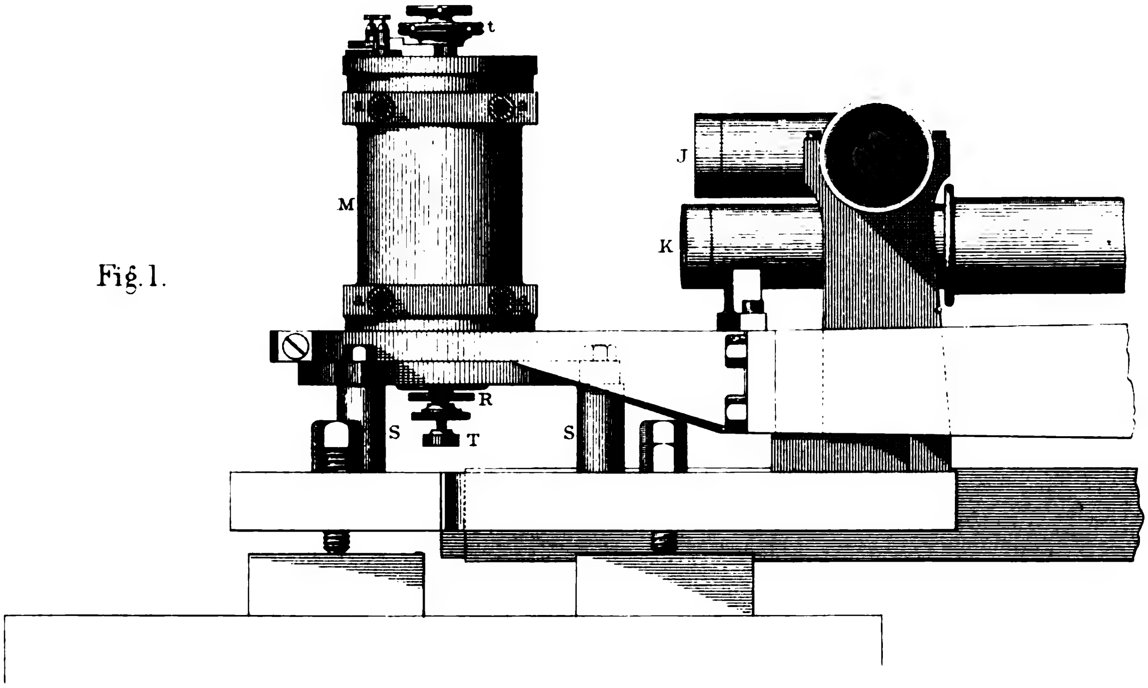

Fig. 1.

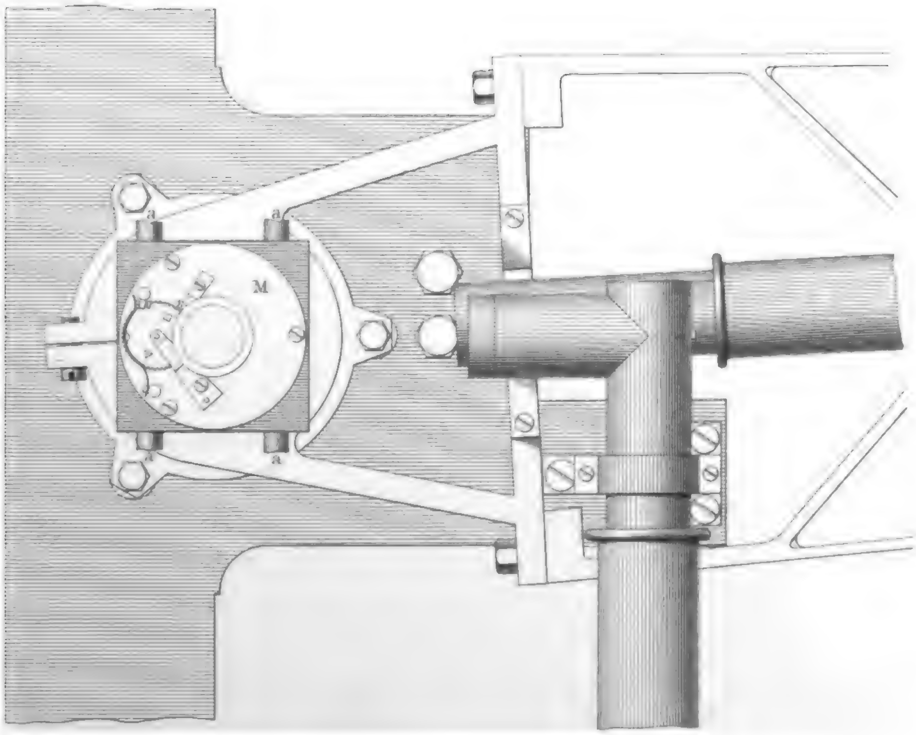

Fig. 2.

0 Scale of Centimeters 20

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Julius Bien & Co. Lith.

Fig. 1

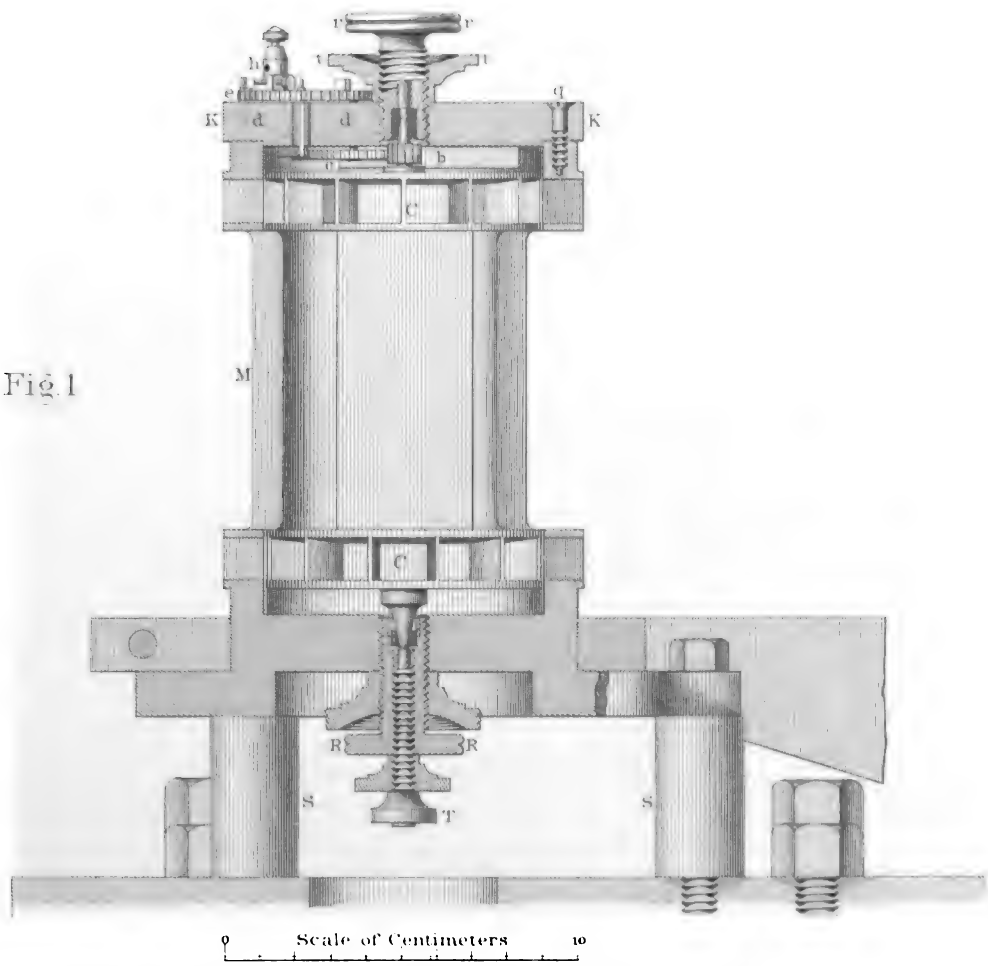
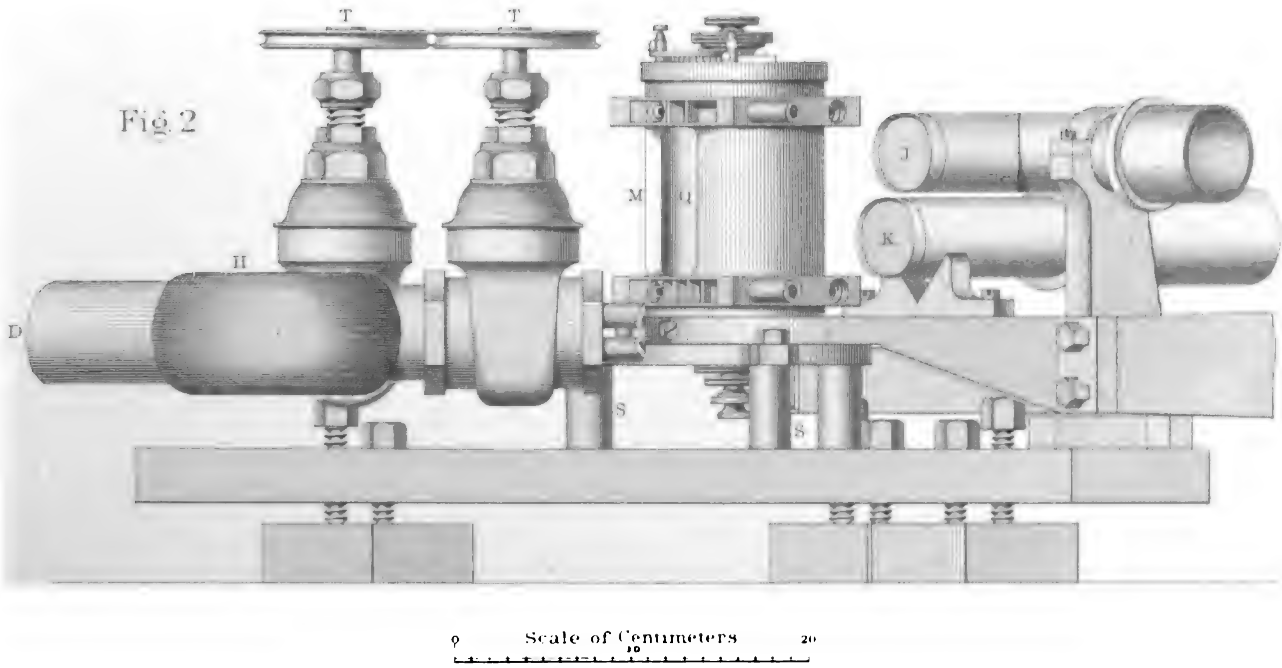


Fig. 2



Roberdeau Buchanan del.

Julius Bien & Co. Lith.

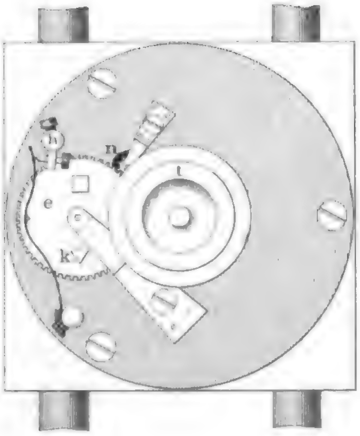


Fig. 1

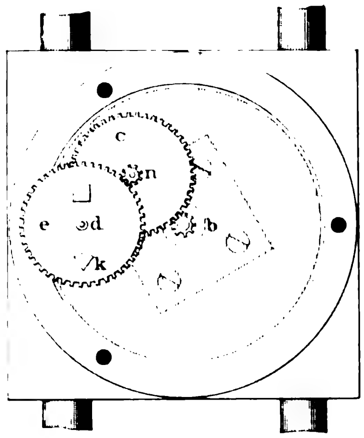


Fig. 2

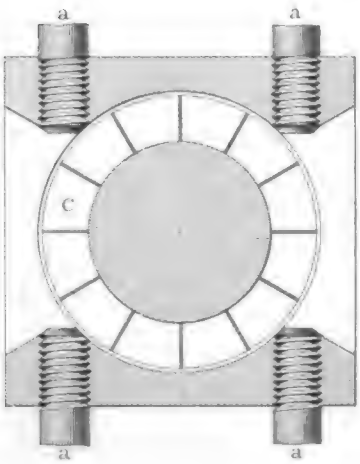


Fig. 3

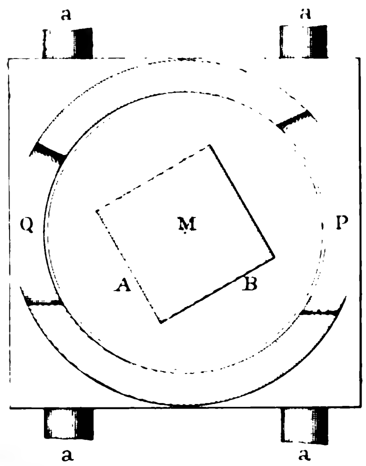


Fig. 4

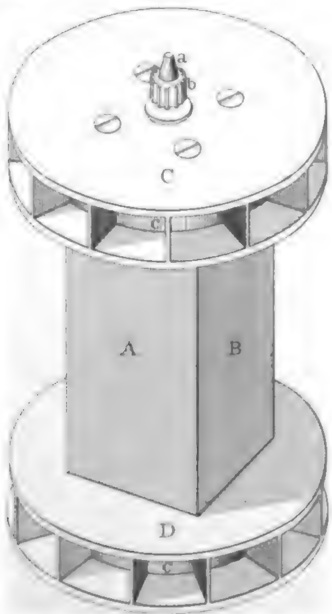


Fig. 5

0 Scale of Centimeters 10

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Julius Bien & Co. Lith.

Fig. 1

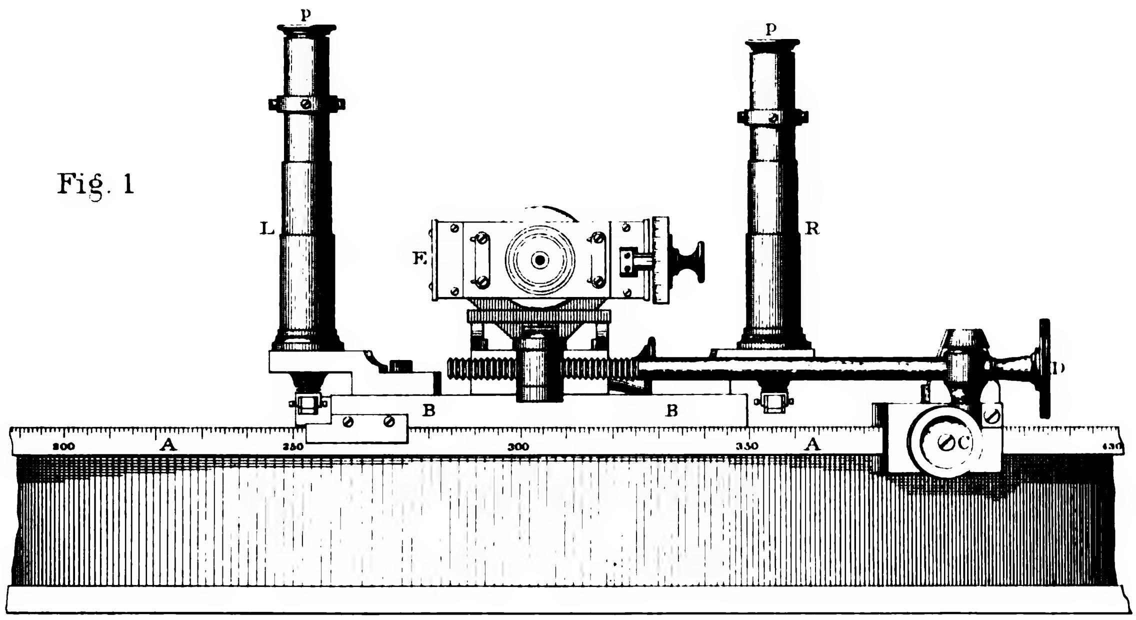
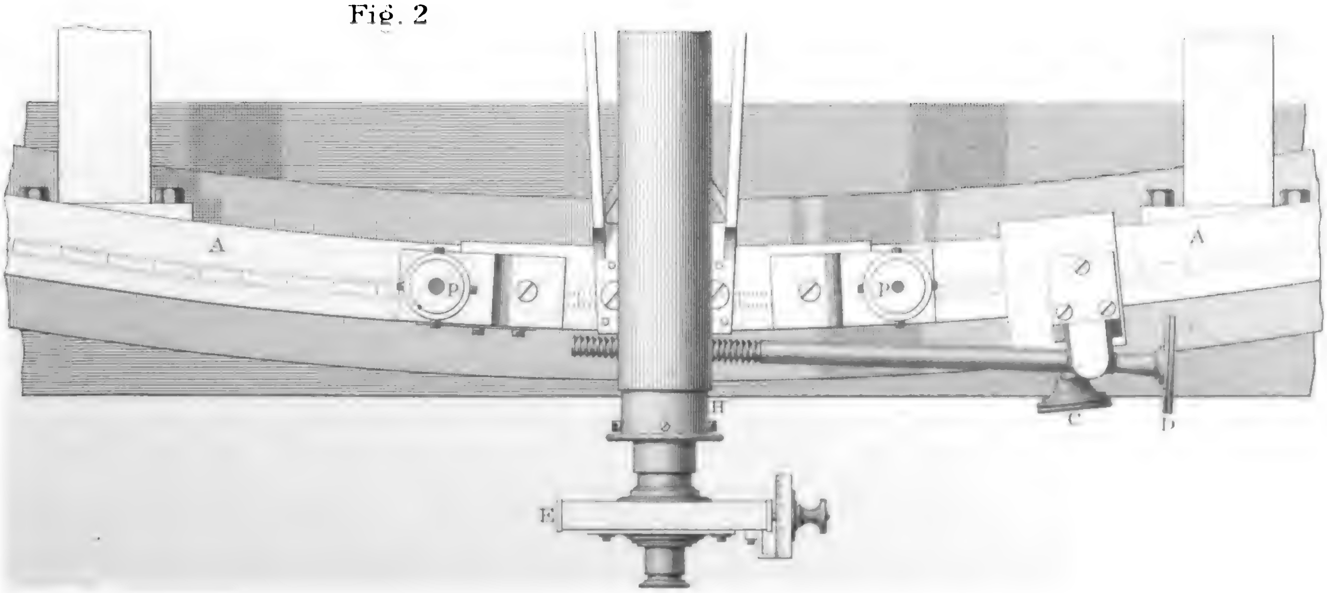


Fig. 2



0 Scale of Centimeters 10 20

Roberdeau Buchanan del

Julius Bien & Co. Lith

SUPPLEMENTARY MEASURES

OF THE

VELOCITIES OF WHITE AND COLORED LIGHT

IN

AIR, WATER, AND CARBON DISULPHIDE,

MADE WITH THE AID OF

THE BACHE FUND OF THE NATIONAL ACADEMY OF SCIENCES.

BY

ALBERT A. MICHELSON,

PROFESSOR, CASE INSTITUTE, CLEVELAND, OHIO.

TABLE OF CONTENTS.

INTRODUCTORY NOTE.

When the first results of my measures between Fort Myer and the Observatory were worked out it was found that they differed appreciably from those of Mr. MICHELSON obtained at the Naval Academy in 1879, and published in Vol. I, Part III, of these Papers. I therefore desired that Mr. MICHELSON (who had in the mean time become professor of physics at the CASE Institute, Cleveland, Ohio) should repeat his determination. Desiring that this determination should be completely independent of any work of my own, I made an application to the trustees of the BACHE Fund for a grant of the money necessary to enable Professor MICHELSON to perfect his apparatus and repeat his work. For the same reason no instructions or suggestions were sent him except such as related to the investigation of possible sources of error in the application of his method. The result of this repetition is discussed in the first of the following reports.

In view of the many questions which have been raised respecting possible deviations of the actual velocity of light under various conditions from those given by the usually accepted theory, it seemed advisable to make a quantitative determination of the velocity of differently colored rays of light through some refracting medium. This work Professor MICHELSON also undertook, with results which seem to show that the accepted theory is correct, with the exception of a probably appreciable difference, as deduced by Lord RAYLEIGH from theoretical considerations, between the group velocity of a system of waves and the ratio of wave length to wave time. The results are embodied in the second of the following reports.

SIMON NEWCOMB,
Sup'd't Nautical Almanac.

NAUTICAL ALMANAC OFFICE,
Navy Department, April, 1885.

FIRST REPORT.

SUPPLEMENTARY MEASURES OF THE VELOCITY OF LIGHT IN AIR AND WATER.

CLEVELAND, *January*, 1883.

SIR: In compliance with your desire I have made a redetermination of the Velocity of Light by the same method as that employed in my previous work.

In order to be able to measure the distance between the two stations without triangulation, the site selected was on a tract of land adjoining the New York, Chicago and Saint Louis Railway.

The building containing the instruments was situated on property belonging to the CASE Institute, and the building containing the fixed mirror was placed on a line parallel with the road and at a distance of 2,000 feet from the former. The measurement of this distance was performed by Professor EISENMANN and myself. His report is here inclosed.

The micrometer used to measure the deflection is the same as that used in previous experiments. It was, however, supported on a brick pier, according to your recommendation. The "radius" was measured by finding the distance from the surface of the mirror to the slit, and therefore the *sine* of the deflection was measured instead of the tangent. It was intended to measure this distance once for all, but it was found convenient to displace the revolving mirror in such a manner that the maximum of light should be reflected to the eye; and it was therefore necessary to measure this quantity every day.

This made the use of straight-edges placed end to end impracticable, and the same steel tape was used as in previous work. It was, however, supported the entire length by a heavy pine timber, so that there was no error either for "stretch" or for "catenary."

As in the previous work, the eye-piece of the micrometer was pointed at the revolving mirror when set at the expected greatest deflection The screw was horizontal, and the angle α is the inclination of the plane of rotation of the revolving mirror with the horizontal.* This angle was measured by reflecting the sunlight from the revolving mirror to the wall of the building, measuring the height in two positions, and dividing the difference by the distance between the two positions.

*A very small error is introduced on account of the angle which the light returning from the fixed mirror makes with the plane of rotation. If this angle is zero, then the angle through which the reflected ray is rotated is twice the angle of rotation of the mirror. If, however, the angle is β, then it can be shown that $\sin \frac{1}{2}x = \sin \theta \cos \beta$, where x and θ are the angles of rotation of reflected ray and mirror, respectively. In the actual case the value of $\cos \beta$ was 0.9999955, so that the error made by placing $\sin \frac{1}{2}x = \sin \theta$, or $x = 2\theta$, is less than five parts in a million.

The revolving mirror was the same as used in previous work, but it was furnished with new sockets, and a further change was made whereby the whole of the revolving part could be more readily removed. It was found to run smoothly, giving no audible sound.

The apparatus for regulating the blast of air was exactly the same as before, but it could not be said to work satisfactorily, as it required rather too close attention to seize upon the few seconds when the speed was exactly right.

The lens was the same one as was employed in previous work. The fixed mirror, however, was slightly concave, and had a diameter of 15 inches.

There was no change in the manner of measuring the speed, except that the standard fork used to determine the rate of the electric fork was an Vt_2 instead of Vt_3. The rate of this standard was found as follows:

The rate of the standard was approximately known from the maker, R **Koeing**, to be 128 v. s. at 68° F. (I use the notation v. s. = *vibrations per second.*

The fork was first compared with a fork actuated by electromagnets (which shall be designated *ef*). This last was weighted so as to give about one beat per second with the standard (*st*), so that *ef* made about 129 v. s.

The *ef* was then compared directly with the pendulum of an astronomical clock beating mean time seconds. The clock was so regulated as to gain less than half a second per day, and this small quantity was neglected.

The two forks were then again compared to make certain that the *ef* had not changed in the interval. The whole operation occupied about five minutes.

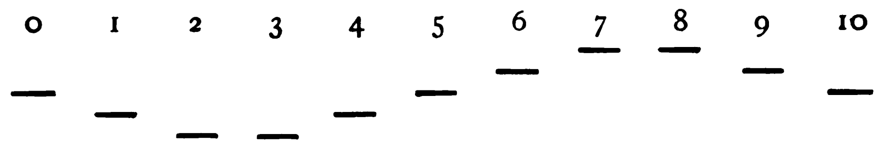
The comparison of the two forks requires no explanation. It may, however, be important to add that the relative position of the two forks was exactly the same as during the observations of the deflection; and, also, that usually these ratings were made just before or just after the observations: so that all possibility of a change of rate of the standard is excluded.

The comparison between the *ef* and the clock was made as follows:

A mirror was attached to one prong of the *ef*, and at some distance in front of this a Geissler tube was placed. This was illuminated by a spark from an induction coil whose primary circuit was broken by a relay, which in turn was worked by the break-circuit of the clock. Hence there was a flash in the tube once every second.

The tube was besides continuously illuminated by light from a window, and its image in the mirror, when the *ef* was vibrating, presented the appearance of a broad band with well defined edges. Against this the flash was projected; and since this last is almost instantaneous, its width would be that of the tube itself.

If, now, the *ef* made an exact whole number of vibrations per second, the flash would always find the fork in the same phase of vibration; consequently its image would always appear in the same position. If, however, the *ef* made, say, 129 1 v. s., then it would occupy successively the positions—



that is, there would be ten flashes between the positions 0 and 10.

Conversely, if ten flashes were counted in one period, the whole number 129 must be increased (or diminished) by $\frac{1}{10}$, and in general, if the number of flashes per period were a, then the *ef* makes $129 \pm \frac{1}{a}$ v. s.

Since, however, the alternate swings of the pendulum are unequal, only the even flashes were counted. Therefore, if the number of even flashes per period $= a$, and the rate of the $ef = n$, and the whole number nearest to $2n$ be denoted by $2N$, then $n = N \pm \frac{1}{2a}$

To determine which sign is to be used, the *ef* is weighted by a very small piece of wax.*

If a is greater than before, the sign is +; if a is less, the sign is —.

In the following table of results:

a = number of (even) flashes per period;
ef = rate of electric fork;
B = number of beats per second between *ef* and *st*.
st = rate of standard fork;
T = temperature, Fahrenheit.

Date.	*a*	*ef*	B	*st*	T
Oct. 16	+17.00	129.030	−1.050	127.980	73.5
16	+17.00	129.030	−1.038	127.992	73.5
18	+ 3.90	129.128	−1.038	128.090	60.0
18	+17.00	129.029	−0.952	128.077	62.2
19	+ 4.65	129.108	−0.996	128.112	56.3
19	+ 3.80	129.132	−1.000	128.132	54.0
20	+11.00	129.045	−0.943	128.102	58.0
20	+12.50	129.040	−0.935	128.105	58.0
21	+12.00	129.042	−0.982	128.060	63.0
21	+11.00	129.046	−0.985	128.061	63.0
21	+15.00	129.033	−0.983	128.050	64.5
24	+ 7.73	129.064	−0.953	128.111	57.0
24	+ 7.27	129.069	−0.952	128.117	56.0
25	+12.00	129.042	−0.953	128.089	59.0
31	− 5.30	128.906	−0.926	127.980	73.5

*During the observations of October 12, 13, 14, and 15 the standard fork had a light mirror attached to one prong. After the 15th this was removed.

The observations with the mirror (excepting the last three) were executed by comparison with an auxiliary pendulum. The results were not concordant.

The last of these experiments (mirror attached) was made immediately before the removal of the mirror, and while the other conditions were precisely the same an experiment was made without the mirror.

The result of the first was	127.888 v. s.
And the second	127.979 v. s.
Difference	0.091 v. s.

Therefore, if 0.091 be subtracted from the rate as deduced from the succeeding observations the result will give the rate of the standard *with mirror attached*.

These observations are represented by the crosses in the following diagram, abscissas representing temperatures, and ordinates excess over 128. The circles represent the unrecorded observations corrected for the effect of the attached mirror.

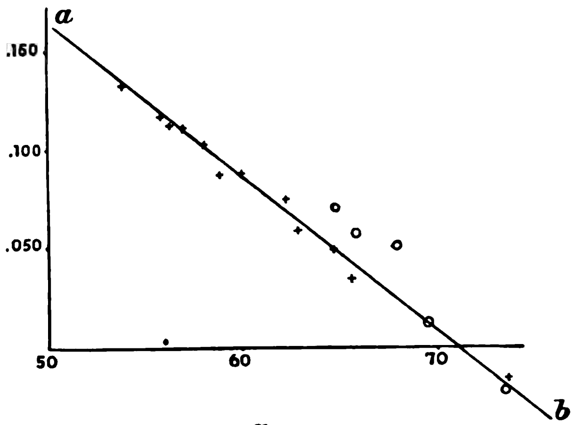


FIG. 1.

The straight line which most nearly represents these observations is $a\ b$. From it we find that the fork makes 128 v. s. at 71°.0 F., and its rate at any other temperature is found by subtracting 0.0079 for each degree above 71.

For the observations made while the mirror was attached to the standard the rate at 71° is 127.909 v. s.

Radius of deflection.—In the measurement of r a mark was scratched on the frame of the revolving mirror, at a distance of about one inch below the center of the mirror, and the distance from this mark to the surface of the mirror was found to be 0.0050 feet.

The division "30 ft." was placed over this mark. The other end (63 + ft.) was placed under the slit, the tape being about one inch below the center of the slit, and the divisions and tenths read off. To the reading thus obtained the distance 0.005 was added.

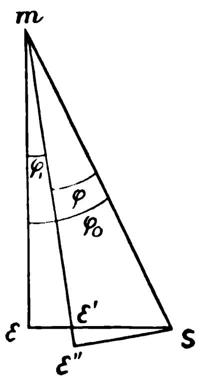


FIG. 2.

Since the eye-piece pointed directly at the mirror only when the deflection was 138^{mm}, a small correction should be applied for the other deflections. This is found as follows:

In the figure let—

m represent the revolving mirror;

s the slit;

e the crosshair of eye-piece for deflection of 138^{mm};

e' the crosshair of eye-piece for deflection d^{mm};

φ the required angular deflection;

φ_0 the maximum angular deflection, or that corresponding to 138^{mm};

φ_1 the difference between φ_0 and φ.

Then

$$\sin\varphi = \frac{e''s}{ms} = \frac{e's\cos\varphi_1}{ms}$$

The values of r and d, given in the table of results, are simply the readings of the tape and the micrometer respectively, without correction. In order to find the angular deflection it is necessary to find the ratio between a foot of the tape and a turn of the screw. This can be deduced from previous observations, but it was thought best to make a new determination. The part of the tape measured was that actually used, namely, from 30 ft. to 63.35 ft.

At 18° C.	33.350	(steel) ft.	=	10161.12 (brass) mm.
At 18° C.	140.0161	turns of screw	=	140 divs. of brass micr. scale.
At 18° C.	140.075	divs. of brass scale	=	139.518 (brass) mm.

From these data we obtain

At 18° C. 1 (steel) foot $= \frac{1}{0.00328212}$ (brass) mm.

At 18° C. 1 turn of steel screw $= 0.995907$ (brass) mm.

Whence correct factor $= 0.00326869$ (for 140 divs.)

To find the factor for any number of divisions or turns, the value of a turn of the screw for the different divisions must be known, and this is given on page 145 of previous work.

This for 140 turns is 0.996307.

If the value for any other number of turns be denoted by T, then the factor $= \frac{0.00326869\,\mathrm{T}}{0.996307}$, or $\mathrm{F} = 0.00328081 \times \mathrm{T}$

The possible error in this determination is 1 in 20,000.

The distance between the marks on the two piers, from Mr. EISENMANN's measurements	= 2049.532 feet.
Distance from cross to mirror, West pier	= −0.225
Distance from cross to mirror, East pier	= +0.048
Distance between mirrors	= 2049.355 feet.
2D	= 4098.71

FORMULÆ OF REDUCTION.

Let $\mathrm{V} =$ velocity of light in air in kilometers per second.

$\varphi =$ angular deflection in seconds.

$\varphi_1 =$ difference between φ_0 and φ (approximate).

$d =$ deflection as read from micrometer.

$d_1 =$ true deflection (in feet of the steel tape).

$r =$ radius (in feet of the steel tape).

$\mathrm{T} =$ value of one turn of screw.

$\mathrm{F} = 0.00328081\ \mathrm{T}$.

$\alpha =$ inclination of plane of rotation with horizontal.

$\mathrm{D} =$ twice the distance between mirrors in feet.

$n =$ number of rotations per second.

Then

$$\sin \varphi = \frac{d_1}{r} = \frac{d\mathrm{F} \sec \alpha \cos \varphi_1}{r}$$

$$\mathrm{V} = \frac{2592000\ \mathrm{D}n}{3280.87\ \varphi}$$

or, putting $\mathrm{F} \sec \alpha \cos \varphi_1 = a$, and $\frac{2592000\ \mathrm{D}}{3280.87} = c$

$$\sin \varphi = a\frac{d}{r}$$

$$\mathrm{V} = c\frac{n}{\varphi}$$

The headings of the columns in the following table of results signify as follows:

$t =$ temperature, Fahrenheit.
B $=$ number of beats of *st* with *ef* per second.
$c =$ correction of *st* for temperature.
$ef =$ rate of "electric" fork.
$n =$ number of turns of revolving mirror per second.
$m =$ micrometer reading of deflected image.
$z =$ micrometer reading of slit.
$d = m - z$.
$\Delta =$ difference between greatest and least values of d.
$e =$ mean error of one determination of d.
T $=$ value of one turn of micrometer screw in mm.
$r =$ radius.
$\varphi =$ deflection in seconds.
$\varphi_0 =$ angular deflections corresponding to $d = 138^{mm}$.
$\varphi_1 = \varphi_0 - \varphi$.
V $=$ velocity of light in kilometers per second, in air.
S $=$ source of light ($s =$ sun, $e =$ electric light).
$no =$ number of observations.
$v =$ distinctness of image (poor $= 1$, fair $= 2$, good $= 3$).
$w =$ weight of the set of observations.
$l =$ logarithm.

Date.	t	B	c	ef	n	m	z	d	Δ	c	T	φ_1	la
Oct. 12	75.0	1.250	—.032	129.127	258.254	138.182	.262	137.920	.15	.038	.99629	0	.5144372
12	75.0	1.333	—.032	129.010	257.871	138.000	.258	137.742	. .	. .	.99629	0	.5144372
12	75.0	1.333	—.032	129.010	258.754	138.500	.267	138.233	. .	. .	.99629	0	.5144372
14	71.0	1.198	.000	129.107	258.214	138.009	.076	137.933	.27	.060	.99629	0	.5144372
16	73.2	1.038	—.017	129.021	258.042	137.927	.027	137.900	.21	.045	.99629	0	.5144372
18	61.5	0.954	+.075	129.029	258.058	137.977	.060	137.917	.19	.049	.99629	0	.5144372
19	56.0	0.988	+.118	129.106	258.212	138.100	.063	138.037	.17	.040	.99629	0	.5144372
19	54.7	1.000	+.129	129.129	258.258	138.130	.063	138.067	.17	.070	.99629	0	.5144372
20	58.0	0.938	+.103	129.041	258.082	137.831	.057	137.774	.25	.056	.99629	0	.5144372
21	64.3	0.983	+.053	129.036	258.072	137.941	.054	137.887	.20	.033	.99629	0	.5144372
24	56.8	0.952	+.112	129.064	258.128	138.068	.058	138.010	.25	.090	.99629	0	.5144372
25	59.0	0.952	+.095	129.047	258.094	137.957	.060	137.897	.09	.032	.99629	0	.5144372
25	59.0	0.952	+.095	129.047	258.094	137.965	.060	137.905	.26	.077	.99629	0	.5144372
26	59.0	0.944	+.095	129.039	258.078	137.931	.058	137.873	.35	.102	.99629	0	.5144372
31	73.0	0.923	—.016	128.907	257.814	137.819	.065	137.754	.12	.035	.99629	0	.5144372
31	73.0	0.923	—.016	128.907	257.814	137.852	.065	137.787	.22	.066	.99629	0	.5144372
Nov. 4	53.0	0.947	+.142	129.089	193.634	103.632	.060	103.572	.20	.055	.99603	11′.6	.5143215
8	56.0	0.936	+.118	129.054	193.581	103.532	.062	103.470	.12	.036	.99603	11′.6	.5143215
8	56.0	0.936	+.118	129.054	193.581	103.534	.062	103.472	.11	.027	.99603	11′.6	.5143215
11	70.5	0.923	+.004	128.927	193.390	103.421	.069	103.352	.09	.027	.99603	11′.6	.5143215
11	70.5	0.923	+.004	128.927	128.927	68.976	.069	68.907	.10	.036	.99585	23′.2	.5142354
14	40.5	0.955	+.241	129.196	129.196	69.115	.045	69.070	.07	.024	.99585	23′.2	.5142354
14	40.5	0.955	+.241	1-9.196	129.196	69.136	.045	69.091	.11	.036	.99585	23′.2	.5142354

Date.	*r*	*ln*	*lr*	*ld*	$l \sin \varphi$	φ	$l\varphi$	*l*V	V	S	*no*	*v*	*w*
Oct. 12	33.350	.4120470	.5230958	.1396272	.1309787	2788.7	.4454018	.4769369	299883	*s*	40	3	7
12	33.350	.4414025	.5230958	.1390664	.1304079	2785.1	.4448408	.4768536	299816	*s*	.	3	5
12	33.350	.4128871	.5230958	.1406117	.1319532	2795.0	.4463818	.4767971	299778	*s*	.	3	5
14	33.350	.4119798	.5230958	.1396681	.1310196	2789.0	.4454485	.4768231	299796	*s*	56	2	3
16	33.351	.4116904	.5231089	.1395643	.1308927	2788.2	.4453239	.4766583	299682	*e*	25	2	5
18	33.356	.4117173	.5231740	.1396178	.1308811	2788.1	.4453083	.4766008	299711	*e*	65	3	4
19	33.354	.4119765	.5231479	.1399956	.1312850	2790.7	.4457132	.4765552	299611	*s*	19	3	6
19	33.356	.4120537	.5231479	.1400899	.1313793	2791.3	.4458065	.4765390	299599	*e*	10	3	2
20	33.355	.4117578	.5231609	.1391421	.1304185	2785.2	.4458564	.4771933	300051	*s*	22	3	3
21	33.355	.4117409	.5231609	.1395234	.1307998	2787.6	.4452305	.4768023	299781	*s*	68	2	9
24	33.355	.4118351	.5231609	.1399106	.1311870	2790.1	.4456198	.4765072	299578	*s*	20	1	1
25	33.356	.4117779	.5231740	.1395549	.1308182	2787.7	.4452460	.4768237	299796	*s*	10	3	10
25	33.356	.4117779	.5231740	.1395800	.1308433	2787.9	.4452772	.4767926	299774	*e*	30	2	2
26	33.355	.4117510	.5231609	.1394792	1307556	2787.3	.4451837	.4768591	299820	*s*	10	1	1
31	33.355	.4113065	.5231609	.1391042	.1303806	2784.9	.4448096	4767888	299772	*s*	15	3	8
31	33.355	.4113065	.5231609	.1392073	.1304837	2785.6	.4449188	4766796	299696	*e*	11	2	2
Nov. 4	33.360	.2869816	.5232260	.0152424	.0063379	2093.0	.3207692	4765040	299573	*s*	30	3	2
8	33.357	.2868627	.5231870	.0148144	.0059489	2091.2	.3203956	.4767588	299748	*s*	20	3	6
8	33.357	.2868627	.5231870	.0148228	.0059573	2091.2	.3203956	.4767588	299748	*e*	46	3	10
11	33.357	.2864340	.5231870	.0143189	.0054534	2088.8	.3198969	.4768288	299797	*e*	20	3	10
11	33.357	.1103439	.5231870	.8382633	.8293119	1392.3	.1437328	.4769030	299851	*e*	20	3	6
14	33.362	.1112491	.5232521	.8392895	.8302728	1395.4	.1446987	.4768422	299809	*s*	6	2	7
14	33.362	.1112491	.5232521	.8394215	.8304048	1395.8	.1448232	.4767177	299723	*e*	20	2	4

The second and third sets of observations were made by setting the micrometer, keeping the image as nearly as possible on the cross-hair, counting meanwhile the number of oscillations of the image of revolving mirror. These observations cannot therefore be weighted in the same manner as the others. The weight given, 5, is only an estimate. The weights in the other observations are deduced from the formula $w = \frac{1}{E^2}$.

The weighted mean from these observations is 299771 kilometers per second, and the probable error $\pm$ 12 kilometers.

The various sources of error in these measurements are discussed in the previous work, and supposing that these errors all affect the result in the same manner, the total error of the final result is found to be less than 60 kilometers.

Value from table - - - - - - - -	299771
Reduction to vacuum - - - - - -	+ 82
Final result - - - - - - - - -	299853 $\pm$ 60 kilometers.

In the previous work two errors were committed: 1st, in neglecting to make allowance for the fact that in measuring *r* the hypothenuse of a triangle was measured instead of the base; 2d, the correction on page 141 for φ should be omitted.

The correction for these two errors reduces the result by 34 kilometers, making

Previous result - - - - -	299910 $\pm$ 50
Present result - - - - - - -	299853 $\pm$ 60

In their work on "Velocity of White and of Colored Light,"* Professors YOUNG and FORBES remark:

"In MICHELSON'S observations the image of the slit was described as indistinct and covering a sensible space. From our results it would appear that the width of his spectrum between mean red and blue would be about 2 millimeters. But it would be a very impure spectrum, and it is only by employing absorptive media, or part of a pure spectrum, to give color to the light used, that we should expect him to detect the difference."

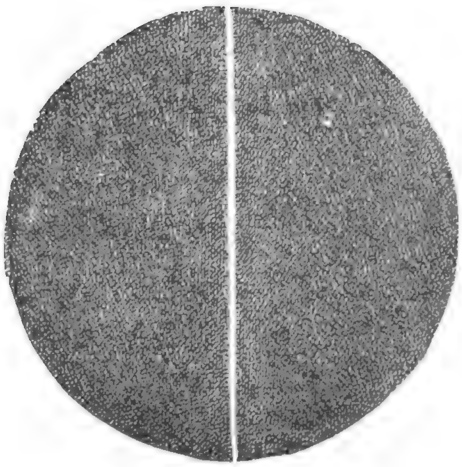


FIG. 3.

The accompanying figure represents the image as seen in the eyepiece, magnified about five times. This, as can be testified by others, is a fairly correct representation of the image under the most favorable circumstances. As a matter of fact the image was, in these exceptional cases, better than in the previous work. The color of the central portion was yellowish, and *both* borders were occasionally pale violet. The width of this image was, in one case, found by actual measurement to be 0.25^{mm} (the actual width of the slit was 0.19^{mm}.)

If now there were such a difference of velocity between the red rays and the blue, the image, drawn to the same scale as in the figure, would have presented a spectrum covering about 10^{mm}.

Again, a table is presented, intended to show that the velocity, as deduced from CORNU'S experiments, increases with the refrangibility of the light used; and another tending toward the same result, consisting of a comparison of the results of CORNU, YOUNG and FORBES, and MICHELSON. Thus—

MICHELSON. Sun near horizon - - - - - - - - -	299940
CORNU. Lime light - - - - - - - - - - - - - -	300400
YOUNG and FORBES. Electric light - - - - - - - -	301382
But from an inspection of the table of my present results it will be seen that the weighted mean of the 318 sun observations is - - - - - - - - - - - -	299865
And of the 267 electric light observations - - - - -	299835

So that the difference, if any there be, is in the opposite direction.

Finally an experiment was made in which a red glass covered one-half the slit. The two halves of the image—the upper white, the lower red, were exactly in line.

* Phil. Transactions of the Royal Society, Part I. 1882.

In conclusion, I would take this opportunity of expressing my obligation to Mr. H. G. Abbey for his valuable assistance in hastening the installation of the apparatus, to several of the students of the Case Institute for their cheerful aid in carrying out the work, and to the Brush Electric Light Company for the loan of their dynamo and light.

Very respectfully, yours,

ALBERT A. MICHELSON.

Prof Simon Newcomb, U. S. N.,
Superintendent Nautical Almanac, Washington, D. C.

POSTSCRIPT.

August 15, 1883.

Sir: In accordance with your suggestion, I have repeated Foucault's experiment for testing the "undulatory theory." The arrangement of apparatus employed was essentially the same as that of Foucault, and therefore needs no detailed description. The liquid used was distilled water, contained in a tube 10.03 feet long. The distance between mirrors was 17.63 feet, and the "radius" was 32.41 feet. Speed of rotation 256 turns per second, except in one experiment (No. 2), in which it was 307 turns per second.

The results obtained confirm Foucault's work, which showed that the velocity of light in water was less than in air. But Foucault made no attempt to obtain quantitative results, whereas the table below shows not merely that the velocity in water is less than in air, but that the ratio between these velocities is equal to the index of refraction of the water.

This for yellow light at ordinary temperature is given as 1.333.

Following is the ratio obtained for $\frac{v}{v_1}$ in six independent experiments:

No. 1.	1.33
No. 2.	1.33
No. 3.	1.34
No. 4.	1.33
No. 5.	1.35
No. 6.	1.30
Mean,	1.330

The error 0.003 is within the limits of errors of experiment.

Very respectfully,

ALBERT A. MICHELSON.

Prof Simon Newcomb,
Superintendent Nautical Almanac.

Prof. A. A. MICHELSON:

DEAR SIR: Appended hereto please find the report of the measurement and reduction of the base used in connection with your Velocity of Light experiments.

The base measured connects the east and west piers. The west pier supports the revolving mirror in your observatory on the CASE Institute grounds, and is located 52 feet north of the nearest rail of the New York Central and Saint Louis railroad track. The east pier supports the fixed mirror; it is approximately 2,050 feet to the east of the west pier and 52 feet north of aforesaid rail.

The line was measured on nearest rail, and afterward transferred to the piers with a theodolite.

The portion of track over which line was measured is straight, has but a slight grade, and lies in a cut of four feet average depth.

The tape used in measuring was the Mississippi River Commission's 300-foot steel tape, U. S. Engineers' No. I, obtained through the courtesy of First Lieut. SMITH S. LEACH, U. S. Engineers, secretary of the Commission.

The following coefficients and constants accompanied the tape and are compiled from the Mississippi River Commission's Office Report $\frac{\text{A. m.,}}{408}$ viz:

"The tape is standard at $36^\circ.75$ F. This value is for tape when supported at intervals of 25 feet, loaded 12 pounds, and without any reduction for catenary, *i. e.*, the indicated distance is the true distance under these conditions."

"Corrections for catenary for 300-foot tape when tension is 12 pounds and length of span between supporting stakes is 25 feet equals 0.8392 millimeter.

"Coefficient of expansion $= 0.00000694 \pm 5$ (in last place).

"Expansion of 299 feet for 1° F. $= 0^{mm}.633 \pm 5$ (in last place).

"Modulus of elasticity $= 27400000 \pm 200{,}000$.

"Extension of tape for each pound of tension $= 1.641$ millimeter."

This tape was originally 300 feet long, graduated at every 10 feet up to the 290-foot mark; from 290 to 299 feet it was graduated into 1-foot intervals, and from 299 to 300 into tenths of feet. At present, it is broken off at the 299.2 mark. Therefore, but 299 feet are used during measurements.

Before beginning the measurements, a × was made on the outside of the upper flange of the nearest rail opposite the west pier. As a further check and to prevent accidental disturbances which might arise from passing trains, a reference stake, 18″ long and 2″ by 2″ at top, was driven flush with ground at 30″ from rail.

A copper tack with (cross) × in its head was then set on perpendicular erected from × on rail. This operation was repeated at the east end of line, after which the measurement was made as follows:

The tape was carefully laid on the top of rail, its rear end, or zero mark, was made to coincide with intersection of × opposite east pier, and then firmly held or clamped. A tension of 12 pounds was then applied at forward end, and on exchange of signals between the observers, stationed at the ends, a × was made on the rail with the sharp blade of a penknife. A thermometer reading was taken simultaneously with the making of the × at the forward end. Signals were again exchanged and tape taken up, carried forward supported at 50-foot intervals to prevent dragging, and again placed on the rail, the operations being repeated as before.

The thermometer from which readings were taken was an ordinary Fahrenheit thermometer graduated into single degrees. Is was first wound with wire and then lashed to forward end of tape.

Two measurements were made: the first from east to west, and second from west to east.

The following gives the notes in tabulated form:

N. B.—It will be observed that owing to the lateness of the hour of measurement, the sun had reached an elevation which threw the embankment's shadow over entire length of line.

Observations.—Cleveland, August 18, 1882.

[First measurement from east to west.]

No. of tape.	Rear end.	Forward end.	Temp. F.	Time.
			°	h. m.
1	0	299	70.0	6 36 p. m.
2	0	299	65.0	42
3	0	299	62.0	50
4	0	299	59.5	55
5	0	299	60.0	7 00
6	0	299	59.0	05
7	0	261′ 0⅝″	58.0	10

[Second measurement from west to east.]

1	0	299	58.0	7 17
2	0	299	56.0	22
3	0	299	56.0	27
4	0	299	54.0	32
5	0	299	54.0	40
6	0	299	55.0	47
7	0	261′ $\frac{17}{32}$″	54.0	57

This gives value of first measurement:

2055.0521 feet, with mean temp. of 61°.93 F.

And of second measurement:

2055.1276 feet, with mean temp. of 55°.29 F.

There was a change of grade opposite to the end of the 4th tape from east end.

		°	′	″
The grade angle for this portion, *i. e.* 1196 feet, was - - - - -	+	0	18	35
And for remaining 289 + feet - - - - - - - - - - - - - -	+	0	09	05

both being measured from the end stations.

The transfer of measurement from rail to piers was made by setting a theodolite over cross on rail and turning off an angle of 90°—three direct and three reverse readings of both verniers being taken—then measuring from the intersection of this line and the base line prolonged to the × on the stone cap of the piers.

The following results were thus obtained:

The × on stone cap of east pier is 0.5596^{m} east or beyond × on rail opposite that end; *i. e.*, the correction is plus.

The × on stone cap of west pier is 2.3502^{m} east of the × on rail at that end; *i. e.*, correction is minus

The angle of depression from west pier to east pier is 00° 08′ 25″.

REDUCTIONS.

Our measurement was made with tape laid on top of rail, therefore having no catenary. Since the tape is standard at 36°.75 F. under the conditions of 12 pounds tension and 25 feet supports, we must find correction of tape for catenary in terms of degrees Fahrenheit as follows:

	mm
Contraction of tape due to catenary with 25 feet supports is - - - - -	0.8392
Expansion of tape due to 1° F. - - - - - - - - - - - - - - - - - -	0.633

hence, the equivalent in terms of temperature is $\frac{0.8392}{0.633} = 1°.32$, which correction is minus.

Therefore $36^{\circ}.75 - 1^{\circ}.32 = 35^{\circ}.43$ F. = temperature at which tape is standard under a 12-pound tension, and supported uniformly throughout, as in our measurement.

Mean temp., 1st meas.	$= 61^{\circ}.93$	and of 2d.	$55^{\circ}.29$
Tape is standard at	$= 35.43$		35.43
Temp. correction 1st meas.	26.50	" " "	19.86

				log's.
Coef. of expan.	=	0.00000694	=	4.8413595
Temp. cor. 1st meas.	=	$26^{\circ}.50$	=	1.4232459
Measured length	=	2055.0521	=	3.3128228
Correction to 1st meas.	=	0.37795	=	9.5774282

				log.
Coef. of expansion	=	0.00000694	=	4.8413595
Temp. cor. 2d meas.	=	$19^{\circ}.86$	=	1.2979792
Measured length	=	2055.1276	=	3.3128389
Cor. to 2d meas.	=	0.28325	=	9.4521776

1st meas.	2d meas.
2055.0521	2055.1276
0.3779	.2832
2055.4300	2055.4108
2055.4108	

2055.4204 = mean of two measurements, reduced for temperature **only.**

Grade correction:

		log.	
1st section	1196 =	3.0777312	
	$\cos 0^{\circ}\ 18'\ 35'' =$	9.9999936	
	=	3.0777248	= 1195.982
2d "	859.4204 =	2.9342057	
	$\cos 0^{\circ}\ 09'\ 05'' =$	9.9999985	
		2.9342042	= 289.418

Measured length cor. for temp. and grade = 2055.400

Transferring measurements to piers, we have the following corrections:

	m.
Correction at east end	= + 0.5596
" " west "	= − 2.3502
Total	= − 1.7906

Reducing this value in meters, using the Clarke value of 39.370423 +, we have:

Log − 1.7906 =	0.2529986	
" common factor =	0.5159890	
	0.7689876	= − 5.8747

Horizontal distance between ×'s on stone caps of piers = 2055.400 − 5.8747 = 2049.5253

Log 2049.5253 =	3.3116533	
$\cos 0^{\circ}\ 08'\ 25'' =$	9.9999987	
	3.3116546	= 2049.532

Which is the slant distance in feet from × on west pier to × on east pier.

Very respectfully,

JOHN EISENMANN.

Case School of Applied Science,
Cleveland, October 14, 1882.

SECOND REPORT.

DETERMINATION OF THE VELOCITY OF LIGHT, AND OF THE DIFFERENCE OF VELOCITIES OF RED AND BLUE LIGHT, IN CARBON DISULPHIDE.

PART I.—*Velocity of white light.*

The requisite apparatus for these experiments was established in the basement of the Chemical Laboratory of the CASE Institute, and the blast of air which actuated the revolving mirror was supplied by a three-inch tin pipe leading from a ROOT blower connected with a gas engine in the basement of the main building.

The blast was regulated in the same way as in the previous experiments, and the whole arrangement of apparatus was essentially the same as in the previous work, so that it would not merit detailed description.

It was surmised that the chief difficulty would be the want of transparency of the liquid in a column of sufficient length to produce the required "deflection." It was found, however, at least in the first part of the work, that there was an abundance of light, using a column of the liquid ten feet long.

The sharpness of the image, however, was anything but satisfactory, and this made the second part of the problem at first seem utterly hopeless. It was found, however, that by sacrificing light by limiting the aperture of the tube by a rectangular opening, the sharpness of the image was decidedly increased. The attempt was made to keep the liquid in violent agitation, but with very unsatisfactory results, and it was finally decided to allow it to remain as quiet as possible, when it arranged itself in horizontal layers of different densities. In this condition, by using the central layers, an image was obtained which gave fairly consistent results.

In the following observations—

$r =$ "radius," or distance from micrometer to revolving mirror.
$a =$ length of air column between mirrors.
$b =$ length of liquid column between mirrors $= 3.07$ meters.
$d =$ linear displacement of image.
$m =$ number of turns per second.
$n =$ ratio of velocity of light in liquid to that in air, which last may be taken at $V = 300\ 000\ 000$ meters.
$M = 1\ 000\ 000$.
$Z =$ reading of micrometer for undeflected image.
$D =$ reading of micrometer for deflected image.

OBSERVATIONS.

No. 1.		No. 2.		No. 3.		No. 4.	
Z	D	Z	D	Z	D	Z	D
115.200	113.961	115.165	113.912	115.160	113.900	115.152	114.510
170	968	164	908	170	895	..	510
200	960	162	910	143	895	..	500
174	965	168	900	152	893	..	495
196	962	153	914	152	902	..	508
182	968	150	900	160	898	..	504
187	962	158	920	146	899	..	512
184	950	115.160	113.909	151	895	..	510
190	948	113.909		159	892	..	485
200	970	$d = 1.251$		150	890	..	500
115.188	113.961	$r = 6.336$		151	113.898	114.503	114.503
113.961		$a = 3.61$		146		$d = .649$	
$d = 1.227$		$m = 256$		160		$r \doteq 6.336$	
$r = 6.336$				152		$a = 3.69$	
$a = 3.61$				146		$m = 128$	
$m = 256$				115.152			
				113.898			
				$d = 1.254$			
				$r = 6.336$			
				$a = 3.69$			
				$m = 256$			

No. 5.		No. 6.		No. 7.		No. 8.	
Z	D	Z	D	Z	D	Z	D
115.155	113.926	115.155	114.270	115.155	114.556	115.180	113.930
..	930	..	261	..	568	140	925
..	925	..	266	..	561	152	920
..	926	..	270	..	580	160	918
..	934	..	280	..	570	160	924
..	933	..	266	..	570	155	925
..	922	..	250	114.568	572	150	923
113.928	930	114.266	262	$d = .587$	114.568	145	928
$d = 1.227$	113.928	$d = .889$	114.266	$r = 6.336$		150	943
$r = 6.336$		$r = 6.336$		$a = 3.69$		168	921
$a = 3.69$		$a = 3.69$		$m = 128$		156	113.926
$m = 256$		$m = 192$				115.162	
						113.926	
						$d = 1.236$	
						$r = 6.336$	
						$a = 3.69$	
						$m = 256$	

OBSERVATIONS—Continued.

No. 9.		No. 10.		No. 11.		No. 12.	
Z	D	Z	D	Z	D	Z	D
115.162	114.240	115.162	113.620	115.152	113.924	115.159	114.232
. .	240	. .	620	182	940	. .	232
. .	250	. .	610	152	924	. .	222
. .	236	. .	624	155	934	. .	233
. .	236	. .	610	134	945	. .	220
. .	248	. .	620	160	925	. .	218
. .	250	. .	625	150	923	. .	232
. .	228	. .	610	115.159	923	. .	222
. .	240	. .	622	113.928	922	. .	226
114.240	230	113.618	616	$d = 1.231$	925	114.226	227
$d = .922$	114.240	$d = 1.544$	113.618	$r = 6.38$	113.928	$d = .923$	114.226
$r = 6.336$		$r = 6.336$		$a = 3.66$		$r = 6.38$	
$a = 3.69$		$a = 3.69$		$m = 256$		$a = 3.66$	
$m = 192$		$m = 320$				$m = 192$	

No. 13.		No. 14.		No. 15.		No. 16.	
Z	D	Z	D	Z	D	Z	D
115.159	114.376	115.159	114.490	115.155	114.284	115.166	114.318
. .	370	. .	510	164	320	. .	326
. .	380	. .	508	170	320	. .	324
. .	362	. .	520	176	302	. .	326
. .	375	. .	512	166	321	. .	334
. .	363	. .	506	167	327	. .	336
. .	366	. .	524	163	320	. .	310
. .	366	. .	512	166	314	. .	316
114.370	373	. .	522	156	326	. .	328
$d = .789$	114.370	114.512	519	175	326	114.324	321
$r = 6.38$		$d = .647$	114.512	115.166	114.316	$d = .842$	114.324
$a = 3.66$		$r = 6.38$		114.316		$r = 3.45$	
$m = 160$		$a = 3.66$		$d = .850$		$a = 3.64$	
		$m = 128$		$r = 3.45$		$m = 320$	
				$a = 3.64$			
				$m = 320$			

OBSERVATIONS—Continued.

No. 17.		No. 18.		No. 19.		No. 20.	
Z	D	Z	D	Z	D	Z	D
115.166	114.496	115.166	114.510	115.166	114.488	115.166	114.503
. .	496	. .	498	. .	504	. .	507
. .	496	. .	520	. .	518	. .	507
. .	496	. .	505	. .	495	. .	507
. .	495	. .	497	. .	496	. .	505
. .	480	. .	504	. .	496	. .	515
. .	490	. .	506	. .	500	. .	516
. .	480	. .	490	. .	503	. .	497
. .	476	. .	503	. .	506	. .	505
114.499	483	114.503	497	114.502	510	114.507	512
$d=$ .667	114.499	$d=$ .663	114.503	$d=$ .664	114.502	$d=$ .659	114.507
$r=3.45$		$r=3.45$		$r=3.45$		$r=3.45$	
$a=3.64$		$a=3.64$		$a=3.64$		$a=3.64$	
$m=256$		$m=256$		$m=256$		$m=256$	

No. 21.		No. 22.		No. 23.		No. 24.	
Z	D	Z	D	Z	D	Z	D
115.166	114.498	115.166	114.494	115.166	114.490	115.166	114.486
. .	490	. .	500	. .	489	. .	486
. .	493	. .	488	. .	486	. .	482
. .	493	. .	486	. .	480	. .	488
. .	510	. .	490	. .	485	. .	478
. .	507	. .	488	. .	492	. .	482
. .	499	. .	488	. .	480	. .	486
. .	507	. .	500	. .	497	. .	496
. .	492	. .	486	. .	490	. .	500
114.498	496	114.492	496	114.490	510	114.486	482
$d=$ .668	114.498	$d=$ .674	114.492	$d=$ .676	114.490	$d=$ .680	114.486
$r=3.45$		$r=3.45$		$r=3.45$		$r=3.45$	
$a=3.64$		$a=3.64$		$a=3.64$		$a=3.64$	
$m=256$		$m=256$		$m=256$		$m=256$	

OBSERVATIONS—Continued.

No. 25.		No. 26.		No. 27.		No. 28.	
Z	D	Z	D	Z	D	Z	D
115.176	114.508	115.170	114.524	115.165	114.350	115.165	114.341
168	516	164	541	. .	368	. .	348
172	508	158	510	. .	371	. .	360
153	505	170	523	. .	370	. .	350
161	520	170	520	. .	354	. .	350
167	511	158	526	. .	359	. .	345
170	521	171	510	. .	360	. .	362
170	510	163	521	. .	360	. .	350
158	519	161	523	. .	354	. .	352
162	499	161	530	114.360	359	114.351	351
115.167	114.512	115.165	114.523	$d = .805$	114.360	$d = .814$	114.351
114.512		114.523		$r = 3.39$		$r = 3.39$	
$d = .655$		$d = .642$		$a = 3.68$		$a = 3.68$	
$r = 3.39$		$r = 3.39$		$m = 320$		$m = 320$	
$a = 3.68$		$a = 3.68$					
$m = 256$		$m = 256$					

No. 29.		No. 30.		No. 31.		No. 32.	
Z	D	Z	D	Z	D	Z	D
115.210	114.378	115.190	114.384	115.176	114.515	115.187	114.356
187	366	. .	386	188	514	. .	374
176	367	. .	397	204	517	. .	360
188	372	. .	370	189	511	. .	371
192	366	. .	382	180	513	. .	370
195	363	. .	380	183	526	. .	362
195	362	. .	385	200	518	. .	369
196	363	. .	388	182	515	. .	358
184	370	. .	386	184	516	. .	351
178	114.367	114.384	384	187	516	114.362	354
115.190		$d = .816$	114.384	115.187	114.516	$d = .825$	114.362
114.367		$r = 3.39$		114.516		$r = 3.39$	
$d = .823$		$a = 3.68$		$d = .671$		$a = 3.56$	
$r = 3.39$		$m = 320$		$r = 3.39$		$m = 320$	
$a = 3.68$				$a = 3.56$			
$m = 320$				$m = 256$			

OBSERVATIONS—Continued.

No. 33.		No. 34.		No. 35.		No. 36.	
Z	D	Z	D	Z	D	Z	D
115.187	114.356	115.206	114.435	115 200	114.192	115.200	113.984
. .	366	221	426	. .	200	. .	974
. .	364	215	432	. .	184	. .	976
. .	354	194	456	. .	189	. .	982
. .	370	200	434	. .	191	. .	963
. .	360	186	435	. .	206	. .	970
. .	366	213	440	. .	186	. .	973
. .	362	192	436	. .	190	. .	980
. .	371	184	437	. .	190	. .	973
114.363	361	191	441	114.191	180	113.975	979
$d=$ .824	114.363	115.200	114.437	$d=1.009$	114.191	$d=1.225$	113.975
$r=3.39$		114.437		$r=5.14$		$r=5.14$	
$a=3.56$		$d=$ .763		$a=3.56$		$a=3.56$	
$m=320$		$r=5.14$		$m=256$		$m=320$	
		$a=3.56$					
		$m=192$					

The time t occupied by the light in traversing the distance between the mirrors is

$$t=\frac{a+bn}{V}-\frac{d}{8\pi rm}$$

Whence

$$n=\frac{\frac{Vd}{8\pi rm}-a}{b}$$

The following table gives the data and calculations. The headings of the columns have the same signification as already assigned, P. (3):

	r	a	m	Md	*log* Md	*log* m	*log* r	$\log \frac{Vd}{8\pi mr}$	No	No — a	n
1	6.336	3.61	256	1227	3.08884	2.40824	0.80182	0.95566	9.03	5.42	1.77
2	6.336	3.61	256	1251	3.09726	2.40824	0.80182	0.97406	9.20	5.69	1.85
3	6.336	3.69	256	1256	3.09899	2.40824	0.80182	0.96581	9.24	5.55	1.81
4	6.336	3.69	128	649	2.81224	2.10721	0.80182	0.98009	9.55	5.86	1.91
5	6.336	3.69	256	1227	3.08884	2.40824	0.80182	0.95566	9.03	5.34	1.74
6	6.336	3.69	192	889	2.94890	2.28330	0.80182	0.94066	8.72	5.03	1.63
7	6.336	3.69	128	587	2.76864	2.10721	0.80182	0.93649	8.64	4.95	1.61
8	6.336	3.69	256	1236	3.09202	2.40824	0.80182	0.95884	9.09	5.40	1.76
9	6.336	3.69	192	922	2.96473	2.28330	0.80182	0.95649	9.05	5.36	1.75
10	6.336	3.69	320	1544	3.18865	2.50515	0.80182	0.95856	9.09	5.40	1.76
11	6.38	3.66	256	1231	3.09026	2.40824	0.80482	0.95408	9.00	5.34	1.74
12	6.38	3.66	192	923	2.96520	2.28330	0.80482	0.95396	8.99	5.33	1.74
13	6.38	3.66	160	789	2.89708	2.20412	0.80482	0.96502	9.23	5.57	1.82
14	6.38	3.66	128	647	2.81090	2.10721	0.80482	0.97575	9.46	5.80	1.89
15	3.45	3.64	320	856	2.92942	2.50515	0.53782	0.96333	9.19	5.55	1.81
16	3.45	3.64	320	842	2.92531	2.50515	0.53782	0.95922	9.10	5.46	1.75
17	3.45	3.64	256	667	2.82413	2.40824	0.53782	0.95495	9.02	5.38	1.75
18	3.45	3.64	256	663	2.82151	2.40824	0.53782	0.95233	8.96	5.32	1.73
19	3.45	3.64	256	664	2.82217	2.40824	0.53782	0.95299	8.97	5.33	1.74
20	3.45	3.64	256	659	2.81889	2.40824	0.53782	0.94971	8.91	5.27	1.75
21	3.45	3.64	256	668	2.82478	2.40824	0.53782	0.95560	9.03	5.47	1.75
22	3.45	3.64	256	674	2.82866	2.40824	0.53782	0.95948	9.11	5.47	1.78
23	3.45	3.64	256	676	2.82905	2.40824	0.53782	0.96077	9.14	5.50	1.79
24	3.45	3.64	256	680	2.83251	2.40824	0.53782	0.96333	9.19	5.55	1.81
25	3.39	3.68	256	655	2.81624	2.40824	0.53020	0.95468	9.01	5.33	1.74
26	3.39	3.68	256	642	2.80754	2.40824	0.53020	0.94598	8.83	5.15	1.68
27	3.39	3.68	320	805	2.90580	2.50515	0.53020	0.94733	8.86	5.18	1.69
28	3.39	3.68	320	814	2.91062	2.50515	0.53020	0.95215	8.96	5.28	1.72
29	3.39	3.68	320	823	2.91540	2.50515	0.53020	0.95693	9.06	5.38	1.75
30	3.39	3.68	320	816	2.91169	2.50515	0.53020	0.95322	8.98	5.30	1.73
31	3.39	3.56	256	671	2.82672	2.40824	0.53020	0.96516	9.23	5.67	1.85
32	3.39	3.56	320	825	2.91645	2.50515	0.53020	0.95798	9.08	5.52	1.80
33	3.39	3.56	320	824	2.91593	2.50515	0.53020	0.95746	9.07	5.51	1.80
34	5.14	3.56	192	763	2.88252	2.28330	0.71096	0.96514	9.23	5.67	1.85
35	5.14	3.56	256	1009	3.00389	2.40824	0.71096	0.96157	9.15	5.59	1.82
36	5.14	3.56	320	1225	3.08814	2.50515	0.71096	0.94891	8.89	5.33	1.74
	Mean value of n . . . . =										1.77
	Weighted mean . . . . =										1.758

According to theory, n should be equal to the refractive index of the liquid, which is about 1.64 for the mean yellow rays (VERDET).

The weighted mean of the observations is 1.758, a result which is about seven per cent. higher than the theoretical value.

It is difficult to account for this considerable difference by attributing it to errors of experiment, for the result, as can be seen by an inspection of the first two columns and the last, is fairly independent of the "radius" or of the speed of revolution of the mirror.

To make sure that no gross mistake was made, a series of experiments was made without using the column of liquid. The result obtained for the velocity of light in air was in error less than two per cent.

Again, the experiment with air alone was made immediately after the experiment with liquid, and the value of n found from the formula

$$\frac{d_1}{d_2} = \frac{\frac{a}{V} + \frac{bn}{V}}{\frac{a}{V} + \frac{b}{V}} = \frac{a + bn}{a + b}$$

was $n = 1.77$.

Still, it is with great diffidence that I state that the velocity of light in carbon disulphide is to that in air as 1.00 to 1.76, with an uncertainty of two units in the second place of decimals.

PART 2.—*Velocities of red and blue light in carbon disulphide.*

In the second part of the work the light passed through a direct-vision prism before reaching the slit. By turning the prism through a small angle either the red or the blue end of the spectrum could be observed. The colors selected were $\lambda_r = .000620$ and $\lambda_b = .000490$.

The observations were made in pairs, alternating in succession the order red-blue to blue red. The micrometer screw was always turned in the same direction.

If d_r represent the deflection for red light and d_b, that for blue; n_r the ratio of velocities in liquid and air for red light and n_b, the same for blue, then

$$\frac{d_r}{d_b} = \frac{\frac{a}{V} + \frac{bn_r}{V}}{\frac{a}{V} + \frac{bn_b}{V}} = \frac{a + bn_r}{a + bn_b}$$

$$n_b - n_r = 2.8 \frac{d_b - d_r}{d_r}$$

The following observations give the values found for $d_b - d_r$ in hundredths of a millimeter and for d_r in millimeters:

No. 1.		No. 2.		No. 3.	No. 4.	No. 5.	No. 6.	No. 7.
+2.4	3.9	1.8	2.9	+1.7	+ .6	+1.5	+ .4	+1.1
1.9	0.0	3.4	1.9	2.0	.2	.3	.4	1.5
.7	3.4	.7	2.0	1.6	−2.4	.9	1.1	2.3
2.3	3.2	2.6	2.0	.1	.3	1.8	.2	1.9
4.1	− .2	.5	1.7	.6	.9	2.5	.5	2.7
3.1	− .8	2.0	.8	2.7	.2	1.5	− .2	2.5
3.5	.6	4.0	. .	1.5	.9	.1	−1.9	0.0
2.9	5.2	.3	. .	1.1	− .3	+1.23	1.7	− .1
− .5	4.4	4.0	. .	3.2	1.7	d_r = 1.55	.7	1.0
8.2	− .2	1.7	. .	0.0	1.8		2.2	1.5
−1.1	− .5	Mean = +2.02		+1.45	.6		.6	.2
−5.3	− .4	d_r =	1.55	d_r = 1.55	+0.41		+0.52	1.0
1.8	−2.1				d_r = 1.00		d_r = 1.25	1.3
.6	1.1							+1.30
3.0	4.1							d_r = 1.25
.8	− .9							
5.2	3.2							
4.3	1.8							
Mean = +1.89								
d_r =	1.55							

No. 8.		No. 9.	No. 10.	No. 11.		No. 12.		No. 13.	
+ .8	0.0	+1.3	+1.4	+1.0	+ .6	+1.5	+1.2	1.0	1.0
2.1	1.4	3.1	.4	.8	− .6	0.0	1.2	.4	2.1
2.5	2.0	1.7	1.3	1.2	1.4	.5	.3	1.9	− .2
1.7	.2	1.8	.9	1.6	.7	− .4	.7	4.0	1.7
2.2	0.0	2.1	1.3	1.0	− .3	.6	− .4	1.5	2.5
1.2	−1.2	2.2	.4	.7	0.0	1.8	.2	1.0	.8
.1	1.1	.7	.6	.2	.7	1.3	0.0	1.0	. .
.6	1.2	2.4	1.8	1.0	.6	1.3	.8	1.5	. .
.3	−1.9	− .6	−1.8	− .2	.8	1.5	.3	1.0	. .
1.2	− .6	3.2	− .5	0.0	. .	1.6	. .	.5	. .
− .8	1.4	2.2	− .2	+0.59		− .2	. .	+1.36	
− .1	.4	2.5	+0.51	d_r = 0.81		+0.83		d_r = 1.24	
1.4	.8	1.7	d_r = 0.82			d_r = 0.83			
1.2	1.2	1.7							
− .3	−1.0	+1.86							
.1	.9	d_r = 1.60							
− .6	− .6								
−1.1	. .								
+0.51									
d_r = 0.65									

In the following table the results are collected together with the data. The letters have the same signification as before:

	r	m	$d_b - d_r$	d_r	$\frac{d_b - d_r}{d_r}$	$n_b - n_r$
1	6.34	320	.0189	1.55	.0122	0.034
2	6.34	320	.0202	1.55	.0130	0.036
3	6.34	320	.0145	1.55	.0094	0.026
4	6.34	213	.0041	1.00	.0041	0.011
5	6.34	320	.0123	1.55	.0079	0.022
6	6.34	256	.0052	1.25	.0041	0.011
7	6.34	256	.0130	1.25	.0104	0.029
8	6.34	128	.0051	0.65	.0079	0.022
9	6.34	335	.0186	1.60	.0116	0.032
10	3.39	320	.0051	0.82	.0062	0.017
11	3.39	320	.0059	0.81	.0073	0.020
12	3.39	320	.0083	0.83	.0100	0.028
13	5.14	320	.0136	1.24	.0110	0.031
	Mean value $n_b - n_r$				=	0.0245
	Theoretical value (VERDET)				=	0.025

If

$$n = 1.77$$

we have

$$\frac{n_b}{n_r} = 1.014 \text{ or } \frac{V_r}{V_b} = 1.014$$

It would appear, then, notwithstanding the rather wide divergences in the separate observations, that we are entitled to conclude from these experiments that orange-red light travels from one to two per cent. faster than greenish-blue light in carbon disulphide.

DISCUSSION OF OBSERVATIONS

OF THE

TRANSITS OF VENUS

IN

1761 AND 1769.

BY

SIMON NEWCOMB.

TABLE OF CONTENTS.

THE TRANSITS OF VENUS IN 1761 AND 1769.

Chapter I.

INTRODUCTORY.

1. The mutations of opinion on the value of the observations made upon the transits of Venus in 1761 and 1769 are noteworthy. For more than a century these observations were looked upon as affording the best data for the determination of the solar parallax, and the future epochs of such transits were anticipated as the only times when valuable additions to our knowledge of distances in the solar system could be made. Now, however, opinion has changed so far in the other direction that some apology may be needed for devoting the time and labor which I have to the present discussion.

The fact appears to me to be that the exaggerated opinion of the accuracy with which transits of inferior planets over the Sun could be observed has resulted, in accordance with a law of human judgment, in impairing the value of the conclusions derived from the observations of these phenomena. It was assumed that the observations were not legitimately liable to a probable error of more than a few seconds, and a temptation was thus offered to treat them on this hypothesis. This led to a habit of too freely rejecting discordant observations, and produced an undue bias in favor of such as could be brought into good agreement with others. It having been found that the results obtained in this way were erroneous to an extent which was not supposed possible, and therefore that the supposed precision of the observations was illusory, there now exists a tendency to look upon them as too inaccurate for further discussion.

But a little consideration will show that we should not consign to oblivion a mass of material so celebrated in the history of science. If we admit at the outset that the probable error of the observed times of contact ranges from 10 to 30 seconds, according to the skill of the observer, the quality of his telescope, and the state of the atmosphere, and that the observations are liable to abnormal errors under the unfavorable conditions which obtained at some of the stations, we shall have a fairly correct

basis on which to judge them. The errors which I have just assigned correspond to errors ranging from $0''.2$ to $0''.6$ in the relative heliocentric positions of the Earth and Venus. A collection of one or two hundred observations of this degree of accuracy at so early an epoch is not to be neglected, especially in view of the scarcity of accurate meridian observations of Venus during the half century following. Even should the parallax derived from them be of little value, the material for the determination of the motions of Venus and the Earth must be alone worth the discussion.

2. There are other considerations pertaining to the subject which have influenced my policy in elaborating the planetary theories. A comprehensive view of the order of progress in determining the constants of astronomy will, I think, show a decided bias in each generation of astronomers towards depending upon a few recent observations supposed to be accurate, to the exclusion of past ones supposed to be affected by undiscoverable sources of error. If we take from time to time during the past century the values of the astronomical constants which have been adopted at each epoch, and compare them with what would have been the result of a judicious weighted mean of all previous determinations, we shall find that in many, possibly in the majority of cases, the latter results would have been the better of the two.

The history of adopted values of the solar parallax affords a case in point. In 1854 was made known HANSEN's celebrated discovery that the parallactic equation of the Moon showed ENCKE's value of the solar parallax to be decidedly less than the true value. In 1862–1863 followed HANSEN's definitive determination $8''.916$, FOUCAULT's $8''.86$ from the velocity of light, and the discussion of observations of Mars by STONE and WINNECKE, leading to values $8''.943$ and $8''.964$.

The result of these investigations was a general tendency to assign to the solar parallax values between $8''.90$ and $9''.00$, entirely rejecting ENCKE's result. What would have been the conclusion had ENCKE's result been combined with these last ones we can not say with precision, because it would depend upon the weights assigned to the several determinations; but, under any judicious assignment of weights, it is certain that the combined result would have been nearer the truth than that adopted.

In 1867 appeared my own discussion from oppositions of Mars and other sources. Influenced by the same bias which had acted upon the others, I entirely rejected ENCKE's discussion, and also applied to the parallactic equation of the Moon, as derived from observation, a small correction, the validity of which I now doubt. The result was a value of the solar parallax which we now well know must have been about $0''.05$ too large. Yet such was the influence of the former bias that I am persuaded that the general opinion among my fellow astronomers was that my result was too small. Had I included ENCKE's determination, with a due weight, the result would probably have been between $8''.79$ and $8''.82$, which we now know would have been nearer the truth, but which would have appeared still less probable than the result I actually reached.

3. Another reason for the present discussion is, that the studies made upon the phenomena of contact, in consequence of the recent transits, should materially influence

the interpretation and discussion of the older observations. It is well known that in the last century most of the observers were unprepared for the optical phenomena attending the interior tangency of the limbs of Venus and the Sun. Chief among the disturbing causes was what is now familiarly known as the "black drop," the appearance of which rendered it impossible to assign any one definitive moment as that of contact.

How this difficulty arose from irradiation was clearly pointed out by LALANDE, and is now so well understood as not to need further description. The result of LALANDE's theory was to lead the astronomers who discussed the observations to divide the contacts into two distinct classes, known, respectively, as "geometrical contact" or "tangency of limbs" and "breaking of the thread of light." The writer has not made an attempt to study exhaustively the history of this subject; but, so far as he is aware, DE FERRER, of Spain, was the first to carefully investigate the differences of time between the two phenomena. This astronomer died in 1818, and the importance of his work would seem not to have been appreciated during his life, since his able and important paper on this subject did not appear until 1833, when it was published by the Royal Astronomical Society of London.*

In this paper is found (page 264) a table of the observed differences in time between the coincidence of the limbs and the formation or breaking of the luminous thread. The interval ranges from 8 to 73 seconds, the mean value being $34^s.1$.

The coincidence of the value of his parallax, $8''.577$, with that afterwards found by ENCKE is worthy of remark.

It would seem that the only observations which he actually employed in deducing the parallax were those of the formation of the luminous thread. Indeed, according to the theory which has been in vogue ever since the time of LALANDE, these are the only observations which correspond to true contact, and therefore are the only ones which should be used. Accordingly, we find that ENCKE relies solely on these observations for the value of the solar parallax which he deduces in his celebrated works on this subject.

In 1864, when it was well established that the accepted value of the parallax was too small, POWALKY attempted to show that by using improved longitudes of the stations, and rejecting certain doubtful observations, a larger value of the parallax could be deduced. His work can not, however, be considered as in any way definitive, and the number of observations which he rejected is too great. His paper therefore threw little light on the difficulties of the problem.†

In 1868 Mr. E. J. STONE discussed the observations of duration made in 1769 at the five stations where both contacts were visible, using observations of apparent contact as well as those of the thread of light. He found that by suitably interpreting the observations, and regarding the interval of time between the two phases of contact as constant, a value of the parallax equal to $8''.91$ would be obtained. His new interpretation consisted in assigning the observed egress at San José and the internal contacts at Tahiti to the class of "apparent contacts," while all his predecessors had

* Memoirs Royal Astronomical Society, Vol. III, p. 253.

† Thesis published at Kiel; translated in the *Connaissance des Temps* for 1867.

considered them as referring to the thread of light. The representation of the observations was remarkably good, as is shown by his table of residuals, which follows:*

Observer.	s.		Mean error. s.
HELL	−1.7	Wardhus	+0.6
SAJNOVICS	+2.8		
WALES	+0.4	Hudson's Bay	−0.1
DYMOND	−0.6		
	−0.9	Kola	−0.9
CHAPPE	+0.5	St. Joseph	−1.6
V. DOZ	−0.4		
MEDINA	−5.4		
GREEN	+5.8	Otaheite	+0.8
COOK	−4.2		

4. The general theory on which all the preceding discussions were founded, though not always full and clearly stated, was substantially this:

Firstly, that the formation of the thread of light at ingress and its breaking at egress mark the true time of tangency of the dark limb of Venus with the bright limb of the Sun.

Secondly, that a definite interval intervenes between this formation or breaking of the thread of light and the moment known as apparent or geometric contact of the limbs, which interval arises from and depends upon irradiation.

Neither of these propositions is a sufficient approximation to the truth. A very little consideration will show that the time of formation of the thread of light at ingress can be no other than the moment when the thread grew sufficiently strong and well marked to be seen by the observer. The time must therefore depend upon the quality of the telescope and the conditions of vision. The greater the atmospheric diffusion of the images, the more imperfect the instrument, and the more hazy the atmosphere, the later will the thread become visible at ingress and the sooner will it disappear at egress. It follows from this that the interval between the two phases of contact must, in a still greater degree, vary with the observer, the telescope, and the conditions of vision.

We may expect that, as a general rule, the earlier the time when the observer saw the breaking of the thread of light at egress the later he would note the moment of apparent geometric contact. The clearer and steadier the air and the sharper the vision the more nearly the two phases would approach each other until, under the best conditions, they should come together.

In his discussion of the transits of Mercury the writer showed that there was no tendency whatever of the observations to group themselves about two distinct phases of contact.† The greater number of observations corresponded to a certain mean phase, from which the number diminished in each direction in fairly good accordance with the usual law of error until the limits of normal error were reached. It must be remarked, however, that this result should not be applied to the case of a transit

*Monthly Notices, R. A. S., XXVIII, p. 255.
†Astronomical Papers of the American Ephemeris, Vol. I, p. 380.

of Venus without further investigation. Owing to the much greater angular magnitude of Venus, as seen in the telescope, the optical phenomena arising from irradiation will be much more sharply defined. The question must therefore be independently investigated in the case of Venus, as it has been in that of Mercury. The whole question is, moreover, rendered difficult by the indefinite and varying character of the phenomena.* As a matter of fact we find that the observations may be divided into three not entirely distinct classes:

Firstly, those in which the formation or extinction of the thread of light is stated to have been observed.

Secondly, cases in which some phenomenon of contact was noted earlier than the formation of the thread of light at ingress or later than its disappearance at egress.

Thirdly, observations in which no statement is made from which it can be inferred that one phase rather than the other was observed.

The first difficulty which arises is, how observations of these different kinds are to be combined. In commencing the work I adopted the conclusion derived from observed transits of Mercury, that only a single mean phase of contact was to be derived from an observation of different phases of the same internal contact by one observer. When the observer noted but a single time it was assumed to refer to this mean phase, unless his description of the phenomenon was inconsistent with that hypothesis. This mean phase would be neither the complete visible formation of the thread of light nor the so-called geometric contact, but a phase between the two, and nearer the former, when the thread of light was about to be formed, or, to speak more exactly, when the light of the cusps seemed to converge towards zero at the expected point of contact.

An examination of the observations soon showed that this method of treatment would be unsatisfactory. The assumption that the mean contact would be found by applying to the observation of the thread of light a fraction of the interval between the two phases rested on too slender a foundation to be adopted without examination. In many cases the thread of light and no other distinctive phase was noted. Some reason was found for believing that in most of the cases where no description of the phenomenon was given the thread of light was noted. I was thus led to revert to ENCKE's original system of interpretation, which seems to have been that the presumption was in favor of the thread of light unless some evidence was shown to the contrary.

But it still remained true that we should expect the thread of light at ingress to be seen later the greater the amount of irradiation. It would therefore not be advisable to assume that an observation made when the Sun was near the horizon, with considerable atmospheric diffusion, would be comparable with an observation made under better conditions. On the other hand, no positive data existed for correcting the one observation so as to make it comparable with the other. It therefore became necessary to introduce the correction as an unknown quantity into the equations of condition. The system on which this is done will be seen in the final discussions of the observations.

* The author may be allowed to quote a former remark of his as still expressing his views on this subject: "It has too generally been assumed that the geometric outlines of Venus and the Sun, considered as mathematical lines, can be noted in observation with the same sort of definiteness and precision as that with which the mind conceives them, and sufficient attention has not been paid to the practical difficulties which the eye meets with in representing this geometric conception. I conceive that the question whether a certain phase can or can not be definitely distinguished and observed by the eye is to be settled by actual trial, and by a consideration of the imperfections of vision, rather than by a consideration of its purely geometric definiteness of form."

5. The question may be asked, why the final result for the solar parallax obtained in the present paper differs so widely from that deduced by ENCKE from the same observations. The completeness and thoroughness of ENCKE's work, with which the writer has been more and more impressed as he proceeded with his own, makes this question all the more pertinent. At the same time he is not prepared to give a definitive answer, for the reason that he has throughout avoided any such comparison of his own work with that of his predecessor as might, by any possibility, bias his judgment in discussing the observations. He entertains the hope that some other astronomer will consider the subject of sufficient interest to make a thorough comparison of the two sets of results. The following remarks are therefore to be regarded rather as guiding threads in such a comparison than as definitive explanations of the difference in question.

Firstly, the great magnitude of the divergence might seem to indicate some systematic difference in the mode of treatment or interpretation. There is, however, but one point in which the difference can be considered systematic, namely, the introduction in the present paper of a correction to the observed time of formation of the thread of light on account of the imperfections of vision. If we consider only the durations observed in 1769 at the five stations where both contacts were visible, we can get no result but that of ENCKE, unless, indeed, we change the interpretation of COOK's observations, as well as of CHAPPE's, as proposed by Mr. STONE. These changes have always seemed to me quite indefensible, in view of the original record of the observations, as given in Mr. STONE's paper, and especially of COOK's diagrams, which he reproduces from the *Philosophical Transactions*. The erroneous result may, however, be explained by the fact that, at the northern stations, where duration was longest, the Sun was very near the horizon, so that ingress was observed too late, and egress too early, as compared with the southern stations. The effect upon the parallax is obvious. The great mass of observations made in Western and Northern Europe upon the ingress of 1769 were all affected by this systematic error, but yet enter into ENCKE's results with a weight proportionate to their number. The weight which they carry in the present discussion is comparatively small, as will be seen by an examination of the concluded results.

Secondly, it may be that I have not assigned the great comparative weight that ENCKE has to certain of the most accordant observations, and that I have paid less attention to the quality of the images and the amount of atmospheric undulation as described by the observers. It is, however, difficult to speak on this point, as ENCKE does not give in detail his judgment upon each separate observation and his reasons for classifying it as he does. By admitting at the outset a probable error of the observations derived from the actual comparison of all the material, it is probable that the present writer has included many observations which ENCKE rejected.

Thirdly, more recent determinations of the longitudes of nearly all the stations would naturally change the results. These changes are of a systematic character only in this way, that they help to counterbalance the mass of faulty observations on which ENKCE's result mainly depends.

CHAPTER II.

OBSERVATIONS OF THE TRANSIT OF VENUS, JUNE 5-6, 1761.

I have deemed it essential to the object of the present discussion to make a copy, so far as practicable, of the original record pertaining to each separate observation. In most cases, however, this can be done only in an imperfect way, owing to the imperfections of the printed statements, and, in some cases, the inaccessibility of the necessary publications.

In the following collection of the observations the stations are arranged nearly in the order of longitude, from east towards west. Some deviations from this order are permitted, in order to bring together observations in the same country, derived from the same published sources. As a general rule I have sought to distinguish the original matter upon which conclusions are founded from my own remarks and discussions by printing the former in smaller type. It is to be understood that, as a general rule, the matter in small type is either an exact transcript from the quoted record or, in a few cases, an abstract from that record, made without reference to the results. In the case of the Swedish observations, however, I was obliged to depend upon the German translation of the Swedish Memoirs in which the originals were published.

In commencing the work of collection I intended to proceed independently of what ENCKE had done. I soon found, however, that nothing which I could discover had escaped his scrutiny, and therefore began to confine myself to ENCKE's list of stations and references, as given in the opening part of each of his works.

In the case of many of the observations of 1761 the only authority extant is a collection by HELL in his *Ephemerides ad meridianum Vindobonensem.* No copy of this collection being accessible in this country I took occasion to make the copies during a visit to Vienna, in 1883. I have therefore been unable to compare the proof-sheets with the original, and to see whether any light could be thrown on some doubtful points by a further examination of the original.

Of course many of the observations are worthless on their face, while a yet larger number exhibit discordances which do not admit of being cleared up, and which necessitate their rejection. I have followed no uniform rule in the treatment of these observations, but have dropped them out, with or without remark, sometimes at one point and sometimes at another.

In the references the following abbreviations are used:

Paris Memoirs: Mémoires de Mathématique et de Physique de l'Académie Royale des Sciences.

N. C. P.: Novi Commentarii Academiæ Scientiarum Imperialis Petropolitanæ.

Schwedische Abhandlungen: Der Konigl. Schwedischen Akademie der Wissenschaften Abhandlungen * * * aus dem Schwedischen übersetzt.

PEKIN.

[*N. C. P.*, XI, p. 524.]

Observer, DOLLIER.—R. P. DOLLIERUS refert, observationem peractam esse tubo 14 pedes longo ad horologium bonæ notæ, cujus motum licet præcedentibus ante observationem diebus ad examen revocare ei non licuerit, didicisse tamen se ait a R. P. BENOIT statum horologii talem fuisse, ut spatio diei Solaris retardaret 16″ 44‴, id quod R. P. ex appulsibus Sirii conclusisse se asseveraverat. Illa ipsa die, qua Venus discum Solis peragrabat, non obstantibus nubibus, ventoque vehementi, sumsit ante ingressum altitudines Solis iisque post egressum correspondentes ex quibus meridies correctus ad horologium prodiit 11^h 56′ 7″ 43‴.

Quoniam observatio Pekinensis fortasse nunc primum in lucem prodit, consultum esse duxi eam verbis ipsius auctoris referre.

* * * Je vis la véritable Vénus, qui venait d'entrer, c' était a très peu près à 9^h 51′ 25″ ou 30″ de tems vrai:

		h.	′	″	‴
Entrée totale de Vénus, horloge		10	6	35	0
Pour défaut du midi	+		3	52	17
Pour retard et équat.	—		0	0	27
Tems vrai		10	10	26	50
Commencement de sortie, horloge		3	56	6	0
Pour défaut du midi	+		3	52	17
Pour retard et équat.	+		0	0	59
Tems vrai		3	59	59	16
Sortie totale de Vénus, horloge		4	14	4	0
Pour défaut du midi	+		4	52	17
Pour retard et équat.	+		0	1	4
Tems vrai		4	17	57	21

SELENGINSK.

[*N. C. P.*, XI, p. 455.]

Observer, RUMOVSKY.—Oculo tandem hebescente, vento tubum aliquantum agitante limbo Solis tantisper fluctuante et tenui nubecula obducto, appulsum limbi Veneris præcedentis ad limbum Solis secundum Horologium meum ad 5^h 7′ 49″, evenire indicabam. In contactu limborum id singulare observabam, quod filum lucidum inter planetæ et Solis limbum interceptum ante expectationem evanesceret ac subito quasi exigua guttula nigra e Venere procedere, limbumque Solis limbo Veneris jungere videretur. Cum Horologium 5^h 8′ 8″ indicaret, Sol per aliquot minuta secunda pleno fulgore radiabat tumque limbus Veneris præcedens ultra Solem prominens lucido annulo cinctus mihi videbatur; attentius vero considerare ac examinare hoc phænomenon non licebat, Sol enim subito per nubem intercurrentem ex conspectu eripiebatur.

In contactu exteriori observando eadem, imo et majora aderant impedimenta, quæ in contactu interiori expertus sum: accidere vero contactum exteriorem limborum Veneris atque judicabam cum Horologium indicaret 5^h 25′ 55″.

Enarro, quæ observavi, neque, quæ forte fuerit causa physica, apparentiam guttulæ nigræ limbum Solis ac Veneris jungentis, annuli ve lucidi Veneris limbum cingentis producens, inquiro.

The results of corresponding altitudes of the Sun for clock correction are shown in the following table:

Times of apparent noon by Rumovsky's clock.

Date.	Clock time.	Mean time.	Clock correction.	Rate.
	h. m. s.	h. m. s.	h. m. s.	s.
June 2	1 43 57.2	23 57 27.6	—1 46 29.6	
				—22.8
3	1 44 29.4	23 57 37.0	—1 46 52.4	
				—21.0
4	1 45 0.2	23 57 46.8	—1 47 13.4	
				—23.9
9	1 47 53.7	23 58 40.7	—1 49 13.0	

It follows that at the time of egress on June 6 the clock correction on mean time was — 1^h 48^m $4^s.7$, giving 3^h 19^m 44^s as the local mean time of contact.

Since 19 seconds after the time of contact was noted the atmospheric line around that portion of the disk of Venus which projects beyond the sun was clearly seen, it would seem that the time of contact was probably well observed. The description is so clear that the observation might be placed in the first class but for the clouds. I shall therefore assign it to Class II.

CALCUTTA.

[*Philosophical Transactions*, 1761, p. 582.]

Observer, WILLIAM MAGEE.—The following are the observations as printed:

	h.	′	″
The appulse uncertain, but very apparent, at	8	11	35
The center of Venus on the Sun's limb	8	16	35
The interior contact at the ingress	8	24	40
Interior contact at the egress	2	15	55
Center of Venus on the Sun's limb at the egress	2	24	0
Total egress	2	32	0

The observer used a stop-watch, the error of which was found by comparing it with the transit of the Sun over the meridian line in the town hall. On the day of the transit, June 6, the watch was thus found to be 4′ 10″ fast, and observations on subsequent days showed it to be gaining nearly 2 minutes each day. The result is therefore:

Time.	Contact II.	Contact III.	Contact IV.
	h. m. s.	h. m. s.	h. m. s.
Watch time . . .	20 24 40	2 15 55	2 32 0
Correction . . .	0 3 52	0 4 22	0 4 22
Apparent time . .	20 20 48	2 11 33	2 27 38
Mean time . . .	20 18 54	2 9 42	2 25 47

Owing to the uncertain determination of time it is doubtful whether this observation is worth using. The apparent time of Contact II is printed 8^h 20^m 58^s.

MADRAS.

[*Philosophical Transactions*, 1761, p. 396.]

Observer, Rev. WILLIAM HIRST.—The total ingress or first internal contact was determined with a precision equal to that of the first external contact at $7^h\ 49'\ 55''$, apparent time.

Mr. HIRST thinks it necessary to take notice of another odd phenomenon. At the total immersion the planet, instead of appearing truly circular, resembled more the form of a Bergamot pear, or, as Governor PIGOT then expressed it, looked like a nine-pin, yet the preceding limb of Venus was extremely well defined. Mr. HIRST suspected this appearance might be owing to their telescopes not being nicely enough set to their focal lengths. Accordingly he took care to try this several times during the transit, but found it not to be the case; for though the planet was as black as ink, and the whole body truly circular just before the beginning of the egress, yet it was no sooner in contact with the Sun's preceding limb than it assumed the same figure as before, at the Sun's subsequent limb, the subsequent limb of Venus keeping well defined and truly circular.

The beginning of the egress or second interior contact was observed only by Mr. HIRST and Mr. CALL, Mr. PIGOT having retired. This phase came on at $1^h\ 39'\ 38''$ p. m., and the total egress, by Mr. HIRST alone, at $1^h\ 55'\ 44''$, apparent time, Mr. CALL unfortunately losing the solar image out of the field of his telescope.

Nothing is said of the determination of clock error.

[*N. C. P.*, XI, p. 569.]

Observers, The JESUITS.—Observatio transitus Veneris per Solem instituta in Madrass Indiæ Orientalis d. 6 Jun, 1761:

	h.	m.	s.
Primus contactus Veneris in discum Solis ante merid.	7	28	28
Immersio totalis Veneris in disc. Solis	7	45	13
Differentia temporis contactum externum inter et totalem immersionem	0	16	45
Contactus limbi prioris Veneris ad limbum Solis, sive initium emersionis post merid.	1	37	1
Emersio totalis	1	53	7
Tempus durationis exitus Veneris ex disco Solis fuit	0	16	6

N. B.—Latitudo urbis Madrass est circiter 13° 8′, et altitudo observatorii supra horizontem circiter 40 pedes. Observatorium fuit in ædibus Gubernatoris. Tempus durationis totius phænomeni, $6^h\ 24^m\ 39^s$.

TRANQUEBAR.

[*N. C. P.*, XI, p. 569.]

Observers, The JESUITS:

	h.	m.	s.
Primus contactus Veneris ad limbum Solis	7	29	39
Immersio totalis	7	46	52
Differentia temporis inter primum contactum et totalem immersionem	0	17	13
Initium Emersionis	1	40	25
Emersio totalis	1	56	34
Tempus durationis Emersionis	0	16	9
Duratio totius phænomeni	6	26	55

Encke gives the name of the observer as Coeurdoux; I do not know on what authority. The observations of the Jesuits, both here and at Madras, are erroneous by several minutes, and seem incapable of correction or use.

TOBOLSK.

[*Paris Memoirs*, 1761, p. 361.]

Observer, Chappe d'Auteroche:

Temps à la pend.			Temps vrai.				Remarques.
h.	′	″	h.	′	″	‴	
6	47	59	6	47	1	0	Le centre n'est pas encore entré.
6	52	49	6	51	51	0	Le Soleil découvert; le centre de Vénus me parait déjà entré.
6	59	44	6	58	46	0	Le Soleil toûjours découvert; j'aperçais la partie du disque de Vénus qui n'est pas encore entrée, et une petite atmosphère en forme d'anneau autour de ce même disque.
7	0	40	6	59	41	44	Je vois encore le petit anneau lumineux, le Soleil parfaitement découvert.
7	1	28¼	7	0	30	14	Entrée totale; j'ai vû le filet de lumière du bord du Soleil, qui a paru comme un éclair, de façon qu'on aura pu saisir cette phase avec l'exactitude la plus rigoureuse. Quelque temps après le Soleil s'obscurcit de nouveau, ce qui ne dura cependant pas long temps; il ne pouvait même être plus serein deux heures avant la sortie.
12	50	23	12	49	20	29	Le bord du Soleil s'obscurcit quoique le ciel soit très-serein et cet astre au centre de la lunette.
12	50	26	12	49	23	29	Contact intérieur décidément de la partie obscure de Vénus et du bord du Soleil.
12	54	50	12	53	47	30	On voit la partie du disque de Vénus qui est déjà sortie, et un anneau en forme de croissant, dont la partie convexe est tournée du côté du bord inférieur de Vénus.
12	57	52	12	56	49	0	Je vois encore le croissant très-bien.
13	4	7	13	3	4	0	Je ne vois plus d'anneau ni la partie du disque de Vénus déjà sortie. Il fait un peu de vent.
13	8	45	13	7	42	16	Sortie totale.

L'anneau me paraît avoir sa principale cause dans le rapport du diamètre de Vénus à celui du Soleil; celui de cette planète étant beaucoup plus petit, devait avoir plus d'un hémisphère éclairé par le Soleil. Le disque de Vénus n'était point parfaitement rond dans sa partie orientale où parut l'anneau, ce qui me fit soupçonner que son diamètre était même plus petit dans ce sens. La lumière de cet anneau était d'un jaune très-foncé auprès du corps de la planète, elle devenait ensuite plus brillante vers la partie la plus éloignée du corps obscur de Vénus, à $6^h\ 59'\ 41\frac{3}{4}''$; la limite la plus obscure de cet anneau me parut toucher le disque du Soleil: je crus même pendant quelque temps que c'était le moment de l'immersion totale; dans cette incertitude je ne voulus pas quitter la lunette, mais j'écrivis immédiatement l'observation, et prêtai de nouveau la plus grande attention à la partie de l'anneau qui n'était pas encore entrée. A $7^h\ 0'\ 30\frac{1}{2}''$ la lumière du Soleil parut avec une telle rapidité, qu'il n'était pas possible de se tromper d'un quart de seconde dans cette phase; et en effet, on voit clairement qu'à cause du fond obscur du ciel et du corps opaque de Vénus, cet effet a du nécessairement avoir lieu dans l'immersion totale, ce qui n'aurait pas

encore été si cette phase n'avait pas été l'immersion du corps de la planète, car la lumière de l'anneau devenant insensiblement plus brillante et se confondant avec celle du Soleil, aurait toujours laissé quelqu'incertitude; aussi celle du Soleil fit disparaître ce qui restait de la lumière de l'anneau, qui me parut s'étendre encore un peu au-delà, où se fit l'immersion totale.

This description of the phenomena of internal contact at ingress corresponds remarkably with what the author himself observed at the Cape of Good Hope in 1882. The only essential difference is that CHAPPE D'AUTEROCHE describes the light of the point formed by the conjunction of the solar cusps at internal contact as increasing so rapidly that he was not in doubt by a quarter of a second, whereas the author was himself in doubt by two or three seconds.

I am in doubt whether the phase observed should be classified as "mean phase" or "thread of light." The only objection to the latter classification is that, where the images are so sharp, as is implied in the above description, there is no noticeable separation of phases. In view, however, of the rapid increase of light, which he describes, I shall, for the time being, call the phase thread of light.

RODRIGUE ISLAND.

[*Paris Memoirs*, 1761, pp. 87, 443.]

***Observer*, PINGRÉ.—Le Soleil s'est levé couvert de nuages; à $6^h\ 43'\ 51''$ du matin, il a paru un peu, Vénus était entièrement sur le disque: la distance des bords les plus voisins était de 10 à 12 secondes au plus; l'entrée ou l'émersion totale pouvait donc être arrivée vers $6^h\ 40'$, ou peu auparavant.**

Le commencement de l'émersion est certainement arrivé à $12^h\ 34'\ 47''$, la fin de l'émersion à $12^h\ 52'\ 23''$, mais cette fin n'est pas assurée, les nuages étaient revenus. A $12^h\ 53'\ 18''$ le Soleil reparaissant, j'ai eu une légère idée que la rondeur de son disque était un peu altérée; mais je n'ai pas eu le temps de m'en assurer, les nuages étaient revenus.

La plus grande distance des bords méridionaux de Vénus et du Soleil a été de $5'\ 57\frac{1}{10}''$. J'ai trouvé le diamètre de Vénus de $55\frac{8}{10}''$.

La latitude du lieu où j'observais, est de $19°\ 40'\ 40''$ méridionale, la longitude de $4^h\ 3\frac{1}{2}'$ environ; car je n'ai pas encore fait les calculs les plus essentiels pour cette partie.

The expression "le commencement de l'émersion est certainement arrivé à, etc.," would seem to imply that the internal contact may have been earlier by several seconds.

The above account is given on page 87 of the Mémoires. But on page 443 of the same volume, in the detailed report, the following table is found:

h.	m.	s.	h.	m.	s.	
0	35	44	0	36	49	Attouchement certain et instantané des bords.
0	53	03	0	54	09½	Vénus, presque sortie, est couverte d'un nuage.
0	53	21	0	54	27½	Je la vois encore, mais bien prête de quitter.

A $0^h\ 54'\ 16''$ de la pendule, ou $0^h\ 55'\ 22''$ temps vrai, le Soleil reparait; M. THUILLIER qui, avec la lunette de neuf pieds, avait vu le contact intérieur au même instant que moi, ne voit plus absolument rien: il fixe le contact extérieur à $0^h\ 54'\ 27\frac{1}{2}''$ ou très-peu de secondes après; et je pense qu'il a raison. A $0^h\ 55'\ 22\frac{1}{2}''$ j'ai eu une très-faible idée de quelque altération dans la rondeur du disque solaire: un nuage survenu aussitôt ne m'a pas permis d'approfondir cette idée, laquelle

d'ailleurs était si légère, que je ne crois pas devoir m'y arrêter: à 0^h 56′ 25½″ il ne restait certainement aucun vestige de Vénus. Un moment après le ciel a fondu en eau: à peine ai-je eu le temps de mettre mes instrumens à couvert.

A good determination of the clock error on apparent time was made on the day of the transit, June 6, showing apparent noon to be at 23^h 58′ 57.″9 by the clock. Unfortunately it had stopped on the preceding evening, so that its rate had to be taken as determined on previous days, when it had lost 2′ 5″.5 per day on mean time. But as the interior contact occurred only 36^m after noon the uncertainty thus arising is slight. I assign the observation to Class II, rather than III, on account of PINGRÉ's experience as an observer.

PETERSBURG.

[*Schwedische Abhandlungen*, 1763, p. 138.]

	Contact III.	Contact IV.
	h. ′ ″	h. ′ ″
Kurganow . . .	10 19 1	10 37 2
Krasilnicow . .	10 19 4	. . . .

[*Ephemerides Vindobonenses*, 1762.]

	I.	II.	III.	IV.
	h. ′ ″	h. ′ ″	h. ′ ″	h. ′ ″
Braun	4 9 20	4 26 20	10 18 58	10 37 4
Krasilnicow . .	4 10 1	4 26 39	. . . .	. . . .
Kurganow . . .	4 9 42	4 26 41	. . . .	. . . .

* * * nam tempus inter initium et finem emersionis secundum meam observationem fuit 18′ 6″. Secundum Krasilnicowii 17′ 56″. Secundum Kurganow 18′ 2″:

	h.	′	″
Secundum observationem meam duratio totalis	6	27	46 [sic]
Krasilnicow	6	26	59
Kurganow	6	27	20

CAJANEBORG.

[*Schwedische Abhandlungen*, 1761, p. 156.]

Observer, PLANMANN.—Um 3 Uhr 59 Min. 56 Sec. bemerkte er zuerst, dass ein Stückchen von der Sonne südöstlichem Rande, wie mit einer Scheere abgeschnitten schien, ohne dass sich eine sonderliche Vertiefung gezeigt hätte. Innerhalb etlichen wenigen Secunden darnach war er der Venus wirklicher Gegenwart versichert.

Um 4 Uhr 18 Min. 5 Sec. schien ihm Venus sich ganz und gar in die Sonne zu senken, indem sich die spitzigen bisher von einander gesonderten Hörner der Sonne, welche die Venus umfasst

hatten, nun zusammen zogen. Obgleich der Pfarrherr FROSTERUS, welcher darauf mit dem dollondischen Fernrohre Acht gab, damit auf 2 Secunden übereinstimmte, so war doch Herr PLANMANN von der Richtigkeit dieses Augenblicks nicht so sicher, als er gewünscht hätte, weil der Sonnenrand durch die rauhigte Luft, wie wolkicht und undeutlich schien. Doch glaubte er nicht weiter als auf 2 oder 3, oder wenigstens nur einige wenige Secunden ungewiss zu seyn.

Um 10 Uhr 8 Min. 59 Sec. fing Venus an durch den Sonnenrand hinaus zu brechen, welcher zuvor durch Annäherung des Planeten immer schmäler und schmäler ward, bis er nun in einem Augenblicke, wie eine gespannte Saite sprang. Diese Beobachtung hält Herr PLANMANN für volkommen gewiss, ob es gleich dem Herrn FROSTERUS schon 40 Secunden zuvor vorkam, als hätte der Planet eine Oeffnung in den Sonnenrand gemacht; aber das leichte Fernrohr das er brauchte, ward durch den starken Wind zu sehr erschüttert, und daher mag wohl der Unterschied rühren.

Um 10 Uhr 26 Min. 22 Sec. sah Herr PLANMANN die letze Spur der Venus in der Sonne.

TORNEA.

[*Schwedische Abhandlungen*, 1761, p. 181.]

Observers, HELLANT, HÄGGMANN, and LAGERBOHM.—Herr Hauptmann LAGERBOHM bemerkte zuerst um 3 Uhr 45 Min. 44 Sec. dass Venus schon angefangen hatte, den südöstlichen Sonnenrand zu berühren, welches ich (HELLANT) und andere gleich darauf ebenfalls wahrnahmen. Um 4 Uhr 3 Min. 54 Sec. glaubte ich, der lichte Ring, den die Sonne um die Venus macht, würde umschlossen und der Planet sei ganz und gar in der Sonne, wenigstens war dieses nach meinem Urtheile um 4 Uhr 3 Min. 59 Sec. gewiss geschehen. Aber Herr Hauptmann LAGERBOHM sagte, er sei sicher, dass er den Ring bis 4 Uhr 4 Min. 1 Sec. offen gesehen habe, da nach seiner Meinung die innere Berührung der Sonne und der Venus geschah.

Als Venus sich dem Ausgange näherte, verdoppelten wir unsere Aufmerksamkeit. Um 9 Uhr 54 Min. 6 Sec. glaubte ich, Venus berühre den Sonnenrand und um 9 Uhr 54 Min. 8 Sec. machte sie sich, meinem Urtheile nach, eine Oeffnung. Herr HÄGGMANN hatte 9 Uhr 54 Min. 18 Sec. aufgezeichnet, aber die, welche im verfinsterten Zimmer beobachteten, sowohl als Herr Hauptmann LAGERBOHM, hatten 9 Uhr 54 Min. 22 Sec. aufgezeichnet, da Venus nach ihrer Meinung auszutreten angefangen hatte.

Um 10 Uhr 11 Min. 58 Sec. glaubte Herr HÄGGMANN, Venus sei völlig aus der Sonne heraus und in eben der Secunde verlor man sie auch im finstern Zimmer. Aber Herr Hauptmann LAGERBOHM sah sie bis 10 Uhr 12 Min. 14 Sec. und ich war sicher, dass sie nicht eher als um 10 Uhr 12 Min. 22 Sec. gänzlich herausging, denn bis auf diesen Augenblick sah ich ein Merkmal von ihr im Sonnenrande.

ABO.

[*Schwedische Abhandlungen*, 1761, p. 158.]

Observers, JUSTANDER and WALLENIUS.—Um 3 Uhr 55 Min. 50 Sec. geschah der gänzliche Eintritt und das ziemlich genau.

Um 9 Uhr 46 Min. 59 Sec. fing Venus an zum Ausgange durchzubrechen und Herr JUSTANDER sieht diese Beobachtung für völlig zuverlässig an.

Um 10 Uhr 4 Min. 42 Sec. trat sie völlig aus, wenigstens sah man nach diesem kein Ueberbleibsel mehr von ihr im Sonnenrande.

Mehr Umstände hat Herr JUSTANDER nicht mitgetheilt. Herr Prof. WALLENIUS beobachtete mit einem guten 3-füssigen Fernrohre, womit er den völligen Eintritt 5 oder 6 Secunden später, aber den Anfang des Austrittes in eben der Secunde wie Herr JUSTANDER sah.

CAPE OF GOOD HOPE.

[*Philosophical Transactions*, 1761, p. 384.]

Observers, Mr. CHARLES MASON and Mr. DIXON.—The egress was observed by them as follows:

Time per clock.		Apparent time.
h. ′ ″		h. ′ ″
2 39 16	The time of internal contact } very clear	21 39 52
56 50	The time of external contact } very clear	21 57 23
2 39 12	The time of internal contact } per Mr. DIXON	. . . .
56 48	The time of external contact } per Mr. DIXON	. . . .

No statement whatever is made respecting phenomena of contact. The clock correction was determined by equal altitudes of fixed stars, especially Antares and α Aquilæ. I have reduced them, with the following result:

	h.	m.	s.		
Internal contact . .	21	38	3.4	MASON	Cape mean time.
	21	37	59.4	DIXON	
External contact . .	21	55	34	MASON	Cape mean time.
	21	55	32	DIXON	

STOCKHOLM.

[*Schwedische Abhandlungen*, 1761, p. 153.]

Observers, WARGENTIN, WILKEN, and KLINGENSTIERNA.—Um 3 Uhr 21 Min. 37 Sec. sah Herr WARGENTIN endlich eine kleine Grube im Sonnenrande, an der Stelle da Venus hinein kommen sollte. Diese Grube blieb beständig an einer Stelle, dadurch, und durch ihre grössere Dunkelheit, unterschied sie sich von den unzähligen anderen herumschwebenden und unbeständigen Ungleichheiten. Innerhalb 11 Secunden darnach war es gewiss, es sei wirklich die Venus. Die Grube schien ihm bereits ziemlich gross, so, dass er glaubte, wenn es vorerwähntes Kochen um die Sonne nicht gehindert hätten, so hätte man die Venus schon eine oder die andere Minute zuvor sehen müssen. Aber aus den Umständen hat er hernachmals geschlossen, Venus könne nicht viel über eine halbe Minute zuvor angefangen haben einzutreten. Herr KLINGENSTIERNA sah sie auch fast um eben die Zeit.

Um 3 Uhr 38 Min. 27 Sec. fing es an Herrn WARGENTIN zu scheinen als befände sich die Venus gänzlich in der Sonne, denn er sah ihre ganze Rundung deutlich, obwohl mit einem schwächeren Scheine an der äusseren Seite, welche zuletzt eintrat. Anfangs glaubte er, dieser schwache Schein sei nichts anderes, als der Glanz der Sonne, welche den Planeten von allen Seiten umgäbe, weil aber der Glanz nicht, seinem Erwarten gemäss, schnell genug zunahm, sondern fast eine ganze Minute gleich schwach blieb, so gab er genau derauf Acht, bis er

Um 3 Uhr 39 Min. 23 Sec. einen stärkeren und lebhafteren Glanz bemerkte, welcher den dunkeln Planeten plötzlich umringte. Die spitzigen gegen einander gewandten Hörner der Sonne, die zuvor die Venus an der äusseren Seite umfasst hatten, gingen da völlig zusammen und schlossen sie gänzlich ein.

Herr KLINGENSTIERNA, welcher erwähnten schwachen Glanz nicht gesehen hatte, versicherte mit vieler Gewissheit, der gänzliche Eintritt habe sich um 3 Uhr 39 Min. 29 Sec. zugetragen. Gleich in dieser Secunde schien es auch Herrn WILKEN, als verliesse Venus den Sonnenrand, so dass alle von dem Augenblicke an sicher waren, sie sei gänzlich in die Sonne getreten.

Als die Venus sich wieder ihrem Austritte näherte, wandten alle ihren grössten Fleiss an, die Augenblicke des Austrittes genau zu bemerken. Um 9 Uhr 29 Min. 40 Sec. schien es Herrn WILKEN, als finge sie an durch den Sonnenrand zu brechen, er zweifelte aber selbst, ob diese Beobachtung richtig sei, theils weil das Telescop bei derselben einiger Erschütterung wegen schütterte, theils auch weil ihm sein Auge trüb und vom Sonnenglanze zu sehr geschwächt war, indem das rothe Glas nicht dunkel genug war. Doch bemerkte er dabei solche Umstände, wie bei den upsalaischen Beobachtungen sind erzählt worden, nämlich dass etwas aus der Venus nach dem Sonnenrande zuschoss. Aber Herr WARGENTIN hat nichts dergleichen gesehen, sondern war völlig gewiss, die Venus habe nicht eher eine Oeffnung im Sonnenrande gemacht als

Um 9 Uhr 30 Min. 8 Sec. da solches plötzlich geschahe, gleichsam als ein zarter Lichtfaden, der zuvor den äusseren dem Austritte sich nähernden Rand der Venus umgeben hatte, im Augenblicke in der Mitte zerrissen und hatten sich seine Enden merklich von einander gezogen. Dieses nun mangelnde Licht war nicht ein fremder Glanz, wie sich zunächst vor dem gänzlichen Eintritte gewiesen hatte und auch nachgehends während des Austrittes bemerkt ward, sondern das eigene gerade Licht der Sonne, sonst hätte es nicht so deutlich und so schnell verschwinden können. Herrn KLINGENSTIERNA'S Beobachtung bestätigt dieses noch weiter, denn er sah den lichten Streifen, welcher die Venus bisher umfasst hatte, noch 3 Secunden später oder um 9 Uhr 30 Min. 11 Sec. bersten, wie von einem stärker vergrösserndem Fernrohre, welches er gebrauchte, zu erwarten war.

Nachdem Venus mehr und mehr austrat, verlohr Herr WILKE sie völlig aus dem Gesichte um 9 Uhr 47 Min. 59 Sec. Freyherr VON SETH sah die letzte Spur von ihr um 9 Uhr 48 Min. 3 Sec. mit einem dollondischen Fernrohre von 3 Fuss, das Herr C. LEHNBERG verfertigt hatte. Herr KLINGENSTIERNA sahe sie um 9 Uhr 48 Min. 8 Sec. Herr WARGENTIN hatte 10 oder 12 Secunden lang noch ein Merkmaal von der Venus zu äusserst am Sonnenrande, wie einen kleinen schwarzen Punkt gesehen, dessen Verschwindung man jeden Augenblick erwartete, und sahe endlich den letzten Blick von ihr um 9 Uhr 48 Min. 9 Sec.

The note of WARGENTIN, at $3^h\ 38^m\ 27^s$, belongs, I think, not to an apparent contact, but to the line of light around the dark limb of Venus.

All the observations of egress evidently belong to the thread of light, with the possible exception of WARGENTIN'S observation of egress, where the employment of the pluperfect tense might suggest a later phase. With this possible exception the observations seem very trustworthy.

HERNOSAND.

[*Schwedische Abhandlungen*, 1761, p. 159.]

Observers, GISSLER and STRÖM.—Die Zeit berichtete man durch tägliche Beobachtungen des Durchganges der Sonne durch eine Mittagslinie, welche der Herr Observator SCHENMARK 1751 im Saale des Gymnasiums gezogen hatte und die man jetzt durch übereinstimmende Sonnenhöhen geprüft hatte.

Um 3 Uhr 38 Min. 26 Sec. senkte sich die Venus ganz und gar innerhalb des Sonnenrandes hinein, wie es Herrn GISSLER schien. Aber nach Herrn STRÖM'S Urtheile war solches 9 Secunden später geschehen. Doch schien es noch nach Herrn GISSLER'S Anmerkung, als befände sich ein spielender und färbender Schatten zwischen den Rändern der Sonne und des Planeten bis um 3 Uhr 39 Min. 23 Sec., da dieser Schatten den Sonnenrand verliess.

Um 9 Uhr 28 Min. 52 Sec. kam es Herrn GISSLER vor als berührte der Planet fast den westlichen Rand der Sonne, aber der Sonnenrand ward nicht eher als um 9 Uhr 29 Min. 21 Sec. durchbrochen.

Um 9 Uhr 46 Min. 35 Sec. verlor Herr GISSLER die letzte Spur der Venus in der Sonne, aber Lector STRÖM sah sie noch 12 Secunden länger.

Noteworthy, but not unprecedented, is the anomaly of GISSLER'S Contact III, tangency of limbs being noted before breaking of the thread of light.

UPSALA.

[*Schwedische Abhandlungen*, 1761, p. 143.]

Observers, MALLET, STRÖMER, and BERGMANN.—Um 3 Uhr 19 Min. 0 Sec. war Venus noch nicht eingetreten.

Um 3 Uhr 20 Min. 45 Sec. bemerkte er, dass ostwärts aussen am untern Sonnenrande, wie wenn da eine Grube wäre, was mehr fehlte, als die vorerwähnten Aushöhlungen betrugen. Er gab solches zu erkennen, und sahe den Rand der Venus ganz wollicht, innerhalb sechs oder acht Secunden, bemerkte er, dass diese Grube ansehnlich in ihrer Breite zunahm, welches die Gegenwärtigen versicherte, Venus sei nur vor kurzem angelangt. Aus dem ersten Anblicke schloss er, die aüssere Berührung der Venus und der Sonne habe sich nicht eher als um 3 Uhr 20 Min. ereignet.

Sobald Venus bis auf drei Viertheile in die Sonne getreten war, bemerkten alle Beobachter einen schwachen Schein oder Rand, der das übrige Viertheil umgab, und die Venus ganz rund zeigte. (Fig. 6.) Herr MALLET sah zugleich im Telescop dass die Sonne kleine Hörner ausschoss, die Venus zu umfassen. Anfangs glaubte er es rühre von wallenden Bewegungen am Sonnenrande her, weil die Dünste übrigens eine kleine Undeutlichkeit verursachten, wie sich allemal bei Gegenständen zu ereignen pflegt, die dem Horizonte nahe sind, oder sonst von zarten Wolken und Dünsten bedeckt werden; als aber Venus näher in die Sonne herauf kam, zeigte sich das Ausschiessen von der Sonne kreisförmigen Figur noch deutlicher, wie es die kleinen Hörner bildete.

Um 3 Uhr 37 Min. 47 Sec. ohngefähr, war das, was bisher ist beschrieben worden, gar merklich an ihr zu sehen. Nachgehends wandte man alle Aufmerksamkeit an, der Venus innere Berührung am Sonnenrande zu beobachten. Als solche geschehen sollte, schien die Venus allen ganz und völlig in die Sonne hinein zu treten, sie hatte eine gleiche Rundung, ausgenommen wo sie dem Sonnenrande am nächsten war, da schien die schwarze Venus länglichter oder als streckte sich eine Erhöhung an ihr, an Gestalt wie ein Wassertropfen, bis an den Sonnenrand. (Fig. 7.) Dieses war allen ein unvermutheter Anblick, war aber sehr deutlich und genau von Herrn STRÖMER beobachtet. Die Venus schien ihm ein Stück in die Sonne hineinzugehen, ohne den Rand zu verlassen, an welchen von der Venus wie ein schwarzes Band ging. (Fig. 8.) Dieses Band ward bald schmäler und riss in einem Augenblick in der Mitte, da sich das eine Ende an den Rand der Sonne und das andere in die dunkle Venus zog, welche ihm da den achten oder sechsten Theil ihres Durchmessers innerhalb des Sonnenrandes schien.

Um 3 Uhr 37 Min. 43 Sec. sagte Herr BERGMANN, er sähe wie Venus den Sonnenrand verliesse, mit welchem sie zuvor vereinigt war. Er hatte 15 Secunden zuvor die Hörner der Sonne so weit von einander gesondert gesehen, dass dar Schein welcher um die Venus glänzte, sich deutlich zwischen ihnen zeigte.

Um 3 Uhr 37 Min. 56 Sec. schien es Herrn MALLET im Telescop, die Hörner der Sonne flössen zusammen, und umschlössen die Venus mit einem sehr schwachen Glanze, so dass er sich nicht zutraute, den Sonnenrand selbst zu sehen, bis Venus weiter in die Sonne hinein gekommen wäre. Indessen bemerkte er diesen Augenblick, von welchem an das Licht der Sonne beständig zusammenhängend schiene.

Um 3 Uhr 38 Min. 2 Sec. hatte der Herr Director MELANDER mit einem 16 füssigen Sternrohre gesehen, wie sich die Ränder der Venus und der Sonne von einander getrennt hatten, und es schien ihm 53 Secunden zuvor, als berührten die Umkreise beider Scheiben einander.

Um 3 Uhr 38 Min. 5 Sec. ist der Augenblick da Herr STRÖMER das beschriebene Band reissen sah.

Um 9 Uhr 28 Min. 0 Sec. schien ihm [STRÖMER] der Rand der Venus mit der Sonne ihrem zusammen zu treffen, und als dieser Augenblick aufgezeichnet war, und der Sonnenrand wieder

Um 9 Uhr 28 Min. 7 Sec. betrachtet ward, war selbige mehr geöffnet als er erwartete. Die Hörner der Sonne schienen ganz stumpf, und sollte man daraus geurtheilt haben, Venus sei noch ganz in der Sonnenscheibe, obgleich der Rand der Sonne dunkel und bedeckt war.

Um 9 Uhr 27 Min. 55 Sec. schien der Rand der Venus dem Herrn MALLET so nahe beim Sonnenrande, dass die Berührung geschehen müsse, wofern nicht der Sonnenrand ausgebogen

gewesen wäre und eine Erhöhung an der Grenze der Sonne gemacht hätte. Dieses schien einige Secunden nach einander zu währen, aber der Schein nahm dergestalt nach und nach ab, dass Herr MALLET nicht genau den Augenblick bemerken konnte, da sich der Sonnenrand öffnete, sondern dieses nur innerhalb 2 Secunden zu bestimmen im Stande war.

Um 9 Uhr 28 Min. 1 Sec. zeigte sich noch ein schwacher Schein des schmalen Sonnenrandes.

Um 9 Uhr 28 Min. 3 Sec. war er so stark geöffnet, dass Herr MALLET sich einbildete, Venus ginge noch ein klein Stück ausser dem Sonnenrande heraus. Man sah die Hörner der Sonne zwerch über, und ein Glanz umgab die Venus und zeigte ihre runde Gestalt ganz klar.

Um 9 Uhr 28 Min. 9 Sec. bemerkte M. BERGMANN, dass Venus die Sonne innerlich berührte, oder dass der Rand der Sonne schwarz ward und sich in Hörner theilte.

Indessen dass Venus aus der Sonne zu gehen fortfuhr, schien anfangs der ganze herausgekommene Theil mit einem schmalen und schwachen Glanze umgeben, nachgehends wie der herausgetrene Theil grösser ward, erstreckte sich der Glanz nur auf einen Theil der Venus.

Um 9 Uhr 46 Min. 29 Sec. war er ganz spitzig, wie eine Degenspitze, und verliess die Sonne in einem Augenblicke. Herr MELANDER hatte eben den Augenblick, und M. BERGMANN eine Secunde später.

CALMAR.

[*Schwedische Abhandlungen*, 1761, p. 160.]

Observer, WICKSTRÖM.— * * * Doch hielt Herr WIKSTRÖM das Auge beständig nach dem Theile des Sonnenrandes gerichtet, wo die Venus erwartet ward, bemerkte aber daselbst keine Veränderung die sich mit Gewissheit der Venus zuschreiben liesse, bis um 3 Uhr 19 Min. 16 Sec. da er wie eine schwarze Spitze, die in den Sonnenrand stäche, sah.

Um 3 Uhr 32 Min. 46 Sec. schien Herrn WIKSTRÖM die Venus völlig eingetreten, aber zwischen ihrem und der Sonne Rande wies sich noch kein Licht.

Um 3 Uhr 33 Min. 1 Sec. fing das Licht an zwischen den Rändern durchzustrahlen und man schloss also mit Gewissheit, sie sei nun völlig eingetreten.

Als sie einige Stunden langsam durch die Sonne fortgegangen war und sich dem Austritte näherte, verschwand endlich das Licht zwischen den Rändern dergestalt, dass Venus gleichsam einen dünnen Rauch von sich liess, der sich an den Sonnenrand anhängte, welcher Rand dadurch an selbiger Stelle seine Rundung verlor, sowohl als Venus die ihrige, dergestalt, dass sie spitzig aussah. (Fig. 8.) Diess geschah gleich um 9 Uhr 23 Min. 40 Sec. Sie ward nachgehends nicht eher an allen Seiten völlig und gleich rund, als um 9 Uhr 24 Min. 33 Sec. Um 9 Uhr 41 Min. 15 Sec. verliess Venus den Sonnenrand völlig.

CARLSCRONA.

[*Schwedische Abhandlungen*, 1761, p. 161.]

Observers, BERGSTRÖM and ZEGOLSTRÖM.—Beim Anfange des Austrittes war die Sonne mit dünnen Wolken bedeckt, so dass man kein gefärbtes Glas nöthig hatte.

Um 9 Uhr 20 Min. 0 Sec. schien es Herrn BERGSTRÖM, als verschwände der schwache Streifen, den er bisher vor der Venus noch vom Rande der Sonne gesehen hatte.

Um 9 Uhr 20 Min. 6 Sec. verschwand der Streifen vor Herrn ZEGOLLSTRÖM'S Augen, so, dass nach seinem Urtheile da die innere Berührung geschah.

Die Zeit der aüsseren Berührung war etwas schwerer recht genau zu bemerken, weil die letzten Ueberbleibsel der Venus sehr schwach wurden, und langsam verschwanden.

Um 9 Uhr 39 Min. 16 Sec. verschwanden sie Herrn BERGSTRÖM;

Um 9 Uhr 39 Min. 21 Sec. Herrn ZEGOLLSTRÖM, welcher alsdann zuerst den Sonnerand völlig rein fand.

LUND.

[*Schwedische Abhandlungen*, 1761, p. 163.]

Observer, SCHENMARK.—Um 9 Uhr 10 Min. 44 Sec. war Venus schon ihrem Austritte so nahe, dass ihr äusserer Rand die Sonne zu berühren schien. Doch war noch wie ein schwacher Schein zwischen beiden, aber indem kam eine Wolke, die auf einige Zeit Sonne und Venus unsichtbar machte.

LANDSCRONA.

[*Schwedische Abhandlungen*, 1761, p. 163.]

Observers, BREHMER, DEHN, and LANDBERG.—Um 9 Uhr 9 Min. 21 Sec. schien Herrn BREHMER alles Licht zwischen der Venus und dem Sonnenrande zu verschwinden, welches Herrn DEHN 3 Sec. später so vorkam, dazu trugen vielleicht die Wolken etwas bei, die im selbigen Augenblick vor die Sonne traten, aber nachdem solche fortgegangen waren, schien es Herrn LANDBERGEN, Venus bräche um 9 Uhr 9 Min. 48 Sec. durch den Sonnenrand. Sie nahm um 9 Uhr 27 Min. 23 Sec. völlig Abschied aus der Sonne.

VIENNA.

[*Ephemerides Vindobonenses*, 1762.]

In Vienna the second interior contact, which occurred in the forenoon, was lost by clouds. The external contact was observed as follows:

A.—In observatorio Cæsareo Regio publico Universitatis.

HELL.—Tubo Newtoniano $4\frac{1}{2}$ ped. cum lente 1 digiti in foco, cujus præstantia æquiparatur tubo dioptrico 30 aut 40 pedum, satis præcise a me observata est, videlicet $21^h\ 43'\ 10''$ solumque dubium paucorum secundorum superfuit.

HERBERTH.—Tubo dioptrico insigni 12 pedum $21^h\ 42'\ 44''$.

M. RAIN.—Tubo insigni dioptrico 9 pedum qui 24 pedum tubum æquat. $21^h\ 42'\ 49''$.

LYFOGORSKI.—Tubo Newt. 3 pedum excellente $21^h\ 42'\ 59''$.

B.—In observatorio Collegio Academici S. J.

Ill. CASSINI de THURY tubo=dioptrico 9 p. proprio, machinæ parallacticæ applicato $21^h\ 42'\ 49''$.

LIESGANIGG.—Tub ooptico tubum 17 vel 18 pedum. Lens objectiva eximiæ claritatis focum habet 11 pedum et 2 digitorum Parisinorum: ocularis 3 digitorum. Temp. astron. vero:

h.	′	″	
21	24	30	circiter videbatur per nubes interior limborum contactus jam contigisse.
21	42	49	dubie;
21	42	51	certe videbatur contigisse exterior . . . contactus.

SCHERFER (SCHERFFUR).—Tel. Newt. ped. 4 $21^h\ 42'\ 35''$.

STEINKELLNER.—Tubo 16 p. $21^h\ 42'\ 14''$.

MASTALIER.—Tubo $5\frac{1}{2}$ p. $21^h\ 42'\ 13''$.

TYRNAU AND LAIBACH.

[*Ephemerides Vindobonenses*, 1762.]

Observer, WEISS (at Tyrnau):

	h.	′	″
Internal contact	9	29	9
External contact	9	47	36
Duration	0	18	27

Observer, SCHOTTLE (at Laibach):

	h.	′	″
Internal contact	9	18	15
External contact	9	36	20
Duration	0	18	5

WETZLAS.

[*Ephemerides Vindobonenses*, 1762.]

Observer, "Illustrissimus Baro FELIX EHRMANS de SCHLUG."—Tubo insigni Newtoniano 4 ped. in arce sua Wezlas prope urbem Crembsium sita. Ad arcis hujus Wezlas differentiam meridianam a Viennensi determinandam præter eclipses solares atque lunares magno sane numero sat. J. obs. . . . . . 4′ 10″ vel 15″ occid. versus lat. 48° 36′ 30″:

	h.	′	″
Cont. int.	21	20	48
Cont. ext.	21	38	50
Duratio	0	18	2

Idem contactus a filio meo tubo quadrantis 6 pedum ita habetur $21^h\ 38'\ 29''$.

The times were from corresponding altitudes of the sun on the days preceding and following, but no numbers are given.

COPENHAGEN AND DRONTHEIM.

[*Paris Memoirs*, 1761, pp. 113–114.]

The observations at these points were made under the auspices of the Danish Government. The authority for the results is a communication from the French minister at the court of Denmark to Lalande, from which the latter prepared the following account of the observations:

COPENHAGEN.—Le ciel fut très-nébuleux à Copenhague le jour du passage; M. HORREBOW ne put observer l'entrée de Vénus, il détermina l'orbite de Vénus par plusieurs observations, dont le calcul est imprimé, et l'Académie en a reçu un exemplaire. A l'égard de la sortie, dont il n'est point parlé dans l'ouvrage imprimé, le commencement fut observé à $2^h\ 3'\ 30''$ sur une pendule réglée sur le temps du premier mobile, la fin à $2^h\ 21'\ 0''$, celle ci est plus exacte que la première, les temps vrais qui y correspondent sont $9^h\ 5'\ 36''$ et $9^h\ 23'\ 3''$, car ayant calculé l'ascension droite du Soleil en temps pour les deux momens d'observations, j'ai trouvé qu'il fallait ajouter à l'heure de la première observation, $19^h\ 2'\ 6''$; et à l'heure de la seconde $19^h\ 2'\ 3''$ pour les réduire au temps vrai. Au reste je suppose dans cette réduction que la pendule était rigoureusement montée sur le temps du premier mobile. M. HORREBOW m'a écrit que la différence ne pouvait être que très-légère, mais qu'au reste il la constaterait parfaitement, en vérifiant la position du mural dont il s'est servi.

The observations are in sidereal time. The reductions to mean time are $19^h\ 0^m\ 15^s$ and $19^h\ 0^m\ 12^s$, so that the record is quite correct in the reduction to apparent time.

DRONTHEIM.—Au mois d'avril 1761, M. le comte de HOLSTEIN, directeur de l'Université et président de l'Académie de Copenhague, chargea, par ordre du Roi, M. O. G. KRATZEINSTEIN, membre de la même Académie, de choisir dans l'Université deux mathématiciens pour aller observer le passage de Vénus en Norvège. M. BUGGE et M. HASCOW partirent à cet effet le 5 de mai, ils arrivèrent le 30 à Drontheim, le 5 juin les hauteurs correspondantes donnèrent le midi vrai à la pendule $0^h\ 59'\ 10\frac{1}{2}''$, mais le jour du passage le ciel fut couvert jusqu'à 9 heures, et l'on ne put observer que la fin et même au travers des nuages: ce fut avec une lunette de huit pieds que M. BUGGE observa le contact intérieur en temps de la pendule $10^h\ 2'\ 10''$ et le contact extérieur $10^h\ 18'\ 58''$, ce dernier est moins exact que le premier, suivant le rapport de l'observateur.

Le ciel ayant été couvert pendant les jours suivans, ce ne fut que le 15 de juin qu'on parvint à prendre encore des hauteurs correspondantes qui donnèrent le midi vrai $0^h\ 54'\ 9''$, la pendule ayant retardé $5'\ 1\frac{1}{2}''$ en dix jours, ou de $30\frac{1}{7}$ par jour, on trouve pour le temps vrai du contact intérieur observé $9^h\ 3'\ 27''$.

LEIPZIG.

[*N. C. P.*, x, p. 480.]

Temp. vero.
h. ' ''

9 7 40. *Observer*, HEINSIUS.—Contactus marginis occidentalis Veneris cum occidentali Solis limbo (situ nempe erecto, prout tubus objecta repræsentare solet) vel initium egressus, prope instare videbatur, siquidem linea tantum luminosa, admodum tenuis, limbum Solis clare jam conspicui, et occidentalem Veneris marginem, interjacebat, et peripheriam disci Solaris continuare videbatur.

9 7 57. Dubitare incipiebam, an non contactus iste jam locum haberet; interim tamen linea lucida valde gracilis ad marginem Veneris occidentalem adhuc se ostendebat.

9 8 7. Contactum nunc celebrari majori certitudine credebam, licet lineola ista luminosa gracillima ad occidentalem Veneris marginem, ast a reliqua disci Solaris peripheria nunc quasi separata, adhuc conspicua effet. Singulare scilicet phænomenum hic se offerebat, quod exactam contactus memorati, respectu momenti temporis respondentis, æstimationem valde impediebat. Linea nempe prædicta lucida, quæ longitudine nonnihil minor cernebatur, quam diameter disci Veneris ad V repræsentati, hujus limbo occidentali instar tangentis *ab* adhærebat, sic, ut Tab. xvi, Fig. 2. abrupta quasi continuationem reliquæ Solis peripheriæ, cujus pars ad AB exhibetur, nunc non referret, sed limbus Veneris occidentalis peripheriam Solis, pauxillum licet, penetrasse videretur. Novum huc accedebat phænomenum non expectatum. Scilicet extra Solem disci alicujus nigricantis partem, quæ notabile disci Veneris segmentum æmulabatur, in situ opposito ad D cernere credebam, ita, ut linea lucida *ab* instar tangentis communis, ejus peripheriam et marginem Veneris occidentalem separaret. Nonnunquam loco segmenti ad D, fascia nigricans, quæ latitudine diametrum disci Veneris æquabat, sic conspiciebatur, ac si discus Veneris umbram, terminis parallelis comprehensam, extra discum Solis projiceret, interjecta tamen ad *ab* lineola. Fallaciam visus hoc phænomenum denotare quisque sibi persuadebit. Interim tamen istud perdurabat usque ad

9 8 27. Quo momento linea lucida *ab*, sæpius memorata, prorsus evanescebat. De hoc disparitionis momento intra duo temporis secunda certus sum. Sane ante 5 secunda occidentali Veneris limbo, in regione puncti contactus, quod linea *ab* antea effecit, flammula aliqua, gracillima licet, adhæserat. Post momentum $9^h\ 8'\ 27''$ nihil lucidi amplius ad occidentalem Veneris marginem discernere potui.

9 8 37. Initium egressus Veneris certissime peractum erat; nec ullum lucis vestigium ad marginem Veneris occidentalem extabat.

I have taken $9^h\ 8'\ 2''$, the mean of the second and third observations, as the moment of contact, whereas ENCKE has taken $9^h\ 8'\ 7''$. The line of light which the observer describes as continuing until $9^h\ 8'\ 27''$ I consider to be the atmospheric line around Venus. If so, the line noted at $9^h\ 8'\ 7''$ as "a reliqua disci Solaris peripheria nunc quasi separata" must have been this Venus line, and contact had passed. The figure showing the phenomena at $9^h\ 8'\ 7''$ gives color to this view. If, on the other hand, the line in question was not seen at all, then contact did not occur until $9^h\ 8'\ 27''$.

No explanation of the optical illusion, which the observer describes, can be given.

MUNICH AND INGOLSTADT.

I have two brochures, bound in one, bearing the respective titles:

(1) Observatio | transitus ♀ per discum ⊙ die astrono | mica 5^{ta} Junii 1761. | In | Observatorio Monacensi, | Cujus elevatio poli $48^h\ 9'\ 55''$ observata est, dif | ferentia vero horaria a meridiano observatorii Parisini supponitur | $36^h\ 50'$ (cum nulla hucusque certa observatione definiri potuerit) | facta. (No place or date.)

(2) Observatio | transitus Veneris per discum Solarem 6 Junii 1761. | Ex Observatorio Collegii Societatis JESU Ingolstadii. | Una cum conclusionibus astronomicis, quantum licuit, inde deductis | P. GEORGIO KRAZ S. J. in Catholica & Electorali | Universitate Matheseos Professore ibidem.

From (1): Ope quadrantis radio trium pedum Parisinorum descripti cui tubus dioptricus $3\frac{1}{2}$ circiter pedum affixus. Micrometrum ejus constat quatuor filis immobilibus in foco tubi sese ad angulos semirectos intersecantibus, et uno mobili horizontali filo paralelle incedenti. Pendulum astronomicum ab artifice Weilheimensi fabrefactum adhibitum fuit, cujus acceleratio supra motum medium ⊙ intra 24 horas erat 10 sec.

	h.	′	″
Tempus penduli die 5ta Junii in meridie vera erat	23	57	53
Die 6ta sequenti	23	58	17

* * * * * * * * *

Contactus limbi ♀ occ. cum limbo ⊙ occid. contigit in tempore penduli $21^h\ 4'\ 0''$ ergo in temp. vero $21^h\ 5'\ 46''$. Emersio totalis ♀ sive contactus ejus limbi orientalis cum limbo ⊙ occid. contigit in tempore penduli $21^h\ 22'\ 2''$ ergo in tempore vero $21^h\ 23'\ 48''$. Differentia emersionum igitur $18'\ 2''$.

From (2): Ut porro circa tempus verum, ad quod omnes observationes fuere reductæ, error non irreperet, mane post quatuor primas observationes, signo dato horologia inter sese comparavi, ubi horologium observatorii ordinarium signabat $5^h\ 38'\ 0''$ normale vero in cubiculo $5^h\ 27'\ 8''\ 30'''$ ut proin tunc discrimen fuerit $10^h\ 51'\ 30''$. Et rursus post finitum transitum primum horol. indicabat $9^h\ 30'\ 0''$ alterum $9^h\ 19'\ 9''\ 45'''$ ubi jam discrimen, erat $10'\ 50''\ 15'''$. Unde, cum horologium normale, centro ⊙ meridianum subeunte, signaret diebus præcedentibus, nimirum

	h.	′	″
Maji 30	11	51	16
31	11	51	22
Junii 4	11	51	49
5	11	51	56

ac proin hisce diebus differentia temporis ab uno meridie usque ad alterum fuerit $6''$ et $7''$ ab octava autem Junii (nam a quinta usque ad hanc diem ⊙ in meridie non adparuit) & sequentibus diebus $9''$ et $10''$ adeoque a quinta ad sextam $8''$, sive eodem die tempus horologii in ipso meridie $11^h\ 52'\ 4''$,

erant tempori observato ea ipsa die circa 9^h demenda $10' \, 50'' \, 15'''$ et vicissim addenda $7' \, 57''$ ac cum proportione horis præcedentibus, ut haberetur tempus verum. Quod autem linea meridiana cum meridiano observatorii omnino congruat, id habeo ex certissimis, & tutissimis mediis, quibus eandem decennio abhinc duxi, & duobus deinceps annis variis intermediis temporibus eodem modo exploravi, ita, ut nunquam integri minuti secundi discrimen deprehenderim. Differt porro meridianus mostri observatorii a meridiano Observatorii Parisiensis $36' \, 10''$ sub altitudine poli $48° \, 45' \, 45''$.

* * * * * * * * *

	h.	′	″	‴
Circa egressum ♀ ex disco ⊙ adparentem tempus contactus limbi ⊙ & ♀ occidentalis	9	4	59	30
Centri ♀ limbum ⊙ occident subeuntis	9	14	6	0
Contactus limbi ⊙ occident, & ♀ orient.	9	23	4	30
Intervallum temporis inter utrumque contactum	0	18	5	0

BOLOGNA.

[*Ephemerides Vindobonenses*, 1762; *Philosophical Transactions*, 1761, p. 399.]

The observations were as follows:

Names.	Internal.			External.			Tubo.
	h.	m.	s.	h.	m.	s.	Ped.
ZANOTTI	21	4	34	21	22	30	2½
MARINUS	21	4	58	21	23	0	10
MATHEUCIUS	21	4	58	21	23	7	22
In conclavi inferiori.							
FRISI	21	4	54	21	22	53	6
CASSALI	21	5	0	21	22	50	8
CANTERZANI	21	4	56	21	22	59	11

The names of the three observers in the "conclavi inferiori" are quoted from ENCKE.

FLORENCE.

[*Ephemerides Vindobonenses*, 1762.]

LEONARDO XIMENES, S. J., all' observatorio di S. Gio. Evangelista e ridota al tempo vero del Meridiano Florentino:

	h.	′	″
Contactus int.	9	4	28.4
Contactus ext.	9	22	56.4
Duratio emersionis	0	18	28.0

Emersionis toto tempore aer erat aliquantum caliginosus ob continuos vapores terrestres, qui discum Solarem obtegebant. Contactus interior mihi videtur admodum accurate observatus, et in contactu exteriore paucorum secundorum solum dubium superest.

Tempore transitus Veneris discus Veneris apparebat bene terminatus et rotundus, sine iride, sine annulo, sive lucido, sive obscuro, sine ulla inequalitate aut prominentiis in circumferentio.

XIMENES also found:

	m.	s.
Diam.⊙ by 10 transits	2	16.87
	=15	48.50
by microm.	15	47.7
Diam. Venus, 4 meas.	0	1′.94

DILLINGEN.

[*Ephemerides Vindobonenses*, 1762.]

R. P. HAUSER, S. J., tubo dioptrico 8 p.:

	h.	m.	s.
Contactus int.	9	0	20
Contactus ext.	9	18	20
Duratio	0	18	0

GÖTTINGEN.

[*Encke, die Entfernung der Sonne, Gotha*, 1822, p. 158.]

ENCKE quotes the observations of TOBIAS MAYER as follows:

Venus e Sole egrediens 1761 *Jun.* 6, *a. m.*

Tempus horolog.			Tempus verum.			
h.	m.	s.	h.	m.	s.	
12	2	12	8	58	26	Contactus interior exacte.
	5	10	9	1	23	Quadrans periph. deest circ.
	7	10		3	23	Triens.
	11	17		7	30	Semissis periph.
	16	44		12	56	Triens periph. restat circ.
	17	45		13	57	Quadrans.
12	20	43	9	16	54	Totalis egressus exacte.

WÜRZBURG.

[*Ephemerides Vindobonenses*, 1762.]

Observer, HUBERT:

Init. em. in tubo Gregoriano	9	1	12
Init. em. in camera obscura	9	1	43
Em. tot. in tubo ast. 3 p.	9	18	0
Em. tot. in camera	9	18	7
Em. tot. in tubo 7½ p.	9	18	35
Em. tot. in tubo Greg.	9	18	49
Duratio in tubo Greg.	0	17	37

SCHWETZINGEN.

[*Philosophical Transactions*, 1764, p. 162.]

Observer, CHRISTIAN MAYER:

	True time.
	h. ′ ″
The interior contact of the western limb of Venus with the western limb of the Sun, observed with DOLLOND's telescope	20 53 8
The moment of the egress wherein the same limb of the Sun, after the interior contact, first appeared corniculated, most accurately observed with the same telescope, was	20 53 35
Whence I concluded that the interior contact happened	20 53 33½

The record in the *Ephemerides Vindobonenses* gives only the second of the observed times, $8^h\ 53'\ 35''$. The interpretation to be put upon the observation is doubtful; but it seems likely that the second time is that when there was a marked separation between the horns. Altogether, I think the mean to be the most probable time of contact.

LYONS.

[*Paris Memoirs*, 1761, p. 473.]

PINGRÉ here remarks: "Le P. BÉRAUD a observé le contact intérieur à Lyon, a $20^h\ 38'\ 44''$: Lyon est marquée, dans la *Connaisance des Temps*, comme étant $9'\ 59''$ à l'est de Paris."

LEIDEN.

[*Philosophical Transactions*, 1761, p. 257.]

Tandem hora $8^h\ 26'\ 50''$ [$8^h\ 36'\ 50''$?], tempore vero, observavi contactum interiorem, sed per nubes tenuiores, ita ut vitrum fumo inquinatum, imo vitra cœrulea et viridia (quæ ex præscripto Cl. De L'ISLE ad manus erant, purissima), seponere debuerim.

MONTPELLIER.

[*Zach, Correspondance Astronomique*, I, 1818, p. 246.]

1761, *le* 5 *Juin, passage de Vénus.*

		h.	m.	s.	
Entrée . .	Cont. extér.	20	34	54	par TANDON lunette de 18 pieds.
		20	35	0	par ROMIEU lunette de 10 pieds.
		20	35	14	par De RATTE lunette de 14 pieds.
	Cont. intér.	20	52	6 à 11″	par TANDON.
		20	53	21	p. ROMIEU, De RATTE, BRUN.

The editor has evidently got the observations inverted, the phase being egress, not ingress. There is probably some mistake in printing the observation of Contact IV by TANDON, since, with his more powerful instrument, he should have been the last to note the contact.

CHÂTEAU DE SAINT-HUBERT.

[*Paris Memoirs*, 1761, pp. 74–76.]

Observer, Le MONNIER.—"J'ai déterminé avec la lunette de 18 pieds le premier contact ou contact intérieur des deux disques à $8^h\ 32'\ 38''$ de la pendule, la circonférence étant un peu ondoyante, ce qui donne $8^h\ 26'\ 23''$ de temps vrai.

"À $8^h\ 51'\ 09\frac{1}{2}''$ dernier contact ou séparation des deux disques fort exactement, ce qui répond à $8^h\ 44'\ 51\frac{1}{2}''$ de temps vrai, et la durée de l'émersion de $0^h\ 18'\ 28\frac{1}{2}''$."

Observer, De la CONDAMINE.—"Instant de la sortie totale à $8^h\ 51'\ 11''$ de la pendule, c'est-à-dire, une seconde et demie plus tard que selon mon observation; M. de la CONDAMINE n'a pas été aussi assuré du premier contact ou contact interne, et la durée de l'émersion n'a été, selon lui, que de $18'\ 07''$ de temps écoulé à la pendule."

The clock was gaining at the rate of 9 seconds and a half per hour on apparent time, which accounts for the difference between the clock corrections applied. LE MONNIER's observation gives no ground whatever for assigning it to any particular phase. It may be regarded as of medium quality. It is deducible from the statement that the other observer noted the internal contact at $8^h\ 33'\ 4''$, clock time, and therefore 26 seconds later than his companion. His doubt about the time may be partly due to its deviation from Le MONNIER, and so hardly justifies the entire rejection of his observation. We may, however, consider it as belonging to the fourth class.

PARIS OBSERVATORY.

[*Ibid.*, 1761, p. 76.]

Observer, MARALDI.—Le ciel était parfaitement serein au temps de la sortie de Vénus du disque du Soleil. J'avais laissé reposer mes yeux pendant plus d'un quart d'heure, et je me suis servi d'une lunette de Campani, de 15 pieds; ainsi je crois avoir observé les phases suivantes avec la plus grande précision qu'il soit possible:

	Temps vrai. h.	′	″	
A	8	28	42	le bord de Vénus concourt avec le bord du Soleil, ou commencement de la sortie.
	8	46	54	contact extérieur ou sortie totale de Vénus.

Observer, BELLÉRI.—M. BELLÉRI a observé le contact intérieur à $8^h\ 28'\ 14''$ et le contact extérieur à $8^h\ 46'\ 40''$, avec ma lunette de 6 pieds.

ENCKE adds observations by ZANONI at $20^h\ 26^m\ 45^s$ and $20^h\ 45^m\ 5^s$, mean time. I do not find them in the Memoirs, and suppose they may be taken from the *Ephemerides Vindobonenses*.

MARALDI's observation may be placed in the same class with Le MONNIER's. His use of the word "concourt," if strictly construed, might be held to indicate the first formation of the dark ligament. But I see no reason for considering that MARALDI really observed anything different from the phase noted by others.

Mr. BELLÉRI's observation should be placed a grade lower on account of being made with a small instrument, and is therefore assigned to the third class.

CONFLANS-SOUS-CARRIÈRE.

[*Ibid.*, p. 80.]

Observer, De la CAILLE.—Le contact intérieur m'a paru se faire à $8^h\ 28'\ 54''$, temps vrai, lequel réduit au méridien de Paris, est à $8^h\ 28'\ 37''$ à $38''$: je n'ai pas hésité long-temps sur la certitude de ce moment. La sortie totale m'a paru à $8^h\ 47'\ 6\frac{1}{2}''$; ce qui réduit au méridien de Paris, serait $8^h\ 46'\ 49''$ ou 50.

M. TURGOT de Brucourt a jugé le contact intérieur 17 secondes après moi, et la sortie totale 20 secondes avant; il se servait d'une lunette de 12 pieds, qui était assez difficile à manier à cause de l'incommodité du lieu; d'ailleurs sa vûe était un peu fatiguée.

M. BAILLY ne put observer le commencement de la sortie, mais il estima la fin 3 à 5 secondes après moi, avec une lunette de près de 6 pieds.

LACAILLE's instrument was a 5-foot DOLLOND telescope of 19 lines aperture.

COLLÉGE DE LOUIS LE GRAND.

[*Ibid.*, pp. 80, 81.]

Observer and observations as quoted by LACAILLE.—Le R. P. de MERVILLE, professeur de mathématiques, observa avec un excellent télescope Newtonien de 6 pieds l'attouchement intérieur des bords du Soleil et de Vénus à $8^h\ 28'\ 40''$ et l'extérieur à $8^h\ 47'\ 4''$; il crut voir autour de Vénus assez constamment une espèce de nébulosité, mais ce phénomène ne lui parut pas assez décidé pour en tirer la preuve d'une atmosphère.

Le R. P. CLOUET, qui observait à côté du R. P. de MERVILLE avec un télescope de 32 pouces qu'il avait construit lui-même, marqua le contact intérieur de Vénus et du Soleil à $8^h\ 28'\ 26''$ et l'extérieur à $8^h\ 46''\ 55''$.

In none of these observations does there appear to be any ground for assigning a special phase. LACAILLE's observations may be placed in the medium or second class on account of his skill and experience, while all the others will be placed in the third.

PALAIS DU LUXEMBOURG.

[*Ibid.*, p. 83.]

Observer, La LANDE.—Je me plaçai dans la situation la plus commode, afin de ne rien ôter à l'extrême attention que je voulais y apporter. L'air était calme, le Soleil bien terminé; enfin toutes les circonstances favorables, lorsque je vis à $8^h\ 28'\ 25''$ ou $26''$ au plus tard, très-certainement et très-exactement, comme un point noir qui se détacha de Vénus pour joindre le bord du Soleil. J'attendis encore quelques momens pour avoir une entière confirmation; mais à $8^h\ 28'\ 30''$, c'est-à-dire $4''$ plus tard, les deux disques étaient très-unis, et l'on ne pouvait plus douter que le moment du contact ne fût passé; en sorte que le moment où je l'ai assigné, ne me parait pas pouvoir être sujet à 2 secondes d'incertitude; aussi l'observation faite par M. MESSIER à l'hôtel de Clugny, avec un télescope catoptrique de 5 pieds, ne diffère que de 2 secondes de la mienne, en ajoutant à celle-ci 2 secondes pour la différence des méridiens, car M. MESSIER observa à $8^h\ 28'\ 30''$.

À $8^h\ 46'\ 46''$ je commençai à croire que Vénus quittait le bord du Soleil; à $8^h\ 46'\ 54''$ j'en étais entièrement assuré, de sorte que je crois pouvoir assigner la sortie à $8^h\ 46'\ 50''$. Mais cette observation ne m'a pas paru susceptible d'une aussi grande précision que la première; M. BAUDOUIN, à l'hôtel de Clugny, avec une très-bonne lunette de 25 pieds, l'observa à $8^h\ 46'\ 46''$, et M. MESSIER

à $8^h\ 46'\ 37''$. Je ne puis avoir sur le temps vrai aucune incertitude, j'ai pris le 5 et le 7, avec le même secteur de 6 pieds, des hauteurs correspondantes de Soleil qui s'accordaient très-bien; et j'avais examiné depuis quinze jours la marche de la pendule, qui était très-régulière.

La LANDE's observation seems to be very exact and satisfactory, and I think, from his description, that the mean of his times noted, or $20^h\ 28^m\ 28^s$, should be taken as that of the fully formed contact, and as most likely to be comparable with the other observations.

PASSY (CABINET DE PHYSIQUE).

[*Ibid.*, p. 99.]

Observers, De FOUCHY, FERNER, and NOËL.—Je ne pus observer le premier contact que M. FERNER et DOM NOËL observèrent à près de deux secondes l'un de l'autre; en prenant un milieu, on aura le premier contact à $8^h\ 28'\ 15''$ de temps vrai à La Muette si l'on y ajoute 14 secondes ½ pour la différence des méridiens à $8^h\ 28'\ 29''$, réduit au méridien de l'observatoire.

J'observai le dernier contact ou la sortie absolue de Vénus à $8^h\ 46'\ 26''$ et M. FERNER à $8^h\ 46'\ 27''$, ce que réduit au méridien de l'observatoire donne l'heure de cette sortie à $8^h\ 46'\ 41'$, et la durée totale de la sortie de Vénus de $18'\ 12''$.

Pendant les derniers phases, et surtout à la sortie, les bords de Vénus et du Soleil paraissaient très-ondoyans, ce qui venait probablement de ce que nous ne voyions l'un et l'autre qu'à travers les vapeurs de la Seine qui était devant nous.

M. FERNER et DOM NOËL qui observèrent le premier contact virent tous deux le disque de Vénus s'allonger lorsqu'il fut à une certaine distance du bord du Soleil, ce que venait, selon toute apparence de ce que Vénus ayant atteint le vrai bord du Soleil, fit disparaître en cet endroit la couronne d'aberration, ou cette augmentation optique qui accompagne ordinairement l'image des corps lumineux dans les lunettes.

It would seem from the statement of FOUCHY that the elongation marked by the two other observers occurred before the time of contact, and may therefore be considered as mainly an optical illusion. Here, as in the other French observations, I see no reason for deciding on a special phase as that observed.

HÔTEL DE CLUGNY.

[*Paris, Académie, Mémoires presentés par divers Savans*, T. VI., 1774, p. 435.]

Observer, M. LIBOUR:

	h.	′	
Commencement de l'émersion ou 1er contact	8	28	31
Fin de l'émersion ou 2e contact	8	46	43
Durée	0	18	12

Cette observation a été faite dans l'observatoire de M. de L'ISLE, à l'Hôtel de Cluni, rue des Mathurins, avec un télescope Newtonien de 4 pieds et demi; j'ai déterminé le temps par les observations du midi, faites les 5, 6 et 7 par des hauteurs correspondantes du Soleil que M. MESSIER avait prises le 4 et qu'il m'a communiquées.

ROUEN.

[*Ibid.*, p. 43.]

Observers, Messrs. BOUIN and DULAGUE.—Vers 8 heures un quart, nous étant aperçus que Vénus était déjà très-près du bord du disque, nous préparâmes les lunettes que nous avions destinées à voir la sortie, en cas qu'il nous fut possible d'apercevoir quelque chose à travers les nuages et les brouillards.

	h.	m.	s.	
A	8	28	12	temps vrai, méridien de Rouen, l'attouchement intérieur était déjà fait.
	8	34	50	la planète avait environ la moitié de son disque hors celui du Soleil.
	8	35	20	nous jugeâmes la sortie du centre de Vénus mieux que la précédente. Enfin à
	8	41	32	M. DULAGUE, avec une lunette de 9 pieds, fixa la sortie totale, et moi avec une de 16.
	8	41	38	je cessai de voir le disque du Soleil échancré. Nous ne fûmes point obligés de nous servir de verre noirci; ainsi le Soleil était dans sa couleur blanche et naturelle.

The first observation seems to have been made after internal contact had passed, and is not used.

BAYEUX.

[*Ibid.*, pp. 133, 134.]

Observer, L'Abbé OUTHIER:

h.	m.	s.	
8	17	00	ou 17^m 10^s le bord de Vénus touche le bord du Soleil.
8	27	00	Vénus sortie à peu-près la moitié.
8	35	15	Vénus sort entièrement; très bien vu.

Nothing is stated about clock or time, and the way the times are given lead us to suspect that OUTHIER could not read individual seconds from his time-piece.

GREENWICH.

[*Philosophical Transactions*, 1761, p. 173.]

BRADLEY being too ill to make observations, those actually made were communicated to the Royal Society by the Rev. NATHANIEL BLISS. There were three observers, BLISS, BIRD, and GREEN. It is stated that all of them agreed in assigning 20^h 19^m 0^s as that of second interior contact. Nearly the same agreement was found for the exterior contact at 20^h 37^m 9^s, apparent time. But the observations were not really independent of each other, and are equivalent to only a single observation.

HACKNEY AND CLERKENWELL.

[*Ephemerides Vindobonenses*, 1762, "*e litteris Domini Birch.*"]

	Tempus verum.	Greenwich.
	h. ′ ″	h. ′ ″
In Platea Scholarum à Heberdem	8 18 29	8 18 56½
In loco Hackney à Ellicot	8 18 45	8 18 56

I have assumed the "Platea Scholarum" to be at Clerkenwell on the authority of ENCKE.

LONDON.

[*Philosophical Transactions*, 1761, pp. 181, 183.]

Observers, SHORT and BLAIR (at Savile House) and CANTON (in Spital Square).

As no statements whatever are given respecting the phenomena, it seems sufficient to quote the results, which are as follows:

(I) at Savile House:

	h.	′	″
Internal contact by Mr. SHORT, through a reflector of 2 feet focus, magnifying 140 times	8	36	21½
Total exit by Dr. BLAIR, through a reflector of 18 inches focus, magnifying 35 times	8	36	12½
Total exit by Mr. SHORT, through a reflector of 2 feet focus, magnifying 140 times	8	37	05½

(II) in Spital Square:

The time, by the clock, of the internal contact was	8	17	4
Of the external contact	8	35	27
Of noon	11	58	24½
Therefore the apparent time of the first contact was	8	18	41
Of the last contact	8	37	4

It may be assumed that the clock corrections were well determined.

CHELSEA.

[*Philosophical Transactions*, 1761, p. 190.]

Observer, SAMUEL DUNN.—The egress was observed as follows:

h.	′	″	
8	16	41.	No diminution of light between the limb of Venus and that of the Sun.
8	16	42.	Slight penumbra or diminution of light near where the contact was to be.
8	16	43.	Penumbra of a gray color near the same place.
8	16	44.	Penumbra almost brown, and the thread of light very narrow and almost lost.
8	16	45.	Penumbra brown, and the thread of light in the contact point indistinct or soft.
8	16	46.	Penumbra more brown, and the touch the smallest possible.

8	16	47.	Penumbra almost black, and the touch a little broader.
8	16	48.	Slight black in the point of contact, and the edges a little broader.
8	16	49.	True black in the point of contact, and the edges a little broader.
8	16	50.	More so. Here I concluded with myself that observers would differ in their judg-
8	16	51.	More so. ments about the moment of contact some seconds of time, or that some
8	16	52.	More so. would estimate the contact sooner than others.

From these observations I conclude that the thread of light, at the point of contact, was so obscured as to be indiscernible at 8^h 16′ 46″, and that true black did not succeed in the same point till 3″ after, namely, 8^h 16′ 49″, and from both of these statements I conclude that the real internal contact was at 8^h 16′ 47″ by the clock, which makes 8^h 16′ 11″ mean time and 8^h 18′ 2″ apparent time at Chelsea and 8^h 18′ 43″ apparent time at Greenwich.

A collection of very fine diagrams is given, showing the phenomena of contact and the gradual progress of the thread of light towards extinction. From these diagrams the time of true contact would seem to be fixed with great precision at 8^h 16′ 49″ clock time. Moreover, it would seem from the description just given that at this time the horns were distinctly separated, since the edges of which he speaks must have separated the horns. On the other hand, he himself fixes the time as 8^h 16′ 47″. On the whole, I think the most probable time to compare with other observers is 8^h 16′ 48″, which gives 8^h 18′ 3″ apparent time.

The external contact would appear to have been observed with equal accuracy at 8^h 35′ 5″ by the clock, or 8^h 36′ 20″ apparent time. If the actual precision of these observations corresponds to the description, they must have been the most accurate made on the transit. But it can not be supposed that an observer could note and record the changes, second by second, as they are given.

SHIRBURN CASTLE.

[*Philosophical Transactions*, 1761, p. 176.]

These observations are given by Nathaniel Bliss, in connection with those at Greenwich, as follows:

Mr. Hornsby at 20^h 15′ 10″ apparent time.

Mr. Phelps 4 seconds later.

The latitude of Shirburn Castle is given as 51° 39′ 22″ and the longitude 4′ 1″ west.

LISKEARD.

[*Philosophical Transactions*, 1761, p. 203.]

Observer, Rev. Mr. Haydon.—The internal contact was observed at 8^h 10′ 0″, clock time, and the total egress at 8^h 29′ 3″. This duration of 19′ 3″ is too great by about a minute. His clock error was well determined, but his method of fixing the moment by the clock was somewhat doubtful, as he had to call out to a boy in a room below when to note the time. There is no certainty that the observation was made in the town, and, altogether, it does not seem worth using.

MADRID.

[*Ephemerides Vindobonenses*, 1762.]

The following extracts are said to be from a work entitled "Observacion del Trànsito de Venus * * * hecha en el observatorio del Colegio Imperial de la Compañía de Jesus de Madrid por el P. CHRISTIANO RIEGER, S. J.," and from letters of said RIEGER.

Emers. cont. interior:

	h.	′	″
Tubo 2 ped. 9 dig. catadipotrico	20	6	53
Tubo majore	20	6	54
Tubo 8 pedum	20	6	56

In Collegio Imperiali et Collegio Nobilium facta observatio interioris contactus a nostris solum 4 sec. differt. In contactu exteriore major habetur discrepantia. Nostræ sic habent

	h.	′	″
Tubo 2 p. 9 dig.	20	24	32
Tubo majore	20	24	33
Tubo 8 p.	20	24	53

Differentia est 21 sec.

	′	″
Duratio emersionis	17	39
	17	36
	17	57

Probably 17′ 36″ should be 17′ 39″.

[*Philosophical Transactions*, 1761, p. 251.]

Observer, P. ANTONIUS EXIMENUS:

	°	′	″
Secundum recentissimas observationes altitudo poli	40	25	0
Differentia temporaria respectu Parisiorum	0	24	8

Facta est observatio cum quadrante duorum pedum cum dimidio, constructo a D. GEORGIO ADAMS, et cum horologio constructo a D. ELLICOT.

	h.	′	″	‴
Die 5ª Junii versabatur Sol in meridiano secundum altitudines correspondentes, ad	11	54	35	0
Debuerat versari secundum ephemeridas D. De la CAILLE, ad	11	58	0	3
Die 6ª secundum altitudines correspondentes, erat Sol in meridiano ad	11	54	50	0
Debuerat esse secundum dictas ephemeridas, ad	11	58	10	28

* * * * * * * * *

His elementis poterant corrigi tempora observationis; consultius tamen visum est incorrecta relinquere; ut quilibet possit illa corrigere iis elementis, quæ ipsi exactiora videantur. Sunt igitur tempora, quæ deinceps notabimus, quæ dabat horologium.

* * * * * * * * *

	h.	′	″
Contactus interior	8	1	44
Contactus exterior		19	23

De contactu exteriore per tria aut quatuor secunda, de interiori vero vix dubitavi.

ENCKE either ignores this observation of EXIMENUS or treats it as identical with the second observation just quoted from HELL'S *Ephemerides Vindobonenses*, which he assigns to XIMENES as observer. Whether he had any other ground for this course than the exact accordance of "durations" and the close accordance of times, I do not know. The fact that he gives the names of the three observers would indicate that he had some other source of information than the extract made by HELL, which does not name any observer except RIEGER. On the other hand, the statement of EXIMINUS that his observation was made with the 2½-foot quadrant seems inconsistent with ENCKE's view.

The clock corrections of EXIMINUS to apparent time, given by the corresponding altitudes, are:

	m.	s.
June 5	+5	25
June 6	+5	10

This gives $+5^m\ 12^s.4$ as the correction at the time of contact, whence we have:

	h.	m.	s.
Interior contact, apparent time	20	6	56
Exterior contact	20	24	35

On the whole, it seems likely that this observation is one of the first two quoted by HELL.

PORTO.

[*Mémoires présentés à l'Académie par divers Savans*, VI, p. 352.]

Observer, M. THEODORO DE ALMEIDA.—L'observation a été faite avec un télescope Gregorien de deux pieds de foyer, un verre vert, et un autre enfumé; la pendule réglée par les hauteurs correspondantes prises avec un quart-de-cercle de deux pieds et demi de rayon.

	h.	′	″
Contact intérieur du bord de Vénus, le 6 juin, temps vrai, au matin	7	44	5
Contact extérieur	8	2	39
Durée de la sortie		18	34
Le diamètre de Vénus			59′

ST. JOHN'S, N. F.

[*Philosophical Transactions*, 1764, p. 279.]

Observer, Professor JOHN WINTHROP (of Harvard College)—Mr. WINTHROP'S account is deficient in data for clock error. It may be inferred, however, from a letter to JAMES SHORT, published in connection with his observations, that his clock error was obtained by equal altitudes made in the most approved way. He could see only the egress, which occurred shortly after sunrise. His account is:

As Venus began now to draw near the Sun's limb I prepared to observe her egress. The interior contact did not appear so perfectly instantaneous as Dr. HALLEY'S papers led me to expect. I was not certain of it till $4^h\ 47'\ 21''$, though I doubted of it at $17''$. The exterior contact I judged to be at $5^h\ 5^m\ 49^s$, doubtful also 3 or $4''$; and so the passage of Venus's diameter $18'\ 28''$.

Chapter III.

OBSERVATIONS OF THE TRANSIT OF VENUS, JUNE 3, 1769.

In addition to the works cited in the preceding chapter reference has been made the following works for observations of the transit of 1769:

I. *Collectio Omnium observationum quæ occasione Transitus Veneris per Solem A. MDCCLXIX per Imperium Russicum institutæ fuerunt.* Petropoli, MDCCLXX.

II. *Voyage en Californie pour l'observation du passage de Vénus le 3 Juin, 1769; Par feu M. Chappe d'Auteroche. Rédigé & publié par M. de Cassini fils.* Paris, MDCCLXXII.

This work contains as an appendix a collection of observations, for some of which it is the sole authority.

III. *Memoirs of the American Academy of Arts and Sciences, Vol. I.* Boston, MDCCLXXXV.

IV. *Transactions of the American Philosophical Society, Vol. I. Second edition*, corrected. Philadelphia, MDCCLXXXIX.

JAKUTSK.

[*Collectio Pet.*, pp. 329, 338, 339.]

Observer, Islenieff.—Die 24 Maii (4 Junii) hora tertia matutina pluit; Sole oriente cœlum undique nubibus erat tectum; horologio monstrante $4^h\ 10'$ sol per nubes apparuit, ac $4^h\ 22\frac{1}{2}'$ vidit Venerem, ac si separaretur a limbo Solis. Observationem hanc ob nubes, quæ brevi post prorsus Solem conspectui eripuere, incertam ipse observator pronunciat.

	Temp. horol.	Temp. ver.
	h. ′ ″	h. ′ ″
Contactus internus in exitu	10 11 22	10 2 35.8
Ante contactum limbus Veneris lucida corona cinctus apparuit, ac ipso momento contactus prominebat tantisper versus limbum Solis. Pro tempore contactus id momentum sumtum est, cum filum lucidum limbos separans subito disparuisset.		
Contactus externus in exitu	10 27 $42\frac{1}{4}$	10 18 $56\frac{1}{4}$
Pro contactu externo id momentum assumtum est, cum exigua in limbo Solis incisura evanuerit, limbusque ejus pristinam rotunditatem recuperaverit.		
Eadem momenta socius tubo astronomico 15 pedum observabat.		
Contactum internum	10 11 23	. . . .
Contactum externum	10 27 42	. . . .

The ingress is worthless, and 5 minutes late. In reference to the discrepancy at egress, the observer stated that he was sure of the seconds of the internal contact but not of the minutes. The latter should probably be diminished by 2.

MANILLA.

[*Chappe, Voyage en Californie*, p. 158.]

Observer, D. E. de RONAS.—The only data are the observed times of egress:

	h.	m.	s.
Interior contact	9	25	45
Exterior contact	9	43	26

PEKIN.

I have no data except the times quoted by ENCKE from LALANDE's Memoir.

BATAVIA.

[*Philosophical Transactions*, 1771, p. 435.]

Observer, MOHR.—Cœlo ita favente, exitum Veneris e disco Solis, telescopio supra dicto (Gregorianum trium pedum), clare, distincte, nec minus accurate, hunc in modum observavi:

1769, 4 *Junii, ante meridiem.*

	h.	m.	s.
Contactus interior sive initium exitus videbatur	8	30	13
Contactus exterior sive exitus totalis visus	8	48	31

It is stated that the clock correction was determined by several equal altitudes of Sun and stars with a 2½-foot quadrant.

ORSK.

[*Collectio Pet.*, p. 277.]

Observer, CHRISTOPHER EULER.—"Veneris contactus cum Sole internus fieri mihi videbatur hor. pend. le PAUTE $5^h\ 22'\ 20''$, initium autem egressus $5^h\ 23'\ 10''$. Contactus vero externus in egressu seu egressus totalis contigit $5^h\ 41'\ 42''$. Cœlum observationi satis favens si nebulam exceperis, quæ aerem obscurum reddebat. Altitudo thermometri 131°. Barometri 27 dig. 6½ lin."

	Temp. pend. LE PAUTE.			Temp. vero astronom.		
Momenta igitur observationum a Cl. EULERO assignata sequentia sunt—	h.	′	″	h.	′	″
Contactus secundus internus	5	22	20	17	17	36
Initium egressus	5	23	10	17	18	26
Contactus secundus externus seu egressus totalis Veneris e Sole	5	41	42	17	36	57

Quum heic pro initio egressus duo momenta assignata sint, merito dubium videri posset, cuinam eorum major fides sit habenda; nobis quidem videtur momentum posterius pro vero initio emersionis Veneris e Sole habendum esse, id quod non solum exinde comprobatur, quod mora Veneris inter binos contactus alibi quoque non major fuerit observata, quam 18′ 30″ quæ tamen si prius momentorum heic allatorum pro vero agnosceretur, fieret 19′ 20″ sed etiam ex ipsa comparatione harum observationum cum iis, quæ in Gurief et Orenburg institutæ sunt, hinc nos quidem non dubitamus asserere, contactum internum in egressu contigisse 17^h 18′ 26″ temp. veri.

The quoted remarks are those of the observer; all the others of the editor. I should have supposed the actual contact to have been the first recorded time rather than the second, and the mean of the two more probable than either. No discussion is of any use until each result is compared with the tabular time.

ORENBURG.

[*Collectio Pet.*, pp. 225, 226.]

Observer, L. Krafft:

Temp. horol.	Temp. ver.	
		Jam accingebat sese ad exitum Veneris observandum tubo *Dollondiano* 12 pedum. Provisis omnibus quæ ad exactum observationem conducere poterant sequentia momenta pro certis reputat.
h. ′ ″	h. ′ ″	
4 54 6	5 5 0.7	Limbus ☉lis et ♀ris tangere sese invicem videntur, distincte conspicuo effectu motus cujusdam tremuli.
4 54 11½	5 5 6.2	Confluxus limborum instantaneus. Hoc momentum pro vero contactu interno reputat.
4 54 16	5 5 8.7	Contactus certo jam præteriit. Partem Veneris e Sole egressam nusquam potuit conspicere.
5 3 0	. . . .	Centrum Veneris in limbo Solis ad sensum apparuit. Tempore exitus cœlum fuit serenius, et sequentem observationem æque exactam, imo exactiorem præcedenti existimat.
5 12 35	. . . .	Cernit adhuc leve vestigium Veneris in limbo Solis.
5 12 39	5 23 34.0	Totalis exitus. Hoc ipso momento limbus Solis, ubi Venus Solem deseruit, æque ac reliqua ejusdem pars undulare incipit.

I take the mean of the recorded times as that of interior contact.

PONOI.

[*Collectio Pet.*, p. 37.]

Observer, J. A. MALLET.—Tunc omnem operam navavi, ut statutum punctum contemplarer, et tandem aliquid cernere inchoavi $9^h\ 50'\ 35''$ mei horologii sed minuto tantum sequenti certus fui Venerem esse. Ad interiorem contactum deinde observandum memet accinxi eumque accuratissime observavi

$10^h\ 9'\ 5''$ mei horologii.

The original data for clock correction are not given. The correction by corresponding altitudes June 3 was found to be $5^m\ 54^s$; correction for rate, $4^s.7$; correction at time of observation, $5^m\ 58^s.7$.

GURIEF.

[*Collectio Pet.*, pp. 198, 199.]

Observers, LOWITS and INOCHODSOW.—Hora $4^h\ 51'\ 16''$ motus tremulus marginum adhuc durat. Veneris figura melius terminata, nigra apparet. Omni licet adhibita attentione, nullum vestigium coloris, vel luminis circa marginem Veneris videre potui, quamvis jam minimæ maculæ Solares et prope marginem Solis sitæ partes lucidiores disci perspicue conspicerentur. Nullum quoque vestigium satellitis Veneris deprehensum.

Hora $4^h\ 54'\ 34''$ Temp. Pend. Initium egressus Veneris, seu secundum contactum internum Veneris cum limbo Solis contigisse pro certo compertum habeo, licet motus tremulus marginum adhuc satis vehemens erat. In hoc contactu nullam mutationem marginis Solis deprehendi, quæ licet undularet, bene tamen terminata erat.

Hora $5^h\ 12'\ 46''$ Temp. Pend. Egressum totalem Veneris e disco Solis seu contactum ultimum in egressu, marginum Veneris et Solis contigisse, certo persuasus sum. Neque heic ullum peregrini luminis indicium.

	Temp. pend. Schelton. d. 24 Maii t. civ.	Temp. vero. d. 23 Maii (3 Junii) t. ast.
	h. ′ ″	h. ′ ″
Ex his igitur colligitur Cel. Prof. LOWITS observasse contactum Veneris internum cum limbo Solis in egressu .	4 54 34	16 52 55
Contactus demum Veneris externus cum Sole pro egressu observatus	5 12 46	17 11 6

Observavit autem D. INOCHODSOW:

	Temp. pend. Paris.	Temp. pend. Londin.	Temp. ver.
	h. ′ ″	h. ′ ″	h. ′ ″
Contactum internum Veneris in egressu .	4 51 34	4 54 22	16 52 42
Contactum Veneris externum in egressu .	5 9 30	5 12 19	17 10 38
Adeoque mora inter utrumque contactum 17′ 56″.			

KOLA.

[*Collectio Pet.*, pp. 151–153.]

Observer, S. RUMOWSKY and two assistants —Prætergressa vero densiore parte nubis, simulac margo Solis superior conspiciendum se præbuit horologio A monstrante . . . $9^h\ 22'\ 0''$ seu $9^h\ 24'\ 15''$ t. v. vidi jam Venerem exigua sua parte Soli incisam.

	Temp. horol.	Temp. verum.
	h. ′ ″	h. ′ ″
Disco Solis extremitate nubis jam jam prætérlabentis obfuscato contactum internum æstimavi limbo Solis Venerisque tremulo . . .	9 39 52	9 42 2
Sole a nube prorsus liberato per exiguum intervallum Venus a limbo Solis remota apparet .	9 40 15″. . 20″	9 42 25″. . 30″

Post introitum Veneris ad medium usque transitus Sol cum Venere multoties prodibat in conspectum; ast semper limbis eorum ita tremulis undulantibusque, et per tam breve temporis intervallum, ut nullæ ad positionem Veneris definiendam suscipi potuerint observationes. Circa primam horam matutinam cœlum undique obtegebatur adeo, ut Sol ad horam tertiam prorsus fuerit inconspicuus; hora demum quarta matutina Sol intra hiatus nubium emergere incipiebat, ac tum illo rariori nubecula tecto, sed limborum undulatione cessante observavi in exitu.

	Temp. horol.	Temp. verum.
	h. ′ ″ ″	h. ′ ″
Contactum internum limborum	15 33 8. . 12	15 35 18.6 22.6
Limbus Solis cum limbo Veneris confusus apparet salva Solis rotunditate	15 33 24	15 35 24.6
Limbus Veneris vix mordere videtur limbum Solis	15 51 20	15 53 30.7
Post modum nubecula intercurrit limbum Solis conspectui eripiens; simulac Sol e nube emersit nullum jam Veneris vestigium	15 52 25	15 54 35.7
Observatio peracta est tubo *Dollondiano* 12 pedum longo.		

Socii OCHTENSKI et BORODULIN idem phænomenon tubo Gregoriano ad horologium B, quia pulsus illius melius ab iis exaudiri poterant, sequentem in modum observarunt.

	Temp. horol.	Temp. verum.
	h. ′ ″	h. ′ ″
Exiguum segmentum Veneris in limbo Solis jam apparet .	9 15 50	9 24 21
Pæterlapsa nube limbus Veneris a limbo Solis sejunctus conspicitur, ita ut inter marginem Solis Venerisque stria lucida ad instar fili tenuissimi appareret	9 33 49	9 42 23
Limbis Solis et Veneris nube tectis contactus eorum internus in exitu	15 26 2	15 35 43

Quotiescunque Sol conspicuus fuit, quæsivimus satellitem Veneris, sed nullum ejus vestigium reperimus.

The printed volume, from which the preceding is a transcript, affords no explanation of the various times of interior contact at egress, nor the means of correcting the obvious typographic error of 10^s in the second of the times. We may, however, take $15^h\ 35^m\ 22^s$ as the observed moment of contact by RUMOWSKY.

WARDHUS.

Considering only the geometric conditions for determining the effect of parallax, one of the most favorable stations in 1769 was that of HELL at Wardhus. The station was near the meridian on which the middle of the transit coincided with midnight, at which time, however, the sun was 3 degrees above the horizon. But the doubts which had frequently been expressed of the genuineness of HELL's observations long made the question whether to introduce them a very embarrassing one to me. It would seem that, very soon after HELL's return from his voyage, LA LANDE, impatient at his failure to publish his observations, expressed strong suspicion of his motives.* HELL offered to exhibit his journal, free from all erasures, but this offer was one difficult to accept. Another writer went so far as to maintain that no observations whatever were made at Wardhus owing to clouds, and that the published observations were pure inventions. The question remained in this unsettled state until 1834, when LITTROW discovered the original journal of HELL's voyage, which had been preserved at Vienna, and published a critical examination of it.† He afterward published a fac simile of the record relating to the transit of Venus. His conclusion was that there were obvious erasures and corrections in the journal, the times of first interior contact and of many other phenomena relating to the transit having been erased, and new ones written in their places, generally in different ink, so that it was very doubtful whether the original recorded times of first interior contact could be discovered.

These results of LITTROW's examination were naturally regarded as conclusive, and it does not appear that any one again scrutinized the manuscript until the writer visited Vienna in 1883, when, more as a matter of curiosity than with the expectation of reaching any definite conclusion, he compared portions of LITTROW's discussion with the original journal. He was soon struck by the circumstance that the descriptions of LITTROW did not accurately correspond to the facts, so far as the color and kind of ink were concerned. Cases in which the same kind of ink was used, but in which more had flown from the pen, were described as those where different ink was used. This naturally led to further investigation, and the conclusion was reached that LITTROW's inferences were entirely at fault. A detailed account of these investigations is given in the Monthly Notices of the Royal Astronomical Society for May, 1883 (Vol. XLIII, p. 371). It will suffice here to give a brief statement of the conclusions, so far as they bear upon the question of using HELL's observations:

(1) With one or two unimportant exceptions, mentioned below, the numbers printed by HELL are identical with those written in the journal at Wardhus, whether altered or unaltered in that journal.

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* ENCKE's *Venusdurchgang von* 1769 is my authority for this statement.

† P. HELL's *Reise nach Wardoe.* Wien, 1835.

(2) With the same exceptions the alterations described by LITTROW, in so far as they exist at all, were made at Wardhus before it was possible to receive other observations, and were not made with any other object than that of giving correct results. Some, in fact, were made before the ink got dry.

(3) The statement of LITTROW, that the original figures of internal contact at ingress were erased and new ones written, is devoid of any foundation whatever.

(4) The only subsequent insertions with different ink relating to the transit of Venus are (1) the time of formation of the thread of light, which is designated in the original by the single word *fulmen*, and (2) a correction of 2^s to SAJNOVIC's time of second internal contact.

(5) LITTROW's mistakes were due to the fact that he was *color-blind to red*, in consequence of which he wholly misjudged the case on first examining the manuscript, and afterward saw everything from the point of view of a prosecuting attorney.

To facilitate a judgment of the phases really observed I give, with each observed time (1), in italics, the original description as written in the journal at the station; (2), in Roman, the description in HELL's printed book.*

Ingress: interior contact.

Clock times. h.	m.	s.	Description in journal.	Printed description.
9	32	35.	*Videtur contactus fieri* .	Limbus Veneris circularem suam formam fere jam recuperare videtur.
9	32	41.	*Contactus certus visus* . .	42^s; censeo circumferentias Veneris & Solis jam perfecte circulares, nec tamen filum lucidum Solis apparet.* *Hoc momentum aliqui observatores habent pro contactu interiore.
9	32	48.	*Fulmen*.	Apparet filum lucidum limbi Solis, Venere jam totaliter ingressa.* *Alii hoc momentum dicunt contactum interiorem, utrique minus recte, ut supra ostendi.
			Pater Sajnovics suo tubo:	Pater SAJNOVICS tubo 10½ pedis ita habet:
9	32	30.	*Contactus dubius* . . .	Videtur Venus circumferentiam suam integram recuperasse.
9	32	45.	*Certissimus ut ajebat* . .	Ingressus totalis Veneris, filo lucido apparente.
9	33	10.	. . . .	D. BORGREWING tubo 8½ ped. Ingressus totalis.

Idem obtinuit D. Borgrewing secunda nempe 10″ *post numerata minuta, sed loco* 32 *minutorum mihi exhibuit* 33′. *Nos, ego et P. Sajnovics post obtentum contactum ad tubum adhuc hærebamus, usque post absolutam numerationem* 33 *minuti, tum accessimus omnes ad horologium, visuri num numerans famulus nobis recte indicasset minuta; adscripsimus nostra minuta prima, idem quoque fecit D. Borgrewing et* 33 *adscripsit, quæ cum mihi serius exhibuisset, nosque ambo ego et P. Sajnovics in adnotandis minutis convenerimus, judicavi erronee a D. B. minuta adscripta esse; interea cum levis hic scrupulus (qui facile per calculum, et comparationem cum aliorum observatorum contactu solvitur) suspendi interea judicium, donec hæc explorata habeam.*

**Observatio transitus Veneris ante discum Solis etc., a R. P. Maximiliano Hell, e S. J., Vindobonæ*, MDCCLXX.

Egress: interior contact.

Clock times.			Description in journal.	Printed description.
			Ego tubo meo ad idem Horologium.	
h.	m.	s.		
15	26	6.	*Videtur aliqua gutta nigra intra limbum Solis et Veneris ante contactum formari.*	Appropinquante limbo Veneris ad limbum Solis, video nigram quasi guttam intra limbum obscurum Veneris & Solis formari.
15	26	12.	*Gutta hæc minui videtur valde.*	Cerno guttam hanc sensibiliter imminui.
15	26	17.	*Disparet, et contactum fieri censeo.*	Gutta hæc momentanee disparet, & veluti diffluit, limbusque Solis & Veneris in unum confluunt,
15	26	19.*	*Certissimus contactus* . .	atque adeo fit contactus interior opticus.
			Pater SAJNOVICS:	P. SAJNOVICS tubo 10 & ½ ped.:
15	26	18.	*Contactus (dubius) certus* .	Contactus interior certus.
		26.*	(*Certus.*)	
15	26	10.	*D. Borgrewing*	D. BORGREWING tubo 8 & ½ pedis contactus interior.

Hic contactus interior mihi adeo momentaneus visus est, ut de uno secundo temporis nullum mihi dubium superfuerit, eandem guttam nigram, quam ego ante contactum cernebam, se quoque observasse ajebat P. SAJNOVICS.

It is in this record of the times observed by SAJNOVICS that the most confusion is shown in the manuscript journal. The following is my description of the record of egress in the journal, and my conclusions from it, as made with the manuscript before me, and printed in the paper already referred to.

The times of HELL's notes of the "gutta nigra" are each increased by 2ˢ; but obviously this correction was made at the time of writing. More serious are the corrections of SAJNOVICS' times. As originally written they read:

Pater Sajnovics contactus dubius	15	26	20
certus			26

So far as can be inferred from the manuscript the first number of seconds might have been 26 as well as 20, only then the two times would have been the same, which is improbable. The last line, "certus . . . . 26" and the word "dubius" were then struck out at the station, and the word *certus* substituted for *dubius*. Whether this was merely a suppression of the "contactus dubius," or included also a change of the "contactus certus" from 26 to 20, we can not say, the manuscript being torn where the top of the 6 belongs; but the latter seems more probable, as otherwise there would have been no object in the change. But this is not all. The 20 or 26 is again changed to 18, and so printed. Moreover, this last change appears to be made with a different ink, and, so far as can be judged from so small a surface, the same ink with which the line "fulmen" was written.

The explanation is too obvious to need more than a statement. An observation of contact is not like one of a star transit, in which the observer must note a moment which he can not alter. It can only be an estimated mean moment for a gradually changing phenomenon extending through a number of seconds. This estimate is liable to change in the mind of the observer as he subse-

*These times are not found in the printed book. The words in parentheses were erased at Wardhus.

quently thinks over the progress of the phenomenon as he saw it. I should be inclined to accept a change of opinion thus reached, if it were not suggested by a comparison with the results of others. Now, SAJNOVICS was the constant companion of HELL, both on the journey and while the observations were going through the press. They, no doubt, discussed their times, and, in consequence of such discussion, SAJNOVICS concluded that his times were late.

In interpreting HELL'S observations we are to take account of his views of the phenomena of contact as set forth on pages 64–69 of his printed book. It may be inferred from this that the mean of the first two recorded times of interior contact at ingress may be taken as a good observation of true geometric contact without such distortion as to interfere with the accuracy of the observation. The second observation would appear to be of a well-formed thread of light. The time when this is seen HELL shows to be later than that of true contact of the limbs. Although the subsequent insertion of this time in the journal may well cause it to be received with a certain amount of suspicion, yet my judgment is in favor of its provisional retention.*

In the case of egress the phenomenon "gutta nigra" has, I believe, always been interpreted as that of a complete cutting off of the thread of light. I am, however, inclined to a different view, although the figure drawn by HELL might be interpreted as showing the thread completely cut off. It is well known that the cutting off of the thread of light is not generally a sudden phenomenon, but occurs through a gradual darkening of the band of light, frequently described by observers, and easily accounted for. With the artificial transit I have frequently noticed, when the atmosphere was unsteady, that some seconds before the final cutting off of the thread of light, dark undulations would show themselves between the two limbs. On the whole, it seems to me that the mean of the three may be taken as the phase most likely to be comparable with the contacts noted by other observers.

The observations of SAJNOVICS are less satisfactory. His first record of ingress may be regarded as an observation of so-called apparent contact. But I doubt whether the second is really an observation of the thread of light, because the words "certissimus ut ajebat" do not indicate that he noticed anything of the kind, while the description in the printed book might very naturally be inserted as a piece of guess work. My inclination is, therefore, to regard the mean of the two observations as a mean contact.

In the case of egress I suspect that the change of the first-observed time of SAJNOVICS from *dubius* to *certus*, and the suppression of his second time, however honestly made, were due to the fact that his last time was later than that of his chief. I therefore judge that the proper time to be adopted as that of SAJNOVICS is $15^h\ 26^m\ 23^s$, the mean of his original recorded times.

In the case of BORGREWING'S observations nothing is to be done but class them with those in which no phase is described. The way in which BORGREWING'S first observation is mentioned in the manuscript journal: "Idem obtinuit D. BORGREWING secunda nempe 10″ * * * *", is difficult of interpretation, because it seems to imply that BORGREWING'S observed time is identical with that of SAJNOVICS. Probably *idem* is merely a hastily written abbreviation for *eundem contactum.*

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* Since writing this I find that, about the beginning of 1770, HELL communicated his observed times to the Swedish Academy through WARGENTIN, and that this time of "fulmen" is the only one given for interior contact. The observation is given thus:

Venus kom hel och hållen in på Solen $9^h\ 34^m\ 10\tfrac{1}{4}^s$. (Kongl. Vetenskaps Academiens Handlingar, 1770, p. 40.)

The clock error has been discussed so fully by LITTROW in the work already referred to that no re-examination is necessary. His results are:

Clock correction, mean time

	′	s.
For ingress	—1	0.9
For egress	—0	59.4

PETERSBURG.

[*Collectio Pet.*, pp. 12–14.]

Observatio egressus Veneris, die 23 *Maii.*

Temp. pend.			Temp. ver.			
h.	m.	s.	h.	m.	s.	
14	52	15	14	49	58	Venus oriente Sole in parte limbi borealis videtur, limbo Solis et Veneris undulante, maleque terminato.
	54	42		52	26	Videtur centrum Solis esse in horizonte. Venus versicolor, nam limbus ejus borealis ruber, australis cærulescens, medium nigrum observatoribus omnibus apparet.
15	25	59	15	23	43	Venus melius terminatum limbum habere videtur et colores disparent, undulatio tamen adhuc notabilis. Contactus interior certo necdum contigit.
	27	50		25	33.7	Domino STAHL contactus interior esse videtur, limbo Solis necdum inciso.
	27	57		25	40.7	Cl. D. Adiunctus LEXELL eundem contactum videt.
	28	0		25	43	Idem contactus Professori MAYER videtur accidere.
15	28	3	15	25	46.7	Contactus internus ob sensibilem incisionem et curvaturam limbi Solis Professori MAYER præteriisse videtur.
	28	4		25	47.7	Eandem limbi incisionem proxime a contactu notat prænobilis D. Professor EULER.
						Contactus exterior ita habet:
15	45	30	15	43	13.7	STAHL . . . . } Contactum exteriorem vident.
	45	40		43	23.7	Cl. D. Adjunctus LEXELL . . . }
	45	47		43	30.7	Prænobilis D. Professor EULER . }
	45	57		43	40.7	Professor MAYER . . . . . . }

From the descriptions of STAHL, MAYER, and EULER it may be inferred that all the observations refer to the breaking of the thread of light.

CAJANEBORG.

[*Schwedische Abhandlungen*, XXXI, 1769, p. 212.]

Observer, PLANMANN.—Die Ränder der Sonne und der Venus schienen sehr zu wallen wegen der Bewegung der Dünste am Horizonte. Ich konnte aber doch der Venus Eintritt in die Sonne sehr genau beobachten. Er geschah

Um 9 Uhr 20 Min. 45 Sec.; in diesem Augenblicke borst das schwarze Band, welches der Venus Körper mit dem Sonnenrand zusammenhängt, nachdem es merklich schmäler geworden war, als 8 Secunden zuvor, und der Venus dunkler Körper ward nun mit dem Glanze der Sonne umgeben.

Um 15 Uhr 32 Min. 27 Sec. geschah der Venus gänzlicher Austritt, indem die schwarze Spitze, die sie gegen das Ende im Sonnenrande bildete, in diesem Augenblick verschwand, worauf dieser Theil des Sonnenrandes eben so wallend erschien als das Uebrige.

The second interior contact was lost by clouds. PLANMANN adds that UHLWYK, with a 3-foot achromatic, observed the total egress at $15^h\ 32^m\ 24^s$.

NORTH CAPE.

[*Philosophical Transactions*, 1769, p. 266.]

Observer, BAYLEY.—June 3, at $13^h\ 46^m\ 40^s$ per clock, or $9^h\ 0^m\ 2^s$ apparent time, the Sun came out from under a cloud, with Venus on it about one-fourth of her diameter; and at $14^h\ 0'\ 41''$, or $9^h\ 14'\ 1''$ apparent time, Venus's outer limb seemed to be in contact with the Sun's limb; but no light of that part of the Sun's limb could be seen, Venus being apparently joined to the Sun's limb by a black ligament, which gradually diminished in breadth, and at $14^h\ 1'\ 36''$, or $9^h\ 14'\ 56''$, the Sun's light broke through it, and Venus and the Sun were, to appearance, perfect (this was certain to about 10 or 15 seconds of time), and the black ligament contracted itself, so that Venus was considerably within the Sun's limb, suppose one-twentieth of her diameter.

During these observations the air was red and hazy, and the Sun's limb very tremulous, and the spots in the Sun very indistinct, and Venus seemed very ill defined when on the Sun. But a better idea will be formed of the bad appearance of Venus at the internal contact, owing to the very hazy state of the air, from the representation of it. (Plate XIII.)

The figure shows atmospheric undulations of such magnitude that an accurate observation was impossible. The fact that Venus was one-twentieth of her diameter within the limb of the Sun would indicate that the second observation was nearly a minute late.

WANHALINNABERG, NEAR ABO.

[*Schwedische Abhandlungen*, XXXI, p. 173.]

Observers, GADOLIN and JUSTANDER.—Auf diesem Berge beobachtete ich mit einem Fernrohre von 20 Fuss.

Um 9 Uhr 0 Min. 25½ Sec. geschah meinem Urtheile nach, der gänzliche Eintritt der Venus. Die Erscheinung verhielt sich folgendermassen: nachdem Venus so weit in die Sonnenscheibe gekommen war, dass man hätte urtheilen sollen, der Rand der Sonne würde sich um die Venus wieder schliessen, so blieben doch beide Theile des Randes von einander weit durch ein dunkles Band abgesondert, das sich von der Venus nachfolgenden Seite an den offenen Sonnenrand

erstreckte. Dies Band ward nach und nach besonders in der Mitte schmäler und schmäler. In dem hier angezeigten Augenblicke ereignete es sich das erste Mal, dass das Band plötzlich in der Mitte wie von einem quer überfliessendem Lichtstrome zerschnitten ward, gleich darauf aber wieder zusammenging. Diese Oeffnung und Zusammenschliessung des Bandes wechselten nachdem immer ab, das Band nahm zugleich nach und nach ansehnlich ab bis man um 9 Uhr 0 Min. 55½ Sec. bemerkte, dass die Oeffnungen des Bandes nicht mehr so kurz nur Augenblicke dauerten, sondern länger währten.

Herr JUSTANDER, der das Fernrohr am Quadranten brauchte, welches nur 3 Fuss lang war, urteilte, der gänzliche Eintritt sei um 9 Uhr 0 Min. 52 Sec. geschehen.

STOCKHOLM.

[*Schwedische Abhandlungen*, XXXI, p. 149–154.]

Observers, WARGENTIN, STRUSSENFELT, FERNER, and WILKE.—Um 8 Uhr 23 Min. 51 Sec. bemerkte ich (WARGENTIN), * * * einen schwarzen Punct, der sich in wenig Secunden, in einen kleinen dunkeln Rand, in den Sonnenrand ausbreitete.

Um 8 Uhr 41 Min. 32 Sec. glaubte ich nach dem Augenmasse und Aussehen, Venus sei ganz und gar in der Sonne, aber sie hing noch mit dem Sonnenrande zusammen, ohngefähr wie die VI. Tafel 3 Fig. vorstellt, bis sie

Um 8 Uhr 41 Min. 47 Sec. sich gleichsam vom Sonnenrande losmachte, indem ein wallender Strahl plötzlich über die Venus hervorschoss und die Oeffnung ergänzte, die sie im Sonnenrande gemacht hatte. Von der Zeit an fing sich das Wallen am Sonnenrande wieder an, welches bisher bei dieser Oeffnung war gehemmt gewesen und Venus bewegte sich frei und ledig durch die Sonnenscheibe.

Herrn FERNER's Beobachtung führe ich mit seinen eigenen Worten an:

Um 8 Uhr 24 Min. 9 Sec. des Abends war der Venus vorhergehender Rand im Sonnenrande ganz wohl sichtbar.

Um 8 Uhr 41 Min. 48 Sec. schien sich der helle Sonnenrand wieder zu ergänzen und Venus war ganz und gar in die Sonnenscheibe getreten.

Der Sonnenrand wallte und zitterte sehr, Venus war zackig und vieleckigt und änderte immerzu ihre Gestalt.

Herr WILKE hat gleichfalls nachstehenden Bericht selbst aufgesetzt.

Um 8 Uhr 24 Min. 6 Sec. ein kleiner schwarzer eintretender Strich, oder ein Tüpfelchen.

Um 8 Uhr 24 Min. 9 Sec. war schon ein ganz deutlich dreieckichter Einschnitt.

Um 8 Uhr 32 Min. 53 Sec. schien der eingetretene Theil des Planeten, durch eine plötzliche Oeffnung in den Wolken (ohne gefärbtes Glas, welches nachgehends weggelassen ward), ganz dunkel, mit ziemlich scharfem Rande, rings herum von einem Ringe umgeben, der überall eine Breite hatte, und mehr weissblass war als die Farbe der Sonne. Nachdem bedeckten dichte Wolken der Sonne obere Hälfte, gaben aber um 2 Uhr 37 Min. 33 Sec. neue Gelegenheit, erwähnten bleichen Ring ganz deutlich zu sehen, da noch ohngefähr ein Viertel vom Umfange des Planeten am Sonnenrande hing. Ich schätzte des Ringes Breite ohngefähr ein Sechstel seines Durchmessers.

Um 8 Uhr 41 Min. 2 Sec. war die Venus mit ihrer ganzen länglichten Rundung in den Sonnenrand getreten und die innere Berührung schien alsdann geschehen zu sein; aber der Venus lichter Ring, blieb noch gleichsam wie eine Aushöhlung im Sonnenrande, und ward bald darauf ganz unsichtbar. Dagegen zeigte sich ein stärkeres Wallen an des Planeten dunkelm Rande, welches hinderte, dass man kein helles Licht zwischen der Sonnen abgesonderten Spitzen vorkommen sah, sondern Venus war noch um 8 Uhr 41 Min. 30 Sec. mit der Sonne durch ein dunkles Band oder einen wallenden Rauch vereinigt, welcher 10 bis 12 Secunden darnach am Sonnenrande anfing

sich aufzuklären, aber der Sonne freien Schein doch nicht durchkommen liess, bis etwa* 8 Uhr 42 Min. 45 Sec. Nach dieser Zeit ward Venus immer mehr eirund. Innerhalb ihren schwarzen wallenden Rändern schien der Kern selbst mit einer dunklen Röthe zu leuchten, bis sie zackicht und verstellt zugleich mit der Sonne sich hinter dichten Wolken verbarg.

Herr STRUSSENFELDT * * * hielt sie ganz eingetreten um 8 Uhr 41 Min. 13 Sec. ob sie wohl nachgehends länger, als eine halbe Minute am Sonnenrande hieng.

HERNOSAND.

[*Schwedische Abhandlungen*, XXXI, p. 225, 226.]

Observer, GISSLER.—Um 8 Uhr 23 Min. oder einige wenige Secunden zuvor fing Venus an sich mit ihrer vorangehenden Rande im Sonnenrande zu zeigen.

Um 8 Uhr 40 Min. 12 Sec. schien fast die ganze Rundung des Planeten innerhalb des Sonnenrandes zu sein, hing aber doch noch fest daran, vermittelst eines schmalen Schattens, den sie mitschleppte, bis 8 Uhr 41 Min. 5 Sec. oder 9 Sec., da dieser Schatten den Sonnenrand verliess und der Sonnenrand ganz rein und hell um die Venus schien.

UPSALA.

[*Schwedische Abhandlungen*, XXXI, p. 156–158.]

Observers, PROSPERIN, STRÖMER, MELANDER, BERGMANN, and SALENIUS.—Um 8 Uhr 38 Min. schien mir (ERICH PROSPERIN) die Krümmung des Planeten mit der Sonne ihrer zusammen zu fallen. Aber, obwohl er nachdem immer tiefer in die Sonne trat, so hing er mit der Sonne durch eine Art von Absatz zusammen, der schmäler und schmäler ward, bis er endlich um 8 Uhr 40 Min. 12 Sec. zerriss. Die Venus sah kurz zuvor aus wie ein Apfel der an seinem Stiele sässe und schwankte, denn das Wallen machte, dass sie hin und her zu gehen schien. Als der Stiel zerriss, war sie schon ein Stück hinein. Während der ganzen Beobachtung war Venus eigentlich nie recht rund, sondern hatte unordentliche Kanten, welches man dem Wallen der Ränder zuschreiben muss. Die Sonne war nun so niedrig, auch in Wolken, dass sich nichts weiter thun liess.

Herr Professor STRÖMER sah die erste Spur der Venus um 8 Uhr 23 Min. 4 Sec., da sie schon ein wenig hinein war. Um 8 Uhr 30 Min. 57 Sec. schien sie etwa zur Hälfte eingetreten.

Um 8 Uhr 39 Min. 58 Sec. schien ihre Rundung die Sonne inwendig zu berühren.

Um 8 Uhr 40 Min. 32 Sec. schloss sich der Sonnenrand um die Venus, doch hatte der Herr Professor zuvor einen etwas matteren Schein zwischen der Venus und dem Sonnenrande gesehen.

Herr Professor MELANDER war der erste unter uns, der der Venus Annäherung an den Sonnenrande bemerkte, um 8 Uhr 22 Min. 1 Sec.

Um 8 Uhr 39 Min. 57 Sec. schien ihm, dem Augenmasse nach, Venus ganz eingetreten, obwohl sich der Rand der Sonne nicht hinter ihr zeigte. Aber

Um 8 Uhr 40 Min. 12 Sec. sah er das schwarze Band bersten, vermittelst dessen Venus am Sonnenrande gehangen hatte.

Herr Professor BERGMANN sah die Venus zuerst um 8 Uhr 22 Min. 45 Sec.

Um 8 Uhr 40 Min. 9 Sec. sah er das schwarze Band reissen und bemerkte dabei eben solche Erscheinungen wie ich.

Herr M. SALENIUS erblickte die Venus zuerst um 8 Uhr 22 Min. 15 Sec.

Um 8 Uhr 39 Min. 16 Sec. merkte er, dass der schwarze Fleck, mittelst dessen Venus am Sonnenrande hing, plötzlich zersprang, so dass der Sonnenglanz sie auf allen Seiten umgab, aber

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*Perhaps a typographical error, giving 42^m instead of 41^m.

der Fleck ging wieder zusammen. Diess geschah in einem Augenblick. Endlich um 8 Uhr 40 Min. 15 Sec. borst der schwarze Band völlig, und der Planet zeigte sich ganz und gar in der Sonnenscheibe.

Wir wachten die ganze Nacht, um beim Aufgange der Sonne bereit zu sein, dass wir nachsehen könnten ob sich noch eine Spur der ausgehenden Venus zeigte, aber es war trüb und ward nicht eher um die Sonne herum heiter, als um 8 Uhr 38 Min. Vormittag.

GREIFSWALD.

[*Philosophical Transactions*, 1769, p. 284.]

Observer, ANDR. MAYER.—Ipsa Veneris facies, distorta nimium, irregularis, atque continuo obnoxia flexui, per reliquum observationis tempus talis erat, qualem figura adjecta exhibet. Paulo ante contactum interiorem, fascia quasi margini Solis alligata visa est, quæ subito soluta $8^h\ 22^m\ 44^s$ temp. ver. docebat ingressum fuisse factum, cujus mora secundum hanc observationem, erat $18^m\ 9^s$.

The figure shows the disc of Venus so distorted, and the limb of the Sun so ill defined, that no observation of contact would appear possible.

ENCKE adds an observation by ROHL at $8^h\ 20^m\ 31^s.5$ mean time. I do not know where he finds it.

LUND.

[*Schwedische Abhandlungen*, XXXI, p. 223.]

Observers, SCHENMARK and NENZELIUS.—Um 8 Uhr 4 Min. 5 Sec. bemerkte ich die erste Spur von des Planeten Antritt an den Sonnenrand. Herr NENZELIUS bemerkte diese Berührung 10 Secunden später, 8 Uhr 4 Min. 15 Sec.

Als der gänzliche Eintritt oder die innere Berührung bevorstand, gab ich (NILS SCHENMARK) mit allem Fleisse auf das Horn der Sonne Acht, das die Venus umfasste, mit dem Vorsatze, es für den eigentlichen Augenblick der Berührung anzunehmen, wenn dieses Horn zusammenlaufen würde. Aber ehe es geschah, und um 8 Uhr 22 Min. 7 Sec. merkte ich deutlich ein schwaches Licht des Sonnenrandes unter der Venus, welches mehr und mehr zunahm; ich konnte nicht anders, als dieses für den rechten Augenblick des gänzlichen Eintritts anzunehmen. Herr NENZELIUS sah dieses schwache Licht 7 Secunden eher um 8 Uhr 22 Min. 0 Sec. Wegen der flatternden Dünste, ward es ihm nachgehends einige Secunden lang unsichtbar, aber er bemerkte es bald darauf wieder ohne dass er genau in Gedanken behalten oder sicher sagen konnte, in welcher Secunde es so klar und beständig geworden, dass man es für der Sonne wirklich hervorkegommenen Rand anzusehen hatte.

Ich lasse es unausgemacht, ob dieser von uns bemerkte schwache Schein in der That gerade vom Sonnenrande gekommen ist oder ob er von einer Brechung der Strahlen in der Atmosphäre der Venus, von einer Beugung des Lichtes herrührte. Meine Schuldigkeit ist, die Beobachtung so anzugeben, wie wir sie bekommen haben.

The doubt of which SCHENMARK speaks can be resolved only by the comparison with other observations. A first glimpse of the completed thread, followed by its disappearance, as described by NENZELIUS, seems to form a good observation of true contact.

PARIS (COLLÉGE DE LOUIS-LE-GRAND).

[*Paris Memoirs*, 1771, p. 504.]

Observers, MESSIER, BADOUIN, TURGOT, and ZANNONI.—"J'attendis le second contact, en laissant reposer ma vue jusqu'au moment que je vis le bord inférieur du Soleil sortir du nuage; le second bord parut ensuite, et je commençai à voir Vénus qui était déjà de plus de moitié entrée à $7^h\ 28^m\ 9^s$ à la pendule, ou $7^h\ 26^m\ 40^s$, temps vrai. Le Soleil approchant de l'horizon se dégageait de plus en plus des nuages, et dès le temps du second contact le lieu du Ciel où se trouvait alors le Soleil était assez serein; mais il y avait beaucoup de vapeurs, et les ondulations excessives empêchaient de voir le disque du Soleil et celui de Vénus bien terminés. À $7^h\ 40^m\ 12^s$ à la pendule, ou $7^h\ 38^m\ 45^s$, temps vrai, le second contact se décida: deux secondes après je vis un filet de lumière très délié entre le bord de Vénus et celui du Soleil, de manière qu'il ne reste dans mon observation qu'une incertitude de deux secondes sur le véritable moment du contact intérieur."

M. BADOUIN, Maître des Requêtes, observait avec moi avec une excellente lunette achromatique de 3 pieds de foyer; elle portait 39 lignes d'ouverture et grossissait cent dix fois; il a marqué le second contact à $7^h\ 38'\ 51''$ de temps vrai.

M. TURGOT, Intendant de Limoges, placé à l'étage au-dessous, a observé le second contact avec un petit télescope Grégorien de 11 pouces de foyer à $7^h\ 38^m\ 50^s$ par le moyen d'une montre à secondes que j'avais mise d'accord avec la pendule quelque temps avant l'observation, et qui fut vérifiée ensuite.

M. ZANNONI, au même endroit, avec un télescope Grégorien de Short, de 3 pieds de foyer, qui grossissait cent huit fois, a marqué le second contact, par le moyen de la même montre à secondes, à $7^h\ 38^m\ 41^s$, temps vrai.

PARIS OBSERVATORY.

[*Paris Memoirs*, 1769, pp. 229, 245, 529.]

Observers, CASSINI de THURY, le Duc de CHAULNES, and du SÉJOUR.—Les nuages nous cachèrent le Soleil jusqu'à $7^h\ 38'$ du soir que nous commençâmes à apercevoir son bord supérieur sortant d'un nuage et Vénus dont le disque ne nous parut pas encore entré sur le Soleil; j'ai jugé le contact intérieur à $7^h\ 38'\ 53''$ avec une lunette de DOLLOND de $3\frac{1}{2}^d$.

M. le Duc de CHAULNES a estimé ce contact 4 secondes plus tard, comme on le verra par le détail de son observation.

M. du SÉJOUR a jugé le contact 10 secondes plus tôt avec une lunette achromatique du Sieur LETANG, qui faisait l'effet d'une lunette de 6 à 7 pieds.

Observer, MARALDI.—Le Soleil a été couvert longtemps avant l'entrée de Vénus sur son disque, et il n'a paru que peu de temps avant l'entrée totale de cette planète. J'ai observé le contact intérieur à $7^h\ 38'\ 50''$ avec une lunette achromatique de 36 pouces, dont l'objectif est composé de trois verres; elle fait l'effet d'une bonne lunette ordinaire de 15 pieds; les bords du Soleil et de Vénus étaient alors assez bien terminés.

Observer, Le Duc de CHAULNES.—Un gros nuage ayant empêché de voir le premier contact le Soleil n'a commencé à être aperçu que lorsque Vénus était déjà entrée aux trois quarts de son disque; les vapeur rendaient les bords du Soleil et de Vénus si ondoyans, que l'on ne pouvait pas juger avec beaucoup de précision des contacts.

Le second contact du bord extérieur de Vénus avec le bord intérieur du Soleil est arrivé à $7^h\ 38'\ 58''$.

COLOMBES.

[*Berlin Memoirs*, 1767, p. 507.]

Observer, BERNOULLI.—Il est vrai que les vapeurs qui s'élevaient de l'horizon rendaient les bords des disques très-mal terminés, le bord de Vénus surtout était comme dentelé; mais voici ce que j'ai remarqué avec précision.

	h.	′	″	
A	7	31	28	de la pendule, je vis qu'une seule des éminences du bord de Vénus touchait encore le bord du Soleil.
	7	31	29	Je voyais la même apparition.
	7	31	30	Je ne pouvais pas dire qu'elle eût cessé, mais ce contact n'était plus que très-faible.
	7	31	31	Enfin j'ai vu distinctement un filet de lumière très-délié entre l'éminence dont je parle et le bord du Soleil le plus proche.

The clock correction to apparent time was $+6^m 43^s$, and seems to have been well determined.

PASSY (CABINET DE PHYSIQUE).

[*Paris Memoirs*, 1769, p. 531.]

Observers, De FOUCHY, de BORY, and BAILLY.—The account seems to have been prepared by BAILLY, who reports as follows:

Enfin à $7^h 38' 33''$ le nuage s'étant éclairci, M. DE BORY annonça que le bord de Vénus avait quitté le bord du Soleil. M. DE FOUCHY, qui l'avait observé comme lui, et qui n'avait attendu à le dire que pour juger, par le mouvement de Vénus, du vrai moment du contact intérieur, M. DE FOUCHY, dis-je, assura, comme M. DE BORY, qu'à $7^h 38' 33''$ Vénus s'était détachée du bord du Soleil, et il assura en même temps qu'il n'y avait pas plus de deux secondes. Nous en avons tous ensuite porté le même jugement; ainsi nous croyons pouvoir dire, avec confiance, que le contact intérieur est arrivé à Passy à $7^h 38' 31''$, ou à $7^h 38' 45\frac{1}{2}''$, temps vrai, réduit au temps de l'observatoire.

The use of the pluperfect tense seems to indicate that the contact occurred just before the Sun appeared. The observation is, however, admitted by ENCKE, though he places it in the second class.

SAINT-HUBERT.

[*Paris Memoirs*, 1769, p. 187.]

Observers, LE MONNIER and CHABERT.—Nous n'avons pu observer à Saint-Hubert que l'entrée du centre de Vénus sur le disque, parce que le ciel venait de s'éclaircir, et le contact interne qui s'est fait à ma lunette achromatique à $7^h 34' 56\frac{1}{2}''$ de temps vrai; avec une lunette de 18 pieds M. de CHABERT a vu le contact interne à $7^h 35' 32\frac{1}{2}''$.

SARON, ROUEN, HAVRE, TOULOUSE.

[*Paris Memoirs*, 1769, pp. 421, 422.]

We have nothing but statements of observed times, quoted by LALANDE, as follows:

Le contact intérieur a été observé à Saron par M. le Président de Saron à $7^h\,44'\,0''$, à 2 ou 3″ près, à cause de l'excessive ondulation et des irrégularités de Vénus et du Soleil; suivant la carte des triangles de la France, il y a 5′ 35″ de différence entre les méridiens de Paris et de Saron, ce qui donne pour le contact réduit au méridien de Paris $7^h\,38'\,25''$.

Cette observation a été faite à Rouen, par M. DULAGUE, à $7^h\,33'\,40''$; par M. BOUIN, à $7^h\,33'\,46''$.

A Caen, par M. PIGOT, à $7^h\,26'\,25''$.

Au Havre de Grace, par M. DIQUEMAR, à $7^h\,30'\,50''$, avec une lunette de 5 pieds.

A Toulouse, M. D'ARQUIER, Correspondant de l'Académie, estima le contact à $7^h\,35'\,8''$, mais le Soleil était fort près de l'horizon et le bord très-irrégulier; M. GARIPUY ne l'observa qu'à $7^h\,35'\,30''$.

BORDEAUX.

[*Paris Memoirs*, 1769, pp. 510, 512.]

LALANDE communicates to the Academy independent observations at two points, by the Abbé FAUGÈRE and Mr. de la ROQUE, as follows:

Par des hauteurs correspondantes le 2 et le 4, M. FAUGÈRE a trouvé qu'au moment du contact intérieur il était $7^h\,27'\,16''$ de temps vrai, ce qui fait $7^h\,38'\,52\frac{1}{2}''$ au méridien de Paris.

* * * * * * * * *

Depuis la lecture de ce Mémoire, M. de la ROQUE, Inspecteur de la jauge des bâtimens de mer à Bordeaux, m'a envoyé une observation qu'il a faite avec un télescope Grégorien de 27 pouces de longeur, il a trouvé le contact intérieur à $7^h\,27'\,5''$, le commencement de l'éclipse de Soleil le 4 à $6^h\,30'\,49''$ du matin; la fin à $8^h\,4'\,11''$; il a réglé son horloge sur des hauteurs correspondantes, prises avec un quart de cercle de bois de 39 pouces de rayon, ainsi que j'y ai souvent invité dans mes écrits les amateurs et les curieux.

CAEN (THE MISSION).

[*Philosophical Transactions*, 1770, p. 262.]

Observers, NATHAN PIGOTT, Esq., his son, and Monsieur DE ROCHFORT.—I prepared myself with all possible care for the observation of the internal contacts, and, though the Sun's limb moved continually up and down with a quick motion, I judged the internal contacts at $7^h\,21'\,44''.5$ by the clock, or $7^h\,26'\,24''.5$ apparent time, and 3″ or 4″ later I saw a thread of light separate the planet from the Sun.

	h	m	s	
Internal contacts by M. de ROCHFORT	7	27	7.5	App. time.
By my son	7	26	55.5	

I find by my register that Monsieur de ROCHFORT judged his observations some seconds too late.

I perceived that Venus, before she separated from the Sun, was considerably stretched out towards his limb, which gave the planet nearly the form of a pear, and, even after the separation of the limbs, Venus was 12 or more seconds before she resumed her rotundity.

These observations of internal contact are, on the whole, more than a minute too early. PIGOTT had left his observatory in charge of some one else and taken a position at the mission as better situated for the observation.

CAEN (OBSERVATORY OF NATHAN PIGOTT).

[*Philosophical Transactions*, 1770, p. 264.]

Observer not mentioned.

Observations of the contacts of the Sun and Venus made in my observatory at Caen with a 17-inch refractor, with 2¾-inch aperture.

Clock.			App. time.			
h.	′	″	h.	′	″	
7	4	40.0	7	9	20.0	Sun well determined; a very small impression appeared on its superior limb; it seemed even doubtful whether the contacts were formed.
7	4	52.0	7	9	32.0	The contacts very certain; this observation excellent; it is thought the contacts could not have been seen sooner than $7^h\ 9'\ 20''$.
7	20	33.0	7	25	13.0	The following limb of Venus seemed to touch that of the Sun; the planet appeared quite round, but soon after seemed to stretch itself out, and to form the tail, mentioned underneath: this observation is thought less certain than the others.
7	23	3.0	7	27	43.0	Internal contacts. By internal contacts must be understood the instant when a sort of tail, such as is represented in the figure, and which joined Venus to the Sun's limb, separated from it so suddenly, that it is impossible there could have been an error of one second. There appeared instantly a considerable distance between the limbs; that distance was not measured, but it might be $\frac{1}{200}$ of the Sun's diameter; and that distance was concluded from the comparison of the apparent length of this tail to the diameter of Venus.

BREST.

[*Paris Memoirs*, 1769, p. 546.]

Observers, M. LE ROY, M. BLONDEAU, M. FORTIN, and M. de VERDUN.—Dans cet état, le contact intérieur de Vénus, qui était notre observation importante, fut observé par M. de VERDUN $7^h\ 11^m\ 37^s$, par M. FORTIN $7^h\ 11^m\ 44^s$, par M. BLONDEAU $7^h\ 12^m\ 4^s$ et par M. LE ROY $7^h\ 12^m\ 7^s$.

Mais, comme ce contact a été estimé 30 secondes plus tard par un des observateurs, et qu'en général les Astronomes de Brest me paraissent avoir été en retard sur ceux de Paris, j'ai cru d'abord qu'il pourrait y avoir une partie de cette différence causée par la longitude de Brest, qui serait peut-être trop forte de quelques secondes dans la Carte de France et dans la Connaissance des Temps; je me suis servi pour la vérifier, de l'éclipse de Soleil, dont la fin fut observée le lendemain à Brest par les mêmes astronomes, avec les mêmes pendules et les mêmes lunettes, à $7^h\ 56^m\ 33^s$, suivant M. LE ROY et M. BLONDEAU; et $7^h\ 46^m\ 4^s$, suivant M. FORTIN.

BESIERS.*

[*Mémoires présentés par Divers Savants*, Tome VI, p. 124.]

Observers, CLAUZADE and DE MANSE:

Pendule non-corrigée. h.	m.	s.	
8	37	45.	M. DE MANSE jugea que le premier bord de Vénus touchait le bord précédent du Soleil—et il assura qu'il ne pouvait s'être trompé que de 4 et 5 secondes quoiqu'il ne se fut servi que de la lunette du quart de cercle, qui n'a que 3 pieds et demi.
8	56	05.	M. CLAUZADE, avec notre lunette de 7 pieds de longueur, jugea que le dernier bord de Vénus séparait du bord du Soleil, et que l'émersion totale était arrivée précisément à ce moment là.

The clock correction was found to be $-5^{m}\ 0^{s}$.

VINCENNES.*

Observer, PROLANGE.—ENCKE takes this observation from the *Journal des Savants* for December, 1761. I have no authority but ENCKE.

GREENWICH.

[*Maskelyne, Greenwich Observations*, I, p. 155.]

Regular circumference of ☉ and ♀ in contact, observed by several persons.

	h.	′	″		h.	′	″	
NEVIL MASKELYNE, 2-foot reflector . . .	12	14	58½	or	7	28	31	apparent time.
MALACHY HITCHINS, 6-foot reflector . . .	12	15	14½	or	7	28	47	
JOHN HORSLEY, Esq., 10-foot achromatic .	12	14	42½	or	7	28	15	
Mr. SAMUEL DUNN, 3½-foot achromatic . .	12	15	55½	or	7	29	28	

Completion of the thread of light, or the internal contact.

	h.	′	″		h.	′	″	
NEVIL MASKELYNE, 2-foot reflector . . .	12	15	50½	or	7	29	23	apparent time.
MALACHY HITCHINS, 6-foot reflector . . .	12	15	24½	or	7	28	57	
Rev. WILLIAM HIRST, 2-foot reflector . .	12	15	45½	or	7	29	18	
JOHN HORSLEY, Esq., 10-foot achromatic .	12	15	55½	or	7	29	28	
Mr. SAMUEL DUNN, 3½-foot achromatic . .	12	16	15½	or	7	29	48	

Mr. PETER DOLLAND and Mr. EDWARD NAIRNE judged the internal contact, and thought the thread of light just ready to be formed, but did not see it completed, at $12^{h}\ 15'\ 48''$, or $7^{h}\ 29'\ 20\frac{1}{2}''$ apparent time.

During these observations the air was very clear, with a west wind, and the Sun very distinct for the altitude.

A little after the internal contact I measured Venus's horizontal diameter with the achromatic object glass micrometer, applied to the 2-foot reflecting telescope, and found it to be $55\frac{3}{4}''$, by a mean of 8 observations, the extremes differing $4\frac{1}{2}''$ from one another, Venus at the same time ill-defined, and her circumference undulating very much.

*These observations belong to 1761, having been accidentally misplaced.

LONDON (MIDDLE TEMPLE).

[*Philosophical Transactions*, 1769, p. 171.]

Observer, J. HORSFALL.—At $7^h\ 26'\ 34''$ he saw a lambent light whirl around the opaque limb of the planet, the Sun being at the time only one-fourth of a degree above the top of the chimney.

LONDON (SPITAL SQUARE).

[*Philosophical Transactions*, 1769, p. 192.]

Observer, JOHN CANTON, M. A., F. R. S.—About half a minute before the total ingress, when the bright cusps of the Sun were at some distance from each other, there appeared a faint light between them, a little lower than the cusps or nearer to the center of the planet. This I observed to increase till the time of the internal contact, which fully convinced me that there is an atmosphere about Venus.

	h.	′	″	
First external contact at	7	8	28½	mean time.
First internal contact at	7	26	59½	" "
Duration of the ingress		18	31	

The observations are expressed by CANTON in Greenwich mean time. The conditions would seem to have been favorable and the time well determined. There is no certain statement of the phase observed; but from the gradual increase of the line of light before contact, it may be inferred that he could scarcely have observed anything but the thread of light.

LONDON (AUSTIN FRIARS).

[*Philosophical Transactions*, 1769, p. 378.]

Observer, ALEXANDER AUBERT:

	h.	′	″	
External contact at	7	8	13	mean time.
Internal contact at	7	26	45	" "
Interval		18	32	

N. B.—At $7^h\ 26^m\ 45^s$ Venus appeared to me in contact with the Sun, and about 6 seconds after I saw the Sun's limb completed.

WINDSOR CASTLE.

[*Philosophical Transactions*, 1769, p. 427.]

Observer, DANIEL HARRIS, F. R. S.—The wind was blowing rather hard, which, together with the smallness of the Sun's altitude, made the limb so very ill-defined and undulating that it is possible there may be an error of 5 or 6 seconds, at least in the time of external contact. The magnifying power was reduced to 55, and by this means the undulating motion of the Sun's limb was greatly reduced, in so much that the error, if any, in the time of the internal contact, by which I mean the completion of the thread of light formed by the Sun's circumference, can not exceed 3 seconds.

Times of the contacts of Venus with the Sun, as observed from the Round Tower, in Windsor Castle, by permission of his Grace the Duke of Montagu, June 3, 1769.

[Latitude 51° 28¼′ N. and longitude 2′ 24½″, in time W. from the Royal Observatory at Greenwich.]

	By the clock.	Mean time.
	h. ′ ″	h. ′ ″
The external contact of Venus with the Sun	7 4 30	7 6 14
The internal contact at	7 22 38	7 24 22
Duration between the contacts, the clock being 1′ 44″ too slow for mean time	. . . .	0 18 8
Venus's diameter measured three different times	. . . .	0 59½

SHIRBURN CASTLE.

[*Philosophical Transactions*, 1769, p. 173.]

Observers, MACCLESFIELD, BARTLETT, and Lady MACCLESFIELD.—At $7^h\ 23^m\ 13^s$ mean time, or $7^h 2\ 5^m\ 28\frac{3}{4}^s$ apparent time (as reduced from sidereal time), his Lordship determined the internal contact, which he judged to happen when the dark penumbra, which was so sensibly perceived between the limbs of the Sun and planet, was lost upon the completion of the thread of light.

At $7^h\ 7^m\ 4^s$ apparent time Mr. BARTLETT first saw Venus upon the Sun, and at $7^h\ 23^m\ 10\frac{1}{2}^s$ mean time, or $7^h\ 25^m\ 26^s$ apparent time, he judged the ingress to happen.

Lady MACCLESFIELD, at $7^h\ 25^m\ 16\frac{1}{2}^s$ apparent time, judged the second internal contact to happen with a reflecting telescope of 6 feet, through which the penumbra before mentioned was hardly to be distinguished.

LEICESTER.

[*Philosophical Transactions*, 1769, p. 238.]

Observer, Rev. Mr. LUDLAM.—At $7^h\ 6^m\ 0^s$, according to the time shown by the clock, a small indenture appeared on the Sun's limb. The increase of it at $7^h\ 6^m\ 14^s$ showed plainly that it was made by the planet.

The internal contact was first noted at $7^h\ 23^m\ 56^s$; at $7^h\ 24^m\ 8^s$ the divided part of the Sun's limb seemed wholly united. The edge of both the Sun and planet were in a continual tremor. At the internal contact the limb of the Sun seemed for several seconds to be alternately united, and again separated by a kind of shootings of the planet.

The clock error was fairly well determined by corresponding altitudes on June 2, and the rate was $2^s.2$ per day gaining. At the time of observation the correction to mean time was $-1^m\ 14^s$.

OXFORD.

[*Philosophical Transactions*, 1769, pp. 175–185.]

Observers, HORNSBY and others.—HORNSBY describes the formation of the ligament, which became narrow and narrower, but does not state when it commenced to appear as such. It was actually broken at $7^h\ 24'\ 13''$, while at $7^h\ 24'\ 23''$ the thread of light appeared equal in breadth to one-tenth of the planet's diameter.

The Rev. Mr. CLARE, Fellow of St. John's College, judged the thread of light to be completed at $7^h\ 24'\ 28''$ having observed the limbs to be in contact several seconds sooner.

Mr. SYKES, with an achromatic refractor of $3\frac{1}{2}$ feet, first saw Venus upon the Sun at $7^h\ 6'\ 0''$, and observed the thread of light to be completed at $7^h\ 24'\ 22''$.

Mr. SHUCKBURGH, of Balliol College, observed there the external contact of Venus with the Sun at $7^h\ 6'\ 8''$ apparent time and the internal contact at $7^h\ 24'\ 25''$; though at $7^h\ 23'\ 16''$ he judged that the center of the planet was removed more than its own semi-diameter from the Sun's limb, or that the true internal contact was then actually passed.

Mr. NIKITIN and Mr. WILLIAMSON made the following observations of the transit with a reflector of 10 inches and a refractor of 8 feet:

	Ext. cont.	Int. cont.
	h. ′ ″	h. ′ ″
Mr. NIKITIN	7 6 44	7 24 $15\frac{1}{2}$
Mr. WILLIAMSON	7 6 29	7 24 $10\frac{1}{2}$

Observer, HORSLEY.—The ligaments detached themselves from the Sun's limb, and the light, as I thought, was visible all around the planet at $7^h\ 51'\ 22''$ by my regulator, and not earlier to my eye, and this I set down as the internal contact.

Mr. JACKSON reckoned the internal contact at $7^h\ 21'\ 51''$ by our regulator. He judged of it, as I did, by the detachment of the ligament, which he saw, as well as I, from the Sun's limb.

KEW.

(House of JOSHUA KIRBY, Esq., in $1^m\ 14^s$ of west longitude.)

[*Philosophical Transactions*, 1769, p. 189.]

Observer, JOHN BEVIS, M. D., F. R. S.—At $7^h\ 28'\ 8''$ the planet was quite entered upon the disc, but was still joined to the Sun's limb by a slender kind of tail, nothing near so black as her disc. At $7^h\ 28'\ 17''$ the said tail vanished at once.

HAWKHILL.

[*Philosophical Transactions*, 1769, p. 339.]

Observers, Lord ALEMOOR, JAMES HOY, and Dr. LIND.—In the internal contact JAMES HOY differed from the other gentlemen and me 2 minutes, he calling it 12 minutes and we 14 minutes; which of us is wrong it will be no difficult matter to determine. In the internal contact we all observed the black ligament or protuberance, which was not broke for some seconds after the regular circumference of Venus seemed to be within the Sun, and the observation we send you was, as near as we could judge, about the time this protuberance was going to break. Lord ALEMOOR also, and he only, observed regular circumferences of the Sun and Venus in contact at $7^h\ 14^m\ 10^s$ mean time:

	Mean time.	
	Ext. cont.	Int. cont.
	h. ′ ″	h. ′ ″
Lord ALEMOOR, 18-inch reflector	6 57 33	7 14 32
JAMES HOY, $3\frac{1}{2}$-foot achromatic, magnifying 150	6 57 30	7 14 35
Dr. LIND, 2-foot achromatic, magnifying 100	6 57 41	7 14 37

Remarks by the Astronomer Royal.—Hawkhill is said by Dr. LIND to be about $1\frac{1}{2}$ miles north-east of Edinburgh. The latitude of the place was determined by meridian altitudes, taken by reflection with the sextant and by the mean of 10 observations, which all agree, within 2 minutes, is 55° 57′ 37″ N.

KIRKNEWTON.

[*Philosophical Transactions*, 1769, p. 345.]

Observer, Rev. Mr. BRICE.—Kirknewton is in latitude 55° 54′ 30″ N. and about 17 miles west of Hawkhill, from measurement on LOWRIE'S map of the environs of Edinburgh.

The clock was examined by taking equal altitudes of the Sun, and found to be 18 seconds slow. Its rate was less than 1 second in 5 days.

	h.	m.	s.
Internal contact clearly seen	7	11	55
18″ added for the clock too slow			18
	7	12	13

GLASGOW (NEAR THE UNIVERSITY).

[*Philosophical Transactions*, 1769, p. 333.]

Observers, Dr. A. WILSON, Mr. P. WILSON, Dr. REID, and Dr. WILLIAMSON.—Planet and Sun undulated a great deal, owing to the state of the air. As the internal contact approached Venus appeared to us to adhere to the Sun's limb by a dark protuberance or neck, both the length and breadth of which varied every moment by constant undulation. Neither did this neck break off instantaneously, but changed its color from black to a dusky brown, till at last the interval betwixt Venus and the Sun's limb appeared quite clear. Each of the observers wrote down his observations on the spot. I reduced them, together with my own, to apparent time from the observations I had made on the going of the clock, and are as follows:

By Dr. WILSON:

	h.	′	″
External contact	6	54	31.4
Venus's center judged to be on the limb	7	1	33.4
Sun's light appeared betwixt Venus and the limb	7	11	56.7

By Dr. WILLIAMSON and Dr. REID:

	h.	′	″
External contact	6	54	28
Internal contact, or when the Sun's light appeared betwixt Venus and the limb	7	12	24

By Dr. REID:

	h.	′	″
Venus's center judged to be on the limb	7	1	24

Dr. REID marked the time when he conceived the internal contact would have happened, if the dark protuberance on Venus had been taken away, and her disc reduced to a circle, viz, $7^h\ 10'\ 24''$.

By Mr. P. WILSON:

	h.	′	″
External contact	6	54	28
Internal contact	7	12	24

His second observation, by which he means the instant when the interval between Venus and the Sun's limb first appeared obvious, was taken down without the least knowledge of what was passing among the other gentlemen who observed.

Latitude of the observatory, 55° 51′ 32″; longitude by corresponding observations, 0^h 17′ 11″ of time from Greenwich west.

CAVAN.

[*Philosophical Transactions*, 1770, pp. 462, 463, 488.]

Observer, Mr. CHARLES MASON:

	h.	′	″	
At	11	17	53	the external contact of Venus and the Sun's limb.
	11	35	30	the contact seemed to be formed, judging by their peripheries.
	11	36	8	internal contact, the thread of light broke out.

Though the air at external contact was not quite so clear as at some times I have seen, yet the Sun's limb appeared well defined, and the spots in the disc very strong, their edges keen and distinct. At the internal contact the air was much changed, and the limb of Venus seemed to cohere to the Sun's limb by a protuberance that appeared like a dark shade, which seemed to prevent my seeing the thread of light for about 40″ longer than I expected.

He seems to have used a sidereal clock. The correction on mean time, as found by double altitudes, was as follows:

	h.	m.	s.
June 2	4	34	21.9
June 3	4	37	56.3

I conclude that at the time of internal contact the correction was $-4^h\ 38^m\ 59^s$.

GIBRALTAR.

[*Philosophical Transactions*, 1769, p. 347.]

Observers, three in number; not named; results communicated by Lieutenant JARDINE:

	h.	′	″
First internal contact with the Sun at	7	7	11
Sun set behind a hill	7	8	3
Clock before mean time		1	8.8

No statement of separate observations is given.

ISLE COUDRE.

[*Philosophical Transactions*, 1769, p. 276.]

Observer, Mr. THOMAS WRIGHT (deputy surveyor of the northern district of America):

Clock. h.	m.	s.	
3	7	48	time when Venus appeared completely round to the eye, and to appearance rather detached, and joined by a small dark thread or ligament, which prevented the rays of light from appearing.
3	8	19	time when the rays of light just appeared, at the internal contact.

The clock correction was well determined by corresponding altitudes, as follows, on mean time:

	m.	s.
June 1, noon	21	45
2, noon	20	47
3, midnight	19	22

NEWBURY, MASS.

[*Memoirs of the American Academy*, Series I, Vol. I, p. 111.]

Observer, WILLIAMS.—At $2^h\ 30'\ 14''$ apparent time I suspected I saw *a small disturbance* on the Sun's limb; but the impression was then so small, irregular, and ill-defined that it was not till after several seconds that I was certain the transit was begun.

Soon after Venus had touched the Sun's limb, the whole of her disc appeared visible; she appeared circular, and was surrounded with a pale glimmering light, not very distinctly defined. From this appearance I concluded it would be impossible to fix upon the precise moment when her limb would be exactly coincident with that of the Sun, and therefore determined to wait till there should appear a final thread of light between them. As the internal contact drew near the thread of light began to form, and seemed to dart on each side of the planet for several seconds without being fixed or settled. At $2^h\ 48'\ 44''$, with a seeming uncertainty of not more than 7 seconds, it became closed and fixed. Venus then appeared wholly within the Sun, separated from his limb by a fine thread of light flowing gently round it. This I fixed upon as the internal contact.

CAMBRIDGE, MASS.

[*Philosophical Transactions*, 1769, p. 356.]

Observer, Prof. JOHN WINTHROP, F. R. S.—The first impression I perceived was at $2^h\ 27^m\ 51^s$ by the clock, the Sun being then perfectly clear. I then rested my eye, which was pretty much fatigued, to prepare it for the total ingress or interior contact. At $2^h\ 45^m\ 15^s$ I began to be doubtful whether the internal contact was not formed, but at 20 seconds was satisfied that it was passed, the Sun's limb being restored to its integrity. During this interval of near 5 seconds there seemed to be a duskiness in the place of contact, my idea of which is well represented by Mr. DUNN'S figure of what he calls the gray contact. The clock was $2'\ 13''$ slow.

PROVIDENCE, R. I.

[*Transactions of the American Philosophical Society*, Vol. I, pp. 95, 96.]

Observer, Mr. BENJAMIN WEST.—Venus was first perceived by making a dent upon the superior limb of the Sun at $2^h\ 29'\ 43''$ apparent time.

The greatest attention was given to the interior contact. This was at $2^h\ 46'\ 35''$ apparent time.

At the moment of interior contact the Sun's altitude was taken with the sextant by Mr. MOSES BROWN and by the stile by Capt. JOHN BURROUGH, and both gave the time with the clocks within 2 seconds. The total ingress was not so instantaneous as I did expect it would be, but the bright cusps of the Sun, as they encompassed Venus, were much more obtuse, and there seemed to be a faint junction of their limbs for at least 4 seconds. The moment this penumbral ligament broke I proclaimed the time. At first I suspected the telescope was not adjusted to a proper focus, but afterwards, by looking at the solar spots, etc., I was convinced of the contrary. During the time we saw Venus upon the Sun she appeared to be surrounded by a ring of a yellowish color; its width was about one-tenth of the diameter of Venus. We saw nothing that might be taken for a satellite.

VILLE DU CAP HAÏTIEN (HAYTI).

[*Paris Memoirs*, 1769, p. 516.]

Observers, PINGRÉ, DE FLEURIEU, DE LA FILIÈRE, and DESTOURES.—The results of the observation of contact are given by PINGRÉ as follows:

Contacts des bords du Soleil et de Vénus.

	Premier contact.		Contact intérieur.	
	Temps de la pendule.	Temps vrai.	Temps de la pendule.	Temps vrai.
	h. ' "	h. ' "	h. ' "	h. ' "
Selon M. DE FLEURIEU	2 26 12	2 26 14½	2 44 43	2 44 45
Selon M. le Chevalier DE LA FILIÈRE	2 26 14	2 26 16½	2 44 39	2 44 41
Selon M. DESTOURES	2 26 18	2 26 20½	2 44 48	2 44 50
Selon moi	2 26 10	2 26 12½	2 44 42	2 44 44

It is stated that DE FLEURIEU noticed the arc of light around that portion of the globe of Venus which did not enter upon the Sun, and that PINGRÉ himself saw it about 2 minutes before the total immersion. There are here no statements respecting the phenomena of contact which will enable us to assign them to any special class, but PINGRÉ states elsewhere that all the observations refer to the thread of light.

BASKENRIDGE, N. J.

[*Transactions of the American Philosophical Society*, Vol. I, p. 125.]

Observer, WILLIAM ALEXANDER, EARL OF STIRLING:

Apparent time. h. ' "	
2 16 00	First discovery of the external contact at the ingress.
2 34 12	Total ingress.

The above account is extracted from his Lordship's letter of June 29, 1770, to the Rev. Dr. SMITH, provost of the College of Philadelphia.

LEWES, DEL.

[*Transactions of the American Philosophical Society*, Vol. I, p. 89.]

Observers, Mr. OWEN BIDDLE and Mr. JOEL BAYLEY.—Report by Mr. BIDDLE.—At the time of the internal contact, agreeable to what was noted by some of the observers at the transit 1761, "the eastern limb of Venus seemed to be united to the limb of the Sun by a black protuberance or ligament, which was not broken by the entrance of the thread of light" until 4 seconds after the regular circumference of Venus seemed to coincide with the Sun's.

For this observation I used a reflecting telescope, magnifying about 150 times, which was in exceedingly good order at that time, and defined the limb of the Sun and spots on its disc very nicely. I had applied a polar axis to it, and had altered the rack work, by which I could keep the same part of the limb in the field with ease.

My companion, JOEL BAYLEY, was not so well provided with a telescope. He had one of DOLLAND's double object-lens refracting glasses of about 4½ feet in length. This, with a ball and

socket, was fixed to a post, which made it very convenient for observation. Thus furnished, we found the contacts take place as follows:

	h.	′	″	
JOEL BAYLEY's external contact was lost by accident, but seen by him after it had taken place at	2	14	30	apparent time.
Internal contact, by JOEL BAYLEY	2	32	8	
External contact, as seen by OWEN BIDDLE	2	14	8	
Internal contact, by OWEN BIDDLE	2	32	8	

These observations are reduced to apparent time. And it must be noted that the time of the internal contact, as given by OWEN BIDDLE, is 4 seconds before the thread of light had broken the dark ligament by which Venus's limb was united to the limb of the Sun, that being the time he estimated the two limbs to be in contact.

It is stated that good corresponding altitudes of the Sun were obtained on the 2d and 3d, but no clock times or corrections are given.

PHILADELPHIA.

[*Transactions of the American Philosophical Society*, Vol. I, p. 45.]

Observer, SHIPPEN.—I therefore carefully observed the progress of the planet, and saw very distinctly as she moved onwards that the illuminated points of the Sun's limb became better defined, and when they approached so near to each other as to be within about 8 seconds of touching, which was at $2^h\ 31'\ 26''$ apparent time, I heard one of the observers call out contact; but as his observation did not seem to agree with the manner which I had fixed for judging of the contact, I continued viewing, with the closest attention, in order to fix the time of contact according to the idea I had formed of it, and at $2^h\ 31'\ 34''$ apparent time I could scarcely distinguish the illuminated points of the Sun's limb to be any longer separate; for in 2 seconds more they appeared to be so far closed as to form a single thread of light on that part of the Sun's limb which a few seconds before had been eclipsed. I therefore conclude that the apparent first internal contact of Venus happened at $2^h\ 31'\ 34''$ apparent time.

This seems to be a good observation of the earliest formation of the thread of light. I think the time which could be compared with other observations is the last one, or $2^h\ 31^m\ 36^s$ apparent time.

Observer, WILLIAMSON.—In determining the internal contact, which I apprehend was done with great exactness, I attended to the instant, when there was a perfect coincidence of the limb of Venus with the limb of the Sun, as when two circles touch internally. This appeared at $2^h\ 31'\ 24''$ apparent time. I expected by the time the assistant had counted another second to have seen light distinctly round the eastern limb of Venus, not such a radiance as had for 7 or 8 minutes rendered that part of the planet visible, but a certain narrow portion of the Sun's limb which had a very distinguishable appearance from the light I have mentioned. The edge of the Sun did not appear so soon; nevertheless I fixed upon $2^h\ 31'\ 25''$ for the precise time of the internal contact, being certain that no part of Venus was then off the Sun. One or 2 seconds more were counted before the Sun appeared distinctly without the limb of Venus. But then it was obvious that Venus did not then touch the Sun's limb in any part, so that the contact was certainly over.

The observation at $2^h\ 31^m\ 24^s$ or 25^s was doubtless that of true contact; but there are so few other observations of this phase that I shall add the first formation of the thread of light, for which we may assume his "1 or 2 seconds" to be at least 3.

Observer, PRIOR.—When the body of Venus was something more than one-third on the Sun, I saw her eastern atmosphere very distinctly reflecting the light of the Sun so strongly on the limb of Venus as to show it well defined; but as it came on the Sun it was entirely lost. The time I note for my internal contact was when the thread of light was distinctly seen all round the body of Venus, which was at $2^h\ 31'\ 28''$ apparent time.

Observer, THOMSON.—At $2^h\ 29'\ 11''$ mean time, or $2^h\ 31'\ 26''$ apparent time, I saw some tremulous rays of light pass from the upper or eastern limb of the Sun to the eye, across, and so as just to touch the upper limb of Venus. Marking that down therefore as the time of contact, I counted 4 seconds, at which time I saw a continued thread of light, like a silver lace, but still with a tremulous motion, round the eastern limb of Venus, whereby it appeared to me that the whole body of Venus was then within the disc of the Sun. The tremulous appearance of the rays of light I at first attributed to my telescope resting against the side of the observatory, but afterwards apprehended might be owing to their passing through the atmosphere of Venus.

Observer, EWING.—About the time that the center of Venus approached the Sun's disc I saw the whole body of Venus, her eastern edge being surrounded with a faint light, which was doubtless occasioned by her atmosphere refracting the Sun's rays. At $2^h\ 29'\ 11''$ mean time, or $2^h\ 31'\ 26''$ apparent time, I saw the internal contact when the whole body of Venus was introduced within the disc of the Sun, and the thread of light completely surrounded her, although not as bright as it became in 2 seconds afterwards.

NORRISTOWN, PA.

[*Transactions of the American Philosophical Society*, Vol. I, pp. 24–29.]

Observer, RITTENHOUSE.—When the internal contact (as it is called) drew nigh I foresaw that it would be very difficult to fix the time with any certainty on account of the great breadth and brightness of the light which surrounded that part of Venus yet off the Sun. After some consideration I resolved to judge, as well as I could, of the coincidence of the limbs, and accordingly gave the signal for the internal contact at $2^h\ 28'\ 45''$ by the clock (when the appearance of Venus and the border of light were as in Fig. 3, Plate 3), and immediately began to count seconds, which any one who has been accustomed to it may do for a minute or two pretty near the truth. In this manner I counted no less than $1'\ 32''$ before the effect of the atmosphere of Venus on the Sun's limb wholly disappeared, leaving that part of the limb as well defined as the rest. From this I concluded that I had given the signal for the internal contact too soon, and the times given by the other observers at Norristown confirm me in this opinion.

I conclude from this description that RITTENHOUSE did not observe any definite phase. It is, I believe, an historic fact that he was nearly overcome by the excitement of the moment

Observer, LUKENS.—When Venus was near one-half of her diameter advanced on the Sun I saw distinctly a border of light encompassing that part of her which was yet off the Sun. This was so bright that it rendered that part of Venus visible and pretty well defined, although not yet entered on the Sun. But towards the internal contact the circular border of light seemed to grow more dusky towards the points where the luminous segments of the Sun's limb were ready to close round the planet. This duskiness did not seem to part wholly from the Sun's limb at the time I apprehended the body of Venus to be wholly entered on the Sun, and then I gave the signal for the internal contact, which was noted by both the persons who counted for me at $2^h\ 28'\ 58''$ by the clock. And I judge at least from $16''$ to $18''$ more, before I saw the Sun's limb clear of this dusky shadow.

The phenomenon not being clearly described, it might be doubtful what phase was observed at $2^h\ 28^m\ 58^s$. But it would seem from a statement of Dr. SMITH (see below) that the first appearance of the thread of light was noted.

Observer, SMITH.—As to the internal contact, the thread or crescent of light coming round from both sides of the Sun's limb, did not close instantaneously about the dark body of the planet, but with an uncertainty of several seconds, the points of the threads darting backwards and forwards into each other in a quivering manner, for some space of time, before they finally adhered. The instant of this adhesion I determined to wait for, with all the attention in my power, and to note it down for the internal contact, which I did at $2^h\ 29'\ 5''$ by the clock, a few seconds later than Mr. LUKENS, who judged in the same way. And even then, though the points of the thread of light seemed to close, yet the light itself did not appear perfect on that part of the limb till about 12″ afterwards, and I apprehended that a person who had waited for the perfection of this final thread of light would have given the contact that number of seconds later than I did, although I was later than the others.

This description seems most precise and accurate. The principal doubt to which it gives rise is whether observers, under less favorable conditions, would not take a phase corresponding to his latest time as that of the first formation of the thread of light.

WILMINGTON, DEL.

[*Transactions of the American Philosophical Society*, Vol. I, p. 126.]

Observer, Mr. WILLIAM POOLE:

[Extracted from a letter to Mr. OWEN BIDDLE, and communicated to the Society December 21, 1770.]

	h.	′	″	
First external contact	2	12	48¾	apparent time.
First internal contact	2	30	20½	

With a refractor of 12 feet, magnifying power about 50 times, Mr. POOLE thinks the external contact was several seconds before the time marked in the margin.

The internal contact was taken just as the Sun's light began to surround the planet, though his limb was not visible beyond the planet till a second or two afterwards.

FORT PRINCE OF WALES, HUDSON'S BAY.

[*Philosophical Transactions*, 1769, p. 480.]

Observers, WILLIAM WALES and JOSEPH DYMOND:

Observations on the transit of Venus, June 3, 1769.

Observations.	Observer.	Time per clock. h. m. s.	Apparent time. h. m. s.
Exterior contact at the ingress	JOSEPH DYMOND . .	0 56 49	0 57 0.6
Do	WILLIAM WALES . .	0 56 56	0 57 7.6
Interior contact at the ingress	. . . do	1 15 10	1 15 21.3
Do	JOSEPH DYMOND . .	1 15 14	1 15 25.3
Thread of light broke at internal contact .	WILLIAM WALES . .	7 0 40	7 0 45¼
Do	JOSEPH DYMOND . .	7 0 43	7 0 48¼
The external contact. Very hazy and the limbs badly defined.	WILLIAM WALES . .	7 18 56	7 19 1¼
Do	JOSEPH DYMOND . .	7 19 15	7 19 20¼

We took for the instant of the first internal contact the time when the least visible thread of light appeared behind the subsequent limb of Venus; but before that time Venus's limb seemed within that of the Sun, and his limb appeared behind hers in two very obtuse points, seeming as if they would run together in a broad stream like two drops of oil, but which nevertheless did not happen, but joined in a fine thread at some distance from the exterior limb of Venus. This appearance was much more considerable at the egress than at the ingress, owing, as we apprehend, to the bad state of the air at that time. We took for the instant of internal contact at the egress the time when the thread of light disappeared before the preceding limb of the planet, from which time WILLIAM WALES took notice that he had told about 24 seconds when the limbs of the Sun and Venus were apparently in contact, a circumstance which he did not venture to attend to at the ingress.

SAN JOSÉ.

[*Voyage en Californie par feu M. Chappe D'Auteroche*, Paris, 1772, p. 94.]

Observer, L'Abbé CHAPPE D'AUTEROCHE:

Temps vrai.	Temps observé à la pendule 2 juin.	
h. ′ ″	h. ′ ″	
23 59 17	23 57 32	*Premier contact*, à la lunette achromatique de 3 pieds, montée sur une machine parallatique. J'aperçais Vénus faisant une petite échancrure sur le bord du Soleil parfaitement terminé. Je ne crois pas que cette première phase s'écarte beaucoup de la véritable, parce que l'échancrure était très-petite.
0 17 27	0 15 42	*Second contact.* À l'entrée totale de Vénus j'observai très-distinctement le second phénomène qui avait été remarqué par la plus grande partie des Astronomes en 1761. Le bord du disque de Vénus s'alongea (voy. Fig. 2) comme s'il était attiré par le bord du Soleil. Je n'observai point pour l'instant de l'entrée totale, celui où le bord de Vénus commençait à s'alonger; mais ne pouvant pas douter que ce point noir ne fit partie du corps opaque de Vénus, j'observai le moment où il était à sa fin; de façon que l'entrée totale ne peut être arrivée plus tôt, mais peut-être plus tard de deux ou trois secondes. Le point noir était un peu moins obscur que le reste de Vénus. Je crois que c'est le même phénomène que celui que j'observai à Tobolsk en 1761.
5 54 50	5 53 9	Premier contact à la sortie avec la lunette de dix pieds. Le Soleil était ondoyant ainsi que Vénus, ce qui rendait cette observation très-difficile. A ce premier contact Vénus s'est alongée plus considérablement que le matin, en s'approchant tout-à-coup du bord du Soleil.
6 13 19	6 11 38	Second contact ou sortie totale. Elle ne me paraît pas être arrivée plus tôt, peut-être 4″ plus tard, mais je n'en suis pas certain.

Pour observer avec toute la précision possible les deux contacts à la sortie, je disposai ma lunette de façon que je ne fusse pas obligé de la remuer vers ces moments. Sans cette précaution j'eusse été dans le cas de perdre de vue Vénus; de prendre le fond du ciel pour le bord du disque de cette planète, et de commettre ainsi une erreur énorme, au lieu qu'en ne quittant pas un instant de vue, au dernier contact, le bord de Vénus qui paraissait un peu plus noir que le fond du ciel, j'eus cette phase avec toute l'exactitude possible.

J'avais chargé M. PAULY d'observer à la lunette de trois pieds les deux contacts de la sortie; il était déjà un peu exercé aux observations. Il observa le premier contact 22″ plus tôt que moi et le dernier 37″ plus tôt. Comme il était à côté de moi, je m'aperçus du moment où il quitta la lunette pour aller à la pendule, et je vis très-bien qu'il fixait trop tôt les moments du premier et du second contact; car je voyais encore Vénus parfaitement lorsqu'il était à la pendule.

I have compared this printed statement of PAULY's observation with CHAPPE's original manuscript, which is preserved in the Paris Observatory. The comparison shows that CASSINI took no pains to give CHAPPE's exact words, and, which is yet more singular, the printed transcript is less explicit than the original, which reads as follows, *verbatim et literatim:*

J'avois chargé mr pauli d'observer a lunette de trois pieds il etoit seulement exercé a prendre des hauteurs correspondantes quil avoit pris 7 a 8 soir il observa le 1er contact a $5^h\ 52'\ 47''$ et le dernier a $6^h\ 11'\ 1''$. Comme il etoit a coté de moy je m'apercus lorsquil quitta la lunette pour aller a la pendule et je vis très bien quil observoit trop tot le 1er contact et je voyois encore venus a la sortie parfaitement lorsquil fut aussi a la pendule.

In his discussion of these observations Mr. STONE assumed that only geometric contact was observed at egress, though the formation of the thread of light was noted at ingress. But it seems to me very clear that the phase of internal contact noted at egress could have been no other than that of the cutting off of the thread of light. That CHAPPE was on the alert for an early phase seems to be conclusively shown by his remark that at $5^h\ 52'\ 47''$ he was aware that PAULY left his telescope to note the clock time, and saw very well that he had observed the contact too soon.

Observers, DOZ and MEDINA.—The only original authority I can find for these observations is in the appendix to CHAPPE's book, where we find the observations made at San José tabulated as follows (p. 159):

Noms des lieux.	Observateurs.	Entrée de Vénus.		Sortie de Vénus.	
		Premier contact.	Second contact.	Premier contact.	Second contact.
		h. m. s.	h. m. s.	h. m. s.	h. m. s.
San Joseph en Californie.	CHAPPE . .	11 59 17	0 17 26.9	5 54 50.3	6 13 19
	DOZ	11 59 14	0 17 25	5 54 47.5	6 12 41
	MEDINA . .	11 59 18	0 17 30	5 54 47.5	6 12 46

The same work contains a complete set of observations made at "*Saint Anne*," in California. I have not, however, succeeded in identifying the station.

TAHITA.

[*Philosophical Transactions*, 1771, p. 410.]

Observers, GREEN, COOK, and SOLANDER:

***Transit of Venus by Mr. Green, with a reflecting telescope of 2-feet focus; magnifying power*, 140 *times*.**

Date.	Observation.	Time per clock.	Apparent time.
		h. ′ ″	h. ′ ″
June 2	Light thus on the ⊙'s limb. (Table XIV, Fig. 1) . .	9 21 45	21 25 40
	Certain. (Fig. 2)	9 22 00	21 25 55
	First internal contact of ♀'s limb and the ⊙. (See Fig. 4)	9 39 20	21 43 15
	Penumbra and ⊙'s limb in contact. (See Fig. 5) . .	9 40 00	21 43 55
June 3	First contact of penumbra, undulating, but the thread of light visible and invisible alternately	3 10 05	3 14 3
	Second internal contact of the bodies	3 10 53	3 14 51
	Second external contact	3 27 30	3 31 28
	Total egress of penumbra, ⊙'s limb perfect	3 28 16	3 32 14

***Transit of Venus by Captain Cook, with a reflecting telescope of 2-feet focus; magnifying power*, 140 *times*.**

June 2	The first visible appearance of ♀ on the ⊙'s limb. (See Fig. 1)	9 21 50	21 25 45
	First internal contact, or the limb of ♀ seemed to coincide with the ⊙'s. (See Fig. 2)	9 39 20	21 43 15
	A small thread of light seen below the penumbra. (See Fig. 3)	9 40 20	21 44 15
June 3	Second internal contact of the penumbra, or the thread of light wholly broke	3 10 15	3 14 13
	Second internal contact of the bodies, and appeared as in the first	3 10 47	3 14 45
	Second external contact of the bodies	3 27 24	3 31 22
	Total egress of penumbra, dubious	3 28 04	3 32 2

The first appearance of Venus on the Sun was certainly only the penumbra, and the contact of the limbs did not happen till several seconds after, and then it appeared as in Fig. 4. This appearance was observed both by Mr. GREEN and me, but the time it happened was not noted by either of us. It appeared to be very difficult to judge precisely of the time that the internal contacts of the body of Venus happened, by reason of the darkness of the penumbra at the Sun's limb, it being there nearly, if not quite, as dark as the planet. At this time a faint light, much weaker than the rest of the penumbra, appeared to converge towards the point of contact, but did not quite reach it. (See Fig. 2.) This was seen by myself and the two other observers, and was of great assistance to us in judging of the time of the internal contacts of the dark body of Venus with the Sun's limb. Fig. 5 is a representation of the appearance of Venus at the middle of the egress and ingress, for the very same phenomenon was observed at both. At the total ingress the thread of light made its appearance with an uncertainty of several seconds. I judged that the

penumbra was in contact with the Sun's limb 10 seconds sooner than the time set down above. In like manner, at the egress, the thread of light was not broken off or diminished at once, but gradually, with the same uncertainty. The time noted was when the thread of light was wholly broken by the penumbra.

Transit of Venus by Dr. Solander, with a 3-foot reflecting telescope.

Observation.	Time per clock.	Apparent time.
	h. ′ ″	h. ′ ″
First external contact plainly convex, a wavering haze seen some seconds before	9 22 11	
Ingress—light seen glimmering under Venus	9 39 33	21 43 28
♀ free from the ☉'s limb	9 40 07	21 44 2
♀'s true limb out	3 27 51	3 31 49
♀'s atmosphere out	3 28 15	3 32 13

The descriptions of the phenomena by all the observers seem to have been biased by the idea that Venus was surrounded by an atmosphere which would appear as a penumbra around the planet. It has, however, been suggested by at least one astronomer that the mirrors of their telescopes were so distorted by the Sun's rays as to destroy the focal adjustment, and that in consequence the planet actually appeared as if surrounded by a haze. We must accept this as a possibility. Yet, this cause could hardly have produced the appearance of the penumbra at external contact, because the Sun's limb would have been blurred equally with that of the planet. I shall take up the observations of interior contacts in the order in which they are printed.

If his Fig. 4 represents what he calls first interior contact, Mr. Green gives a representation of Venus which is not surrounded by any penumbra and is wholly within the Sun's disc; but the image of the planet is connected with the Sun's limb by a black rectangle, wholly unlike the usual black drop, being tangential to the limb. But it can be nothing else than a representation of the black drop as reproduced from memory. We may therefore regard the representation as that of a geometric contact.

In Fig. 5, which represents what he calls the contact of the penumbra, this black rectangle has wholly disappeared, and the Sun's limb is complete. This observation must therefore be either that of the thread of light or of some later phase.

At egress the statement that the thread of light was visible and invisible alternately would seem to imply a good observation of the breaking of the thread at $3^h\ 14'\ 3''$.

Captain Cook's figures are substantially identical in their general features with those of Mr. Green. At $21^h\ 43'\ 15''$ he had some sort of a geometric contact; at $21^h\ 44'\ 15''$ the first formation of the thread of light, as would appear from the description, although the figure represents the Sun's limb as entirely complete His observation of egress at $3^h\ 14'\ 13''$ seems also quite explicit.

Dr. Solander gives no figures and says nothing about the problematical geometrical contact. But his words, "light seen glimmering under Venus," can not, I think,

refer to anything but the formation of the thread of light. We have therefore the results in apparent time:

Ingress—formation of the thread of light:

	h.	′	″
Mr. GREEN	21	43	55
Captain COOK	21	44	15
Dr. SOLANDER (light first seen)	21	43	28
Dr. SOLANDER (Venus free from Sun)	21	44	2

Egress—rupture of thread of light:

Mr. GREEN	3	14	3
Captain COOK	3	14	13

These conclusions seem to me the only ones that can be legitimately derived from the descriptions and drawings of the phenomena by the observers, irrespective of any agreement with other observations. They agree with the interpretation of ENCKE, but are wholly at variance with that of STONE, who assigns them to the class of apparent contacts. The considerations which led him to take this course are:

It appears to me clear that the penumbra *as dark or nearly as dark as the planet* is nothing more nor less than a part of the planet itself. If this be so, COOK and GREEN have both observed contacts, and not the forming and breaking of the black-drop. COOK states that at the egress he observed the time when the thread of light was wholly broken off by the penumbra.

CHAPTER IV.

GEOGRAPHICAL POSITIONS OF THE STATIONS.

As a general rule, the latitudes of the points of observation are so easily determined that no important question need be raised respecting their accuracy. For the purpose of the present work an accidental error of 2 or 3 minutes in the latitude of any station is of no serious importance. It may be fairly assumed that all the latitudes determined by the observers, or otherwise known a century ago, have a smaller limit of error than this. No discussion has therefore been devoted to them.

In the case of the longitudes it is different. The absolute time of each observation should be known within 2 or 3 seconds, and it is but recently that the longitudes of distant points could be determined with this degree of accuracy. It is therefore necessary to enter upon a special investigation of this subject. In many cases the positions are so well known from readily accessible data that no special investigation is necessary. In other cases an independent result is to be derived.

For convenient reference a classification of the positions has been made, depending upon the character of the discussion necessary to obtain a sufficiently accurate result. First in order are placed those points for which it was unnecessary that a special investigation should be made. Then follow in order the stations which had to be discussed individually:

I. *Stations whose longitudes are well determined by telegraphic signals or other modern methods.*—In this case the only serious doubt which can arise is, whether the point determined telegraphically or otherwise is sufficiently near the place of observation. This question is one which does not in all cases admit of a satisfactory decision. Commonly, the point accurately determined is an observatory, and, as a general rule, the longitude of the place of observation has been assumed to be that of the observatory, unless, from local or other data, it seemed probable that such was not the case. When the observatory was on a meridian decidedly different from what seemed to be the average central meridian of the town, it was assumed that the latter should be taken as the place of observation, and a small correction was applied accordingly. These corrections are so small and unimportant that no statement or discussion of them has been deemed necessary. In the column of reference in the table at the end of the present chapter all well-known stations are designated by the numeral I.

II. *Russian and Siberian stations.*—In order to obtain the latest information on this subject a request for information was addressed to Director STRUVE, of the Pulkowa Observatory. The information was carefully collected by the late Dr. WAGNER, whose communications on the subject will be found in the appendix.

III. *Scandinavian stations.*—The positions of most of these were supplied by Dr. GYLDEN from the results of official surveys.

IV. *German stations.*—Dr. AUWERS kindly supplied a list of the ascertained or probable positions of those German stations which were not otherwise well known

V. *Jakutsk, Selenginsk, and Ponoi.*—The observations at Jakutsk are so uncertain that no determination of the longitude from the observed occultations has been made. ENCKE's longitude from the eclipse of the Sun has been adopted.

For Selenginsk I have also accepted ENCKE's position. The occultations observed by RUMOWSKY are both doubtful. For Ponoi there is no modern determination.

VI. *Manilla and Batavia.*—The longitude of the cathedral at Manilla and of the time-ball observatory at Batavia were determined telegraphically by Commanders GREEN and DAVIS, U. S. N., with the result:*

	h.	m.	s.	
Manilla	8	3	52.21	E.
Batavia	7	7	14.47	E.

I have no information of the position of the point of observation at Manilla. At Batavia OUDEMANS informs me that it was $7^s.5$ east of the time-ball.

VII. *Pekin.*—This city covers so much ground, and contains within its limits so many points of observation, that great care is necessary to ascertain where each point lay and to avoid confusing one point with another.†

The principal points which have been used in astronomical determinations are the following:

The "Collegium Lusitanorum" of the Jesuits.

The American transit of Venus station.

The "Collegium Gallorum" of the Jesuits, which is the present site of the cathedral known as Pe-thang.

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*Telegraphic determinations of longitude in Japan, China, etc. Washington, Bureau of Navigation, 1883.

†In commencing to determine the location of the Jesuit observing station in Pekin, I attempted to derive a result from the recorded statements of the authors who had investigated the longitude of that or other points within the city. The discordance in the various statements on the subject is very remarkable. An important point to be determined was the difference of longitude between an establishment commonly known as the "Royal Observatory" at Pekin and the "college" of the French Jesuits, where the latter are supposed to have made their observations. In the *Astronomische Nachrichten* (Vol. VIII, p. 258) WURM gives a short note on the subject, in one part of which it is stated that the observatory is 12^s in time *West* of the college, while in another sentence he states that it is 12^s *East*. In the *Philosophical Transactions* for 1774 MASKELYNE published a communication upon an eclipse observed at Pekin, in which he says that the Royal Observatory is 14^s *West* of the Jesuits' College. But the account which he communicates yet later states that it is *East*. So far as can be determined from the above data the balance of authority is in favor of the *West* direction. But this conclusion seemed to be completely reversed by GEORGE V. FUSS (*Ast. Nach.*, XII, p. 103), who actually observed at Pekin, and reduced his determination of longitude from the Jesuit College to the Royal Observatory by *subtracting* $12^s.9$ from the *East* longitude of the former point. This proceeding, by an astronomer upon the ground, whose knowledge rested on personal observation, ought to have decided the question in favor of the easterly direction. And yet the true direction is *West*, as was learned by an examination of FRITSCHE's map in the *Mélanges Mathématiques et Astronomiques*, of St. Petersburg (Vol. IV), and from the triangulations made by the French and American transit of Venus expeditions in 1874.

The "Russian Cloister," in which was a magnetic house, where FUSS made his observations.

The French transit of Venus station of 1874.

The present Russian magnetic observatory.

The ancient "Chinese Royal Observatory," situated on the wall near the southeast corner of the city.

Among the determinations of the longitudes of one or another of these various points six are worthy of consideration. I have gone over and corrected some of these, in order to get the result which seemed the most probable from the data employed. Each of the results was reduced to the "Coal Hill," a point near the center of the city to which the other points were referred in the triangulations made by the French and American parties in 1874. The two triangulations exhibited a perfect agreement, thus leaving no doubt about the identity of the different points of observation.

I deem it unnecessary to go into the details of each of the six special determinations of longitude. The most reliable of all, that of WATSON's, from numerous occultations, is only a provisional result, derived from a preliminary reduction; but I do not conceive that the final discussion will result in any change of more than a fraction of a second.

Separate results for the longitude of Pekin.

(1) POWALKY re-reduces several old occultations, already used by WURM, from which I find for the—

		h.	m.	s.
Collegium Gallorum		7	45	40.5
Reduction to Coal Hill	+			3.0
Longitude of Coal Hill		7	45	43.5

(2) FUSS, from 10 ☽ culminations, with a transit 13 inches long, for the—

		h.	m.	s.
Old Russian Cloister	$\lambda'=$	7	36	9.9
Reduction to Coal Hill	—			2.0
Reduction to Greenwich	+		9	21.0
Longitude of Coal Hill		7	45	28.9

(3) FRITSCH, 1868–'69, from Moon culminations finds longitude of the—

		h.	m.	s.
Russian Magnetic Observatory		7	45	56.4
Reduction to Coal Hill	—			8.4
Longitude of Coal Hill		7	45	48.0

(1) *Astronomische Nachrichten*, Vol. LXI, p. 292.
(2) *Ibid.*, Vol. XII, p. 103.
(3) *Mélanges Mathématiques et Astronomiques*, Vol. IV, p. 743.

(4) FLEURIAIS, 1874, from ☽ culminations—

French transit of Venus station of 1874 .	$\lambda'=7$	36	23.3
Reduction to Coal Hill	—		3.3
Reduction to Greenwich		9	21.0
Longitude of Coal Hill	7	45	41.0

(5) WATSON, 1874, from 5 culminations of ☽'s limb I—

American transit of Venus station . . .	7	45	28.6
Reduction to Coal Hill	+		4.1
Longitude of Coal Hill	7	45	32.7

(6) WATSON—

From occultations of stars	7	45	29.5
Reduction to Coal Hill	+		4.1
Longitude of Coal Hill	7	45	33.6

Of these six results (2) and (5) can hardly receive any weight at all, while (6), of which the probable error is about a second of time, is entitled to a greater weight than all the rest together.

The results of (3) and (4) are so large that I have rapidly examined the processes by which they are deduced.

In (3) FRITSCH applied a correction to the Moon's tabular R. A., of which the mean value is $-0^s.20$. But I have found that for his mean epoch the actual correction is $-0^s.33$. This change will diminish his longitude by 4 seconds.

Arranging FLEURIAIS's results by observer and limb they are:

	h.	m.	s.
FLEURIAIS, I	7	36	25.4
FLEURIAIS, II			16.2
LAPIED, I			21.5
LAPIED, II			12.9
Mean	7	36	19.0

a result $4^s.3$ less than that already cited. The latter is derived from 5 culminations only, observed on dates when the Paris and Greenwich observatories gave accordant corrections to the tables. I prefer the mean of the separate results for each limb, including all the observations.

(4) *Mission de Pekin.* French Papers relating to transit of Venus, Vol. II, Part I, p. 162.

Applying these corrections, the results, together with the weights to which they severally appear entitled, are as follows:

	h.	m.	s.	
WATSON, occultations	7	45	33.6.	Weight = 20
FLEURIAIS, ☽ culminations	0	0	36.7.	Weight = 3
FRITSCH	0	0	44.0.	Weight = 2
POWALKY, occultations	0	0	43.2.	Weight = 2
Mean	7	45	35.4.	

Nothing is gained by assigning any other probable error than $\pm 1^s$ to this result.

Reducing to other stations in Pekin the results for the east longitudes from Greenwich are:

	h.	m.	s.
1. The "Collegium Lusitanorum" of the Jesuits . .	7	45	30.3
2. The American transit of Venus station of 1874 .	7	45	31.3
3. The "Collegium Gallorum" of the Jesuits (site of the present Cathedral Pe-thung)	7	45	32.4
4. The Coal Hill	7	45	35.4
5. The old Russian Cloister	7	45	37.4
6. The French transit of Venus station of 1874 . .	7	45	38.7
7. The Russian Magnetic Observatory	7	45	43.8
8. The Ancient Chinese Royal Observatory . . .	7	45	44.6

VIII. *Madras.*—I have at hand no exact data for determining the difference of longitude between the Madras Observatory and the points where the observations were made. On the admiralty charts Fort George, which is near the meridian of the center of the town, is 9^s east of the observatory, for which

$$\lambda = -5^h\ 21^m\ 0^s$$

It may therefore be assumed that, for the point of observation,

$$\lambda = -5^h\ 21^m\ 9^s$$

IX. *Rodrigues.*—PINGRÉ says of the position of his station:

Cette isle a environ 15,000 toises de longueur de l'est-nord-est à l'ouest-sud-ouest, sur près de 5,000 de largeur. C'est sur la côte septentrionale, à 5,000 toises environ de la partie la plus orientale, dans le lieu nommé *Enfoncement de François le Guat* que j'ai fait mes observations. (*Paris Memoirs*, 1761, pp. 414, 415.)

This statement shows that he landed on the coast of Mathurin Bay, the principal harbor of the island. No such name as he gives is, however, found on the admiralty chart of the island. There is a little village at the landing place, and an old fort at Point Venus, about 0'.5 of arc to the northeast, where an English party observed the transit of 1874. These would seem to be the probable limits of the position to be assigned to PINGRÉ's station.

The English party found for the latitude of Point Venus

$$\varphi = -19^\circ\ 40'\ 22''$$

The center of the village is placed 15″ farther south on the chart, so that, for it

$$\varphi = -19^\circ\ 40'\ 37''$$

PINGRÉ made 40 determinations of latitude, of which the mean results are:

	°	′	″
From 16 measures south	19	40	40.3
From 24 measures north	19	40	37.1

The close accordance of the result with the latitude of the village gives color to the supposition that he observed at this point. For its longitude we have:

		h.	m.	s.
Longitude of Point Venus (English Transit of Venus, Report 1874, p. 365)		−4	13	43.5
Village west of Point Venus		+		1.5
PINGRÉ's station	$\lambda =$	−4	13	42.0

X. *Wardhus*—I am indebted to Prof. H. MOHN, of Christiania, director of the Meteorological Institute, for an account of expeditions made by him during the years 1876–'78 to determine the positions of certain points in the northern part of Norway, including Wardhus. It will suffice to give his concluded results for the position required, which is as follows:

		°	′	″		h.	m.	s.
Astronomical station in Wardhus .	$\varphi =$	70	22	23.4;	$\lambda =$	2	4	31.07
Reduction to HELL's station . . .	−			11.6	+			3.26
Position of HELL's station . . .		70	22	11.8		2	4	34.33 ± 0ˢ.75

XI. *North Cape.*—This being the name by which the station on the island of Magero, occupied by BAYLEY in 1769, has been known in astronomical literature, I have retained it. In connection with his paper in the *Philosophical Transactions* BAYLEY gives a map of the island, with his position marked upon it. I sent a tracing of this map to Professor GYLDÉN, with a request to compare it with the latest map which the official surveys accessible in Stockholm could supply. It appeared from his reply that there had been no recent astronomical determination in the region. The tracing which he sent me, and the British admiralty charts, show the actual conformation of the coasts to be so different from that laid down by BAYLEY that the position of the latter could not be inferred except from the latitude. This element he determined to be 71° 1′. On the coast line which he might have occupied there is but one point as far south as this, and this point, being in a cove, may be taken as that of observation. The only authorities for its longitude are the admiralty charts of Sweden and Great Britain, which give

$$\lambda = -1^h\ 43^m\ 38^s$$

The eclipse of the Sun on June 3 was observed by BAYLEY as follows:

June 3, 1769, at $1^h\ 48^m\ 4^s$, the clouds clearing away, I saw the Sun and the Moon had made a small impression or notch in the Sun's limb; by observing the increase of the eclipse I suppose it began 4, 5, or 6 seconds sooner than I first saw it, or at $1^h\ 48^m\ 0^s$ per clock or $20^h\ 59^m\ 19^s$ apparent time, nearly.

Clouds came on, so that I saw the Sun no more until $3^h\ 38^m\ 0^s$ per clock, and it broke away very clear, and continued clear to the end, which was at $3^h\ 48^m\ 19^s$ per clock, or $22^h\ 59^m\ 17^s$ apparent time. The air being very clear, the end seemed certain to about 2 seconds.

Common experience shows that an estimate of the kind described, in connection with the beginning, commonly requires to be more than doubled. I have therefore adopted $20^h\ 59^m\ 11^s$ apparent time as the most probable moment of actual tangency. For the times thus inferred Prof. REUEL KEITH computed the quantities shown in the following table.

The Greenwich mean times are found by applying a provisional longitude, $1^h\ 44^m\ 6^s$, to the observed mean times.

The Moon's longitude, latitude, and parallax were computed from HANSEN's tables for these Greenwich mean times, and the Sun's place from LE VERRIER's tables.

From the geocentric positions the apparent positions of the Moon were computed by the method given in my *Researches on the Motion of the Moon* (appendix to the Washington Observations for 1876). But the parallax used in passing to the apparent position of the Moon is the difference of parallaxes of Sun and Moon.

The difference of longitude and latitude between the centers of the Sun and Moon, as seen from the station at the two assigned moments, is hence inferred, and hence D, the apparent distance of the centers.

The sum of the apparent semi-diameters is given in the same column with D. Were the provisional longitude in perfect accord with the observations, these two should agree. Their difference, divided by the partial derivative of D as to the time, obtained on the supposition that the local mean time remains constant, gives a correction to the longitude, the application of which to the provisional longitude gives the longitude found in the last column:

Local and Greenwich mean times, June 3, 1769.	☽'s tabular geocentric longitude and latitude.	Eq. hor. parallaxes of ☽ and ☉.	☽'s apparent longitude and latitude.	☉'s longitude and latitude.	Differences, ☽—☉.	Distance of centers and sum of semi-diameters.	Resulting longitude.
h. m. s.	° ′ ″	′ ″	° ′ ″	° ′ ″	″	″	h. m. s.
20 57 1.4	73 9 59.6	61 16.7	73 17 59.1	73 49 11.1	—1872.0	1969.1	1 43 46.1
19 12 55.4	+ 0 59 14.9	8.7	+ 0 10 9.7	— 0.1	+ 609.8	1958.4	
22 57 8.1	74 25 52.5	61 15.9	74 25 57.2	73 53 58.2	+1919.0	1956.6	60.1
21 13 2.1	+ 0 52 24.7	8.7	+ 0 6 21.6	— 0.1	+ 381.7	1959.5	

The most probable result of an observed eclipse is, I think, generally obtained by giving double weight to the last contact; but this eclipse exhibits the anomaly that the observed is greater than the tabular duration, showing some error in one of the phases, and not justifying my subtraction of 8^s from his observed time.

We may therefore put

$$1^h\ 44^m\ 0^s$$

as the longitude derived from the observation of the eclipse.*

XII. *Cape of Good Hope.*—No specific statement of the location of the station is made. A journal is given showing that the parties arrived in Table Bay April 27, carried the instruments ashore May 2, and set the clock going May 4. There can be no serious danger of error in supposing that the point of observation was near the present custom-house. I find by a local map that this point is about $12^s.3$ west from the Royal Observatory.

I therefore adopt for the position of the station: Longitude, $1^h\ 13^m\ 42^s$ east; latitude, $33^\circ\ 55'.6$.

XIII. *Florence.*—The astronomical observatory, known as that of Florence, is on the hill of Arcetri, which I find by a local map to be $1'.5$ south from the center of the city. No reduction is therefore necessary to the longitude, which I assume to be the same as that of the observatory.

XIV. *Bordeaux.*—Two stations have been designated by this same name. The one supposed to be in the city, and occupied by DE LA ROQUE; the other 8,800 toises south of Bordeaux and 400 toises east of Château Trompette.

From geodedic data LALANDE considers the longitude of FAUGÈRE's station near Château Trompette to have been $11^m\ 36\frac{1}{2}^s$ west of Paris; but ENCKE adopts $11^m\ 34.5^s$. For Bordeaux (St. André) the *Connaissance des Temps* gives the longitude $0^h\ 11^m\ 38.6^s$; while for DE LA ROQUE's station ENCKE adopts $0^h\ 11^m\ 37^s$. There being no convenient way to test or correct these numbers, I have adopted ENCKE's positions as being probably founded on the best available data.

XV. *Madrid.*—The position of the old observing station relative to the present observatory, was communicated by Signor MERINO, director of the latter.

XVI. *Lisbon.*—The longitude of the present Royal Observatory was found by Commander GREEN to be $36^m\ 45^s$. By measurement on an admiralty chart I find the center of the city to be 11^s to the east, making

$$\lambda = 0^h\ 36^m\ 34^s$$

XVII. *St. John's.*—WINTHROP makes no clear statement of the position of his station. All he says upon the subject is this: "As this town is bounded by high mountains towards the sun-rising, so that no house in it would answer our end, we encamped on an eminence at some distance, from whence we could see the Sun presently after his rising." We can not therefore infer even the direction of his station from the town.

Correspondence with the United States consul at St. John's elicited a thorough discussion of the subject from Captain ROBINSON, harbor master of that port, which renders it highly probable that the station was near the present site of Fort Townsend, on an eminence rising above the western boundary of the town. This point is $5^s.8$ west from Chain Rock Battery, which has been taken as the standard point of refer-

*Owing to the loss of some of the computations I can not fully verify the tabular numbers on which this result depends.

ence in the marine surveys of Newfoundland. For the longitude of the latter we have the following results:

(1) Admiral BAYFIELD, in 1844, made a chronometric determination of Chain Rock Battery—Halifax. I have no knowledge of the work, but, correcting the result for the correction of BAYFIELD's adopted longitude of Halifax, the U. S. Hydrographic Office gives for Chain Rock Battery

$$\lambda = 52^\circ\ 40'\ 54''$$

(2) The U. S. Coast Survey* finds for the longitude of Heart's Content, the landing of the Anglo-American telegraph cables, $53^\circ\ 22'\ 25''$. On the admiralty chart of Heart's Content the longitude is given $53^\circ\ 22'\ 11''$ (Chain Rock Battery being $52^\circ\ 40'\ 47''$). Assuming that this implies a connection of the two points, the common correction required is $+14''$ making for Chain Rock Battery

$$\lambda = 52^\circ\ 41'\ 1''$$

The difference of the results is only $7''$. We therefore conclude:

		h.	m.	s.
Chain Rock Battery	$\lambda =$	3	30	44
WINTHROP's station	$\lambda =$	3	30	50

XVIII. *Isle Coudre.*—The observer at this station seems to have been most unfortunate in his determination of his position, his printed computations being full of blunders. Modern surveys, however, leave little doubt on the subject. There is an anchorage and landing place on the northwest side of the island which seems to be the only point of note, and at which it may be supposed the observations were made. The U. S. Hydrographic Office has supplied the following chain of longitudes from Quebec to this landing place, showing its longitude to be $4^h\ 41^m\ 39^s$:

	°	′	″
Quebec Observatory west of Greenwich . . .	+71	12	18.9
WOLF's monument east of Quebec Observatory .	—		2.0
Isle Coudre east of WOLF's monument	—	47	39.0
Whence, Isle Coudre, west of Greenwich .	70	24	37.9

XIX. *Cambridge, Mass.*—The grounds of Harvard University, in or near which it may be presumed WINTHROP's observations were made, lie 3^s of time east of the present Cambridge Observatory. This gives $4^h\ 44^m\ 28^s$ for the longitude to be applied.

XX. *Providence, R. I.*—No data whatever are given for fixing the point in Providence at which Mr. WEST made his observations. The position of the Unitarian Church in that State is given by the Coast Survey as follows:

$$\varphi = 41^\circ\ 49'\ 26''$$
$$\lambda = 71^\circ\ 24'\ 19''$$
$$= 4^h\ 45^m\ 37^s.3$$

*Report for 1880, Appendix VI, p. 14.

XXI. *Cape Haïtien.*—The identification of the station occupied by PINGRÉ (called by him *Cap François*) and the fixing of its longitude, have not been free from difficulty. There are two points called Cape Français on the island; but a very slight examination shows that neither of them could have been the one occupied. Moreover, the latitude given by PINGRÉ himself, and repeated in the *Histoire* (19° 57′), would have placed the station several miles out at sea.

From an examination of all the conditions, and the remarks made by PINGRÉ on his station, there seems little doubt that it was near the town now called *Ville Cap Haïtien*, but which was also known as the *Ville Cap François.* This town is in latitude 19° 46′, and, as PINGRÉ says his station was a little to the north of the town, the most probable explanation of the latitude he gives is that he made a mistake of 10′ in writing and printing it.

The lack of precision in the published data on which the longitude of this point depends was such that a special application for assistance was addressed to the late Colonel PERRIER, of the *Bureau des Longitudes.* I was thus placed in communication with M. ANTOINE D'ABBADIE, to whom I became indebted for a full description of the various determinations bearing on the question. From the data supplied by him, and from the publications of the U. S. Hydrographic Office, I have been enabled to prepare the following chain of longitudes, terminating at the point in question:

		h.	m.	s.
Fort St. Pierre, Martinique—Greenwich, determined telegraphically by Commander F. M. GREEN, U. S. Navy		4	4	44.8
Fort de France—Fort St. Pierre	—			26.8
Fort d'Islet (Au Prince)—Fort de France	+		45	8.0
Cap Haïtien—Fort d'Islet	—			37.8
Cap Haïtien—Greenwich		4	48	48.2

XXII. *Lewes, Del.*—The position of the village of Lewes, Del., is well determined by the U. S. Coast Survey. The only question which can arise is, whether the station occupied by BIDDLE and BAILEY, and called by them Lewis Town, was identical with the present village of Lewes. The evidence in the affirmative seems entirely satisfactory. The station is described by the observers as 29″.0 south and 3′ 16″.8 west of the Provincial Light-house on Cape Henlopen.*

This corresponds accurately to the situation of Lewes relative to the present lighthouse. This correspondence of the two differences of position affords strong evidence of the identity of the former light-house with the present one.† The position of the latter is

$$\begin{aligned} \varphi &= 38^\circ\ 46'\ 42'' \\ \lambda &= 75\ \ \ 5\ \ \ 3 \\ &= 5^h\ 0^m\ 20^s.2 \end{aligned}$$

* *Transactions of the American Philosophical Society*, Vol. I, p. 88.

† This identity is established by the records of the U. S. Light-house Board.

The latitude assigned to the light-house by the observers is 38° 47′ 8.″ The difference of 26″ is not greater than the possible error of their observations. Adopting this as the position of the light-house, that point in the town of Lewes where the observations were made will be situated in

$$\begin{aligned} \varphi &= 38^\circ\ 46'\ 12'' \\ \lambda &= 75\ \ \ 8\ \ \ 20 \\ &= 5^h\ 0^m\ 33^s.3 \end{aligned}$$

By a survey connecting their station with Philadelphia the observers found it to be 1ˢ of time east of the latter point. This gives the longitude only 2ˢ of time greater than the above result, which may therefore be adopted as the definitive longitude.

XXIII. *Philadelphia.*—The observatory used by the astronomers of the American Philosophical Society in the latter part of the last century, was in the State House Square. Its position is given by the Coast Survey* as follows:

Latitude	39° 56′ 52″.4
Longitude	75 9 3.0
	= $5^h\ 0^m\ 36^s.20$

XXIV.—*Norristown*—Norristown was the home of RITTENHOUSE, and the point where he made his observations. It was connected with the observatory in Philadelphia by a survey which showed it to be 52″ of time west of that point. The meridian thus determined passes through the present position of Norristown, as given on modern maps, so that the only uncertainty which can affect it is that of the particular point in the town where RITTENHOUSE's observatory was situated. This uncertainty is so small that I have not deemed it necessary to investigate the question further, and have therefore adopted the longitude:

Norristown . . . . $5^h\ 1^m\ 28^s$

XXV. *Fort Prince of Wales, Hudson Bay.*—The longitude of this station has to depend entirely upon the occultations observed by DYMOND and WALES. They are printed as follows in the *Philosophical Transactions* for 1769, p. 479:

*Report U. S. Coast Survey for 1874, p. 133.

Date.	Time per clock.	Apparent time.	Occultations of the fixed stars by the Moon, etc., observed.
1768.	h. ′ ″	h. ′ ″	
Sept. 21	7 2 9 7 2 16	7 6 52 7 6 59	ρ Capricorni immerged behind the Moon's dark limb.—J. D. and W. W.
1769. Mar. 15	11 21 6	11 24 34	ζ Geminorum immerged behind the Moon's dark limb (very exact).—W. W.
	12 8 44	12 12 11	Ditto emerged (perhaps about 5″ sooner).—J. D.
29	16 54 0 16 53 58	16 46 22 16 46 19	ν^2 Sagittarii immerged behind the Moon's bright limb.—W. W. and J. D.
Apr. 9	10 29 21	10 20 27½	τ Tauri immerged behind the Moon's dark limb.—J. D.
10	15 38 44 15 39 14	15 29 39 15 30 9	Jupiter's first satellite immerged close to the body of the planet.—W. W. and J. D.
Aug. 11	9 16 47	9 10 22½	The star, No. 43, of Ophiuchi, in Mr. FLAMSTEAD's catalogue, immerged behind the dark limb of the Moon (very faint).
	10 14 56	10 8 31	B, in the same constellation and catalogue, immerged.—J. D.
	10 14 54	10 8 29	Ditto, per W. W.—N. B.—The immersion happened towards the northern limb of the Moon, so very near the intersection of light and darkness as to render the observation doubtful to 2 or 3″.

The difference of 7ˢ in the first occultation shows that it was missed by one of the observers. The balance of probability being in favor of the latest observation, I shall provisionally use it.

The occultation of March 29 being behind the bright limb of the Moon, is unreliable. The last one of August 11 is also unreliable. We have left the following four occultations, of which all but the first seem entirely reliable. The reduced observations will be as follows:

	h.	m.	s.	
1768, September 21 . .	6	59	31;	immersion of ρ Capricorni.
1769, March 15 . . .	11	33	26;	immersion of ζ Geminorum.
1769, April 9	10	21	47;	immersion of τ Tauri.
1769, August 11 . . .	9	15	10;	immersion of B. A. C., 5868.

The separate values of the longitude derived by Professor KEITH are as follows:

	h.	m.	s.
From occultation of ρ Capricorni	6	16	34.7
From occultation of ζ Geminorum			38.8
From occultation of τ Tauri			37.8
From occultation of B. A. C., 5868			33.8
Mean	6	16	36.3

This result differs only a second from that derived by POWALKY* from the same observations.

XXVI. *San José*, the position of CHAPPE's observing station, was located and determined by Assistant GEORGE DAVIDSON, U. S. Coast Survey, in 1873. His result† is:

$$\varphi = +23^\circ\ 3'\ 37''.28 \pm 0''.47.$$
$$\lambda = 7^h\ 18^m\ 48^s.99 \pm 0^s.42$$

XXVII. *Tahiti, Point Venus.*—I have been unable to refer to all the original data from which the longitude of this point may be derived. It seems that two occultations were observed there by Captain COOK in 1773 and 1774. ENCKE says these were computed by TRIESNECKER in the *Ephemerides Vindobonenses*, 1806, p. 306, with the result:

$$\lambda' = 10^h\ 7^m\ 12^s$$

whence

$$\lambda = 9^h\ 57^m\ 51^s$$

Two occultations were also observed at the same point by PREUSS‡ in 1824, April 24, only one of which was correct. These and COOK's occultations are computed by POWALKY (*Astronomische Nachrichten*, Vol. LXI, p. 292), with the results:

	h.	m.	s.
1773, August 28, immersion of β^2 Capricorni . . .	9	57	55.1
1773, August 28, emersion of β^2 Capricorni . . .			53.2
1774, April 26, immersion of γ Libræ			55.5
1824, April 4, immersion of an anonymous star . .			60.5

If we reject the emersion, which must have taken place at the Moon's bright limb, and give half weight to the last determination, owing to the want of a more accurate position of the star than that given by BESSEL's zones, the result will be

$$\lambda = 9^h\ 57^m\ 55^s.4$$

The *Connaissance des Temps* for 1888 gives

$$\lambda = 9^h\ 57^m\ 57^s$$

for the light-house at Point Venus, but gives no indication from which it can be learned on what data this result depends. The presumption seems to be that the above result is as accurate as any that can be derived from existing data.

**Astronomische Nachrichten*, Vol. LXI, pp. 293, 294.
† Report U. S. Coast Survey for 1874, Appendix No. X.
‡ *Astronomische Nachrichten*, Vol. IV, p. 131.

The adopted geocentric co-ordinates of the points of observation are collected in the following table. In the names of the stations I have very generally followed ENCKE or other investigators, although there may be several cases in which the name assigned is not the usually accepted one.

In the last column the Roman numerals refer to the parargraphs of the present chapter, in which the longitude is discussed or the authority for it given. Where the authority is ENCKE it is to be understood either that no more recent determination is available or that ENCKE's result is presumed to be correct.

It will be remarked that the numeral I is applied in a general way to all the longitudes which were supposed to be so well known that no discussion was necessary. Most of those not found in the list of known observatories have been taken from the *Connaissance des Temps.*

Table of adopted geographical positions of stations.

Name of station.	Transit observed.	Latitude.	Logarithm of— $\rho \sin \varphi'$	Logarithm of— $\rho \cos \varphi'$	Longitude (+ west; — east).	Authority and reference.
		° ′			h. m. s.	
Jakutsk	1769	+62 1.8	9.9443	9.6725	−8 38 58	V (ENCKE).
Manilla	1769	+14 35.4	9.3983	9.9859	−8 3 52	VI (GREEN).
Pekin	1761–1769	+39 55.0	9.8048	9.8855	−7 45 32	VII.
Batavia	1769	− 6 10.2	9.0283*n*	9.9975	−7 7 22	VI.
Selenginsk	1761	+51 6.1	9.8890	9.7988	−7 6 25	V (ENCKE).
Calcutta	1761	+22 33.2	9.5810	9.9656	−5 53 21	I.
Madras	1761	+13 4.1	9.3514	9.9887	−5 21 9	VIII.
Tobolsk	1761	+58 12.3	9.9275	9.7228	−4 33 6	II (WAGNER).
Rodrigues	1761	−19 40.6	9.5245*n*	9.9741	−4 13 42	IX.
Orsk	1769	+51 12.2	9.8897	9.7979	−3 54 15	II (WAGNER).
Orenburg	1769	+51 45.8	9.8930	9.7925	−3 40 24	Ibid.
Gurief	1769	+47 7.0	9.8627	9.8336	−3 27 42	Ibid.
Ponoi	1769	+67 4.5	9.9626	9.5918	−2 44 30	V (ENCKE).
Kola	1769	+68 52.7	9.9681	9.5580	−2 12 13	II (WAGNER.)
Wardhus	1769	+70 22.2	9.9723	9.5276	−2 4 34	X (MOHN).
Petersburg	1761–1769	+59 56.5	9.9354	9.7009	−2 1 14	I.
Cajaneborg	1761–1769	+64 13.6	9.9527	9.6395	−1 50 56	II (WAGNER).
North Cape	1769	+71 1.0	9.9740	9.5136	−1 44 0	XI.
Tornea	1761	+65 50.9	9.9584	9.6131	−1 36 35	II (WAGNER).
Abo (Wanhalinna)	1769	+60 29.2	9.9378	9.6937	−1 29 31	II (WAGNER).
Abo (the town)	1761	+60 26.9	9.9376	9.6941	−1 29 6	I.
Cape Town	1761	−33 55.6	9.7442*n*	9.9194	−1 13 42	XII.
Stockholm	1761–1769	+59 20.6	9.9327	9.7086	−1 12 16	I.
Hernosand	1761–1769	+62 38.0	9.9467	9.6637	−1 11 46	III (GYLDEN).
Upsala	1761–1769	+59 51.5	9.9350	9.7019	−1 10 32	I.
Tyrnau	1761	+48 22.6	9.8714	9.8231	−1 10 22	IV (OPPOLZER).
Vienna Observatory (old)	1761	+48 12.6	9.8703	9.8245	−1 5 32	I.
Calmar	1761	+56 39.6	9.9199	9.7411	−1 5 25	IV (GYLDEN).
Carlscrona	1761	+56 9.7	9.9174	9.7468	−1 2 21	Ibid.

Table of adopted geographical positions of stations—Continued.

Name of station.	Transit observed.	Latitude.	Logarithm of— $\rho \sin \varphi'$	Logarithm of— $\rho \cos \varphi'$	Longitude (+ west; — east).	Authority and reference.
		° ′			h. m. s.	
Wetzlas	1761	+48 36.2	9.8730	9.8212	—1 1 38	IV (Oppolzer).
Laibach	1761	+46 2.9	9.8551	9.8423	—0 58 3	IV (Oppolzer).
Greifswald	1769	+54 5.8	9.9065	9.7692	—0 53 21	Encke.
Lund	1761–1769	+55 41.9	9.9150	9.7519	—0 52 45	I.
Landscrona	1761	+55 52.2	9.9159	9.7501	—0 51 20	Encke.
Copenhagen	1761	+55 41.0	9.9150	9.7521	—0 50 19	I.
Rome	1761	+41 53.9	9.8223	9.8723	—0 49 55	I.
Leipzig	1761	+51 20.3	9.8905	9.7966	—0 49 28	IV (Auwers).
Munich	1761	+48 8.4	9.8698	9.8251	—0 46 17	Ibid.
Ingolstadt	1761	+48 45.9	9.8740	9.8198	—0 45 41	Ibid.
Bologna	1761	+44 29.5	9.8432	9.8539	—0 45 23	I.
Florence	1761	+43 47.5	9.8377	9.8590	—0 45 2	XIII.
Dillingen	1761	+48 34.6	9.8728	9.8214	—0 41 59	IV (Auwers).
Drontheim	1761	+63 25.6	9.9497	9.6518	—0 41 37	IV (Gylden).
Göttingen	1761	+51 32.0	9.8916	9.7947	—0 39 46	I.
Würzburg	1761	+49 47.7	9.8808	9.8108	—0 39 44	IV (Auwers).
Schwetzingen	1761	+49 23.1	9.8782	9.8145	—0 34 18	IV (Auwers).
Lyons	1761	+45 45.8	9.8530	9.8444	—0 19 17	I.
Leiden	1761	+52 9.3	9.8953	9.7887	—0 17 56	I.
Montpellier	1761	+43 36.7	9.8363	9.8604	—0 15 31	I.
Saron	1769	+48 34.1	9.8727	9.8215	—0 14 56	Encke.
Beziers	1761	+43 20.5	9.8342	9.8624	—0 12 51	I.
Vincennes	1761	+48 50.7	9.8746	9.8191	—0 9 45	Encke.
Conflans-Sous-Carrière	1761	+48 49.4	9.8744	9.8193	—0 9 38	Encke.
Hôtel de Clugny	1761	+48 51.1	9.8745	9.8190	—0 9 23	I.
Paris (Coll. de Louis le Gd.)	1761–1769	+48 51.0	9.8746	9.8191	—0 9 23	I.
Paris (St. Généviève)	1761	+48 50.8	9.8746	9.8191	—0 9 23	I.
Paris (Luxemburg)	1761	+48 51.0	9.8746	9.8191	—0 9 21	I.
Paris (Royal Observatory)	1761–1769	+48 50.2	9.8745	9.8192	—0 9 21	I.
Paris (Ecole Militaire)	1761	+48 51.1	9.8745	9.8190	—0 9 13	I.
Passy	1761–1769	+48 51.3	9.8746	9.8191	—0 9 7	Encke.
Colombe	1769	+48 55.5	9.8751	9.8184	—0 9 1	Encke.
St. Hubert	1761–1769	+48 53.0	9.8748	9.8188	—0 7 25	Encke.
Toulouse	1769	+43 36.6	9.8364	9.8605	—0 5 46	I.
Rouen	1761–1769	+49 26.3	9.8785	9.8140	—0 4 23	I.
Havre	1769	+49 29.3	9.8789	9.8135	—0 0 26	I.
Greenwich	1761–1769	+51 28.7	9.8913	9.7952	0 0 0	I.
Hackney	1761	+51 3.0	9.8887	9.7993	+0 0 11	Encke.
London (Middle Temple)	1769	+51 30.8	9.8915	9.7950	+0 0 15	I.
London (Spital Square)	1761–1769	+51 31.2	9.8916	9.7949	+0 0 17	I.
London (Austin Friars)	1769	+51 30.8	9.8915	9.7950	+0 0 20	I.
London (Clerkenwell)	1761	+51 31.0	9.8915	9.7949	+0 0 27	I.
London (Savile House)	1761	+51 30.8	9.8915	9.7950	+0 0 30	I.
Chelsea	1761	+51 29.5	9.8914	9.7951	+0 0 41	I.

Table of adopted geographical positions of stations—Continued.

Name of station.	Transit observed.	Latitude.	Logarithm of— $\rho \sin \varphi'$	$\rho \cos \varphi'$	Longitude (+ west; — east).	Authority and reference.
		° ′			h. m. s.	
Kew	1769	+51 28.1	9.8913	9.7954	+0 1 14	I.
Caen (Mission)	1769	+49 11.2	9.8769	9.8162	+0 1 23	I.
Caen	1769	+49 11.2	9.8769	9.8162	+0 1 25	I.
La Trompette	1769	+44 40.7	9.8447	9.8526	+0 2 13	XIV (ENCKE).
Bordeaux	1769	+44 50.2	9.8460	9.8515	+0 2 16	I.
Windsor	1769	+51 28.3	9.8913	9.7953	+0 2 21	I.
Bayeux	1761	+49 16.6	9.8775	9.8154	+0 2 49	I.
Shirburn Castle	1761–1769	+51 39.4	9.8924	9.7936	+0 3 56	I.
Leicester	1769	+52 38.0	9.8981	9.7840	+0 4 35	I.
Oxford	1769	+51 45.6	9.8930	9.7926	+0 5 0	I.
Hawkhill	1769	+55 57.6	9.9164	9.7490	+0 12 38	Map.
Kirknewton	1769	+55 53.3	9.9160	9.7498	+0 13 40	Map.
Madrid	1761	+40 24.8	9.8094	9.8822	+0 14 50	XV.
Glasgow	1769	+55 51.5	9.9158	9.7501	+0 17 4	I.
Leskeard	1761	+50 26.9	9.8850	9.8049	+0 17 51	Map.
Brest	1769	+48 23.4	9.8715	9.8230	+0 17 58	I.
Gibraltar	1769	+36 7.2	9.7679	9.9078	+0 21 25	I.
Cadiz	1769	+36 31.5	9.7721	9.9055	+0 25 12	I.
Cavan	1769	+54 51.7	9.9106	9.7611	+0 30 11	I.
Agromonte	1769	+41 9.0	9.8158	9.8774	+0 34 26	= OPORTO?
Oporto	1761	+41 9.0	9.8158	9.8774	+0 34 26	Admiralty chart.
Lisbon	1761	+38 42.6	9.7937	9.8929	+0 36 34	XVI (GREEN).
St. John's	1761	+47 34.0	9.8659	9.8299	+3 30 50	XVII.
Isle Coudre	1769	+47 24.7	9.8648	9.8312	+4 41 39	XVIII.
Newbury	1769	+42 48.0	9.8298	9.8662	+4 43 27	U. S. Coast Survey.
Cambridge	1769	+42 22.8	9.8264	9.8692	+4 44 28	XIX.
Quebec	1769	+46 48.3	9.8605	9.8362	+4 44 49	I.
Providence	1769	+41 49.4	9.8217	9.8730	+4 45 37	XX.
Cape Haïtien	1769	+19 47.0	9.5267	9.9738	+4 48 48	XXI.
Baskenridge	1769	+40 42.2	9.8119	9.8803	+4 58 11	U. S. Coast Survey.
Lewes	1769	+38 46.2	9.7943	9.8925	+5 0 33	XXII.
Philadelphia	1769	+39 56.9	9.8052	9.8852	+5 0 36	XXIII.
Norristown	1769	+40 9.7	9.8071	9.8838	+5 1 28	XXIV.
Wilmington	1769	+39 44.5	9.8033	9.8865	+5 2 12	U. S. Coast Survey.
Hudson Bay	1769	+58 47.5	9.9302	9.7156	+6 16 36	XXV.
St. José	1769	+23 3.6	9.5901	9.9640	+7 18 49	XXVI.
Tahiti	1769	—17 29.2	9.4749n	9.9796	+9 57 56	XXVII.

CHAPTER V.

TABULAR ELEMENTS TO BE CORRECTED BY OBSERVATION.

1. The tabular elements were, in the first place, derived from LE VERRIER'S Tables of the Sun and of Venus,* but were corrected before the final comparison with observations. The heliocentric elements derived directly from the tables are as follows:

Transit of 1761.

Greenwich mean time.	Earth's— Absolute longitude.	Earth's— Radius-vector.	Earth's— Latitude.	Venus's— Longitude.	Venus's— Radius-vector.	Venus's— Latitude.
h.	° ′ ″		″	° ′ ″		″
June 5, 13	255 25 13.35	1.0154301	—0.55	255 18 1.05	0.7263135	—166.95
16	32 23.60	4439	—0.56	29 54.81	3271	—209.25
19	39 33.85	4577	—0.57	41 48.57	3407	—251.55
22	46 44.10	4714	—0.58	53 42.33	3542	—293.84

Transit of 1769.

June 3, 6	253 17 54.53	1.0151435	—0.07	253 11 41.72	0.7261547	+299.67
9	25 5.05	1592	—0.05	23 35.81	1689	257.37
12	32 15.57	1748	—0.03	35 29.90	1831	215.07
15	39 26.10	1904	—0.01	47 24.00	1974	172.77

2. The comparison with observation was made by the heliocentric method, analogous to BESSEL'S Theory of Eclipses, developed in my *Discussion of Transits of*

*Annales de l'Observatoire de Paris, Mémoires, Tomes IV and VI.

*Mercury.** Referring to that paper for the development, it will suffice here to present the formulæ and results. We put

l, b, r, l', b', r' = the heliocentric co-ordinates of the planet and of the Sun, respectively.

c, ω = the angular distance of the centers of the Earth and planet, as seen from the center of the Sun, and the position-angle of the great circle joining them.

c_1, ω_1 = the same quantities, relative to the planet and any point on the Earth.

r, r_1 = the radii of either cone circumscribing Venus and the Sun, at the center of the Earth and at the point of observation, respectively.

f = the angle of either cone.

R, R' = the linear radii of the planet and Sun.

τ = the local sidereal time.

Π = the Sun's equatorial horizontal parallax.

ε = the obliquity of the ecliptic.

ρ, φ' = the geocentric co-ordinates of the place of observation.

The analytic condition which determines the tabular time of contact is

$$c_1 = \mathrm{r}_1$$

c_1 and r_1 being functions of the time and of the co-ordinates of the place.

The formulæ for computing these quantities are as follows: For the center of the Earth c and ω are determined, with all necessary precision, from the equations

$$c \sin \omega = b' - b$$
$$c \cos \omega = l - l'$$

l and b being first corrected for aberration in the way subsequently described.

Determine the auxiliary quantities, h and α, from the equations

$$h \sin \alpha = \cos \varepsilon \cos l'$$
$$h \cos \alpha = \sin l'$$

Then

$$p = h\Pi\rho \cos \varphi' \cos (\tau + \alpha) - \Pi\rho \sin \varphi' \sin \varepsilon \cos l'$$
$$q = \Pi\rho \sin \varphi' \cos \varepsilon - \Pi\rho \cos \varphi' \sin \varepsilon \sin \tau$$

$$c_1 \cos \omega_1 = c \cos \omega + p$$
$$c_1 \sin \omega_1 = c \sin \omega + q$$

$$\mathrm{r} = \frac{\cos c}{\cos f} \left\{ \frac{\mathrm{R}' - \mathrm{R}}{r} - \frac{\mathrm{R}'}{r'} \sec c \right\} \text{ (for interior contact)}$$

$$\mathrm{r} = \frac{\cos c}{\cos f} \left\{ \frac{\mathrm{R}' + \mathrm{R}}{r} - \frac{\mathrm{R}'}{r'} \sec c \right\} \text{ (for exterior contact)}$$

$$\mathrm{r}_1 = \mathrm{r} - \rho\Pi \sin \mathrm{S} \cos z$$

S being the Sun's angular apparent semi-diameter and z its zenith distance.

* Astronomical Papers, Vol. I, Part VI, p. 422.

3. *Numerical data.*—The adopted numerical value of the Sun's semi-diameter is that derived from the discussion of the transits of Mercury:

$$R' = .004\ 653\ 16$$

The decision upon the semi-diameter of Venus to be adopted is more difficult, owing to the irreconcilable discrepancies between different determinations of that element. For the present purpose it was deemed advisable that the adopted value should depend upon measures made in transit across the face of the Sun, and that double-image measures should be taken rather than those with a filar micrometer. These two conditions are, in fact, those which come nearest to the conditions of contact in a transit. Two such determinations were available, that of Auwers at Luxor, in 1874, and my own in South Africa, in 1882. The resulting diameters of Venus, reduced to the distance unity, are

	″
Auwers	16.96
Newcomb	16.88
Adopted mean	16.92

whence

$$R = .000\ 041\ 02$$

We thus derive the following values of r upon the plane passing through the center of the Earth perpendicular to the line joining the centers of the Sun and Venus:

	″
1761. Ingress: internal contact	r = 364.59
1761. Egress: internal contact	364.58
1761. Egress: external contact	387.88
1769. Ingress: internal contact	364.615
1769 Egress: internal contact	364.595
1769. Egress: external contact	387.898

In all cases $r_1 = r - 0''.04 \cos z$, z being the Sun's zenith distance at the time of contact.

In the paper referred to it is shown that the effect of aberration may be corrected by taking for a moment t

The Sun's absolute co-ordinates for the moment t

Venus's absolute co-ordinates for the moment $t - \tau_3$

τ_3 being the time required for light to pass from Venus to the Earth, and then considering the problem as a purely geometric one.

We have for all the phases of both transits

$$\tau^3 = 143^s.8 = 2^m.397$$

$$\text{Motion of } l = 3.''966 \text{ per minute}$$

$$\text{Motion of } b = -0.''235 \text{ per minute}$$

Hence, the corrections to l and b on account of aberration are

$$\delta l = -9''.51$$
$$\delta b = +0.56$$

We thus have the following values of $l - l'$ and $b' - b$:

Greenwich mean time.	$l - l'$	$b' - b$
h.	$''$	$''$
1761, June 5, 13	−441.81	+165.84
16	−158.30	+208.13
19	+125.21	+250.42
22	+408.72	+292.70
1769, June 3, 6	−382.32	−300.30
9	− 98.75	−257.98
12	+184.82	−215.66
15	+468.39	−173.34

A preliminary comparison with some of the observations showed that the errors of these values were too large for convenience and accuracy in comparing with observation. The following provisional corrections were therefore applied:

$$\text{To } l - l' = c \cos \omega, \text{ correction} = +0''.25$$
$$\text{To } b' - b = c \sin \omega, \text{ correction} = -2.00$$

Thus, we have the following provisional values of $c \sin \omega$ and $c \cos \omega$:

Transit of 1761.

Greenwich mean time.	$c \cos \omega$	$c \sin \omega$	Greenwich mean time.	$c \cos \omega$	$c \sin \omega$
h. m.	$''$	$''$	h. m.	$''$	$''$
14 20	−315.55	+182.64	20 10	+235.71	+264.86
21	−313.98	+182.87	11	+237.29	+265.10
22	−312.40	+183.11	12	+238.86	+265.33
23	−310.83	+183.34	13	+240.44	+265.57
24	−309.25	+183.58	14	+242.01	+265.80
25	−307.68	+183.81	15	+243.59	+266.04
26	−306.10	+184.05	16	+245.16	+266.27
27	−304.53	+184.28	17	+246.74	+266.51
28	−302.95	+184.52	18	+248.31	+266.74
29	−301.38	+184.75	19	+249.89	+266.98
30	−299.81	+184.98	20	+251.46	+267.21
			21	+253.04	+267.45
			22	+254.61	+267.68
			23	+256.19	+267.92
			24	+257.76	+268.15

Transit of June 3, 1769.

Greenwich mean time.	$c \cos \omega$	$c \sin \omega$	Greenwich mean time.	$c \cos \omega$	$c \sin \omega$
h. m.	"	"	h. m.	"	"
7 25	−248.16	−282.32	13 10	+295.35	−201.20
26	−246.59	−282.08	11	+296.92	−200.97
27	−245.01	−281.85	12	+298.50	−200.73
28	−243.44	−281.61	13	+300.07	−200.50
29	−241.86	−281.38	14	+301.65	−200.26
30	−240.29	−281.14	15	+303.22	−200.03
31	−238.71	−280.91	16	+304.80	−199.79
32	−237.14	−280.67	17	+306.37	−199.56
33	−235.56	−280.44	18	+307.95	−199.32
34	−233.98	−280.20	19	+309.53	−199.09
35	−232.41	−279.97	20	+311.10	−198.85
36	−230.83	−279.73	21	+312.68	−198.62
37	−229.26	−279.50	22	+314.25	−198.38
38	−227.68	−279.26	23	+315.83	−198.15
39	−226.11	−279.03	24	+317.40	−197.91
40	−224.53	−278.79	25	+318.98	−197.67

For the provisional value of the solar parallax we take

$$\Pi = 8''.848 \div r' = 8''.715$$

The numerical expressions for p and q are, then, as follow:

1761. Ingress: $p = [9.940]\ \rho \sin \varphi' - [0.9381]\ \rho \cos \varphi' \cos (\tau + 13^\circ\ 22')$
1761. Egress: $p = [9.933]\ \rho \sin \varphi' - [0.9382]\ \rho \cos \varphi' \cos (\tau + 13^\circ\ 9')$
1769. Ingress: $p = [9.997]\ \rho \sin \varphi' - [0.9375]\ \rho \cos \varphi' \cos (\tau + 15^\circ\ 20')$
1769. Egress: $p = [9.991]\ \rho \sin \varphi' - [0.9375]\ \rho \cos \varphi' \cos (\tau + 15^\circ\ 7')$

1761. Ingress and Egress: $q = [0.9028]\ \rho \sin \varphi' - [0.5405]\ \rho \cos \varphi' \sin \tau$
1769. Ingress and Egress: $q = [0.9028]\ \rho \sin \varphi' - [0.5404]\ \rho \cos \varphi' \sin \tau$

4. The tabular times were obtained by successive approximations. The principal elements of the last approximation are given in the following exhibit.

The Greenwich mean time of computation is an approximate value, generally obtained from a previous computation. Sometimes, however, especially in the case of exterior contacts, this time is used only to the nearest minute or the nearest 10 seconds, being derived by applying the duration of ingress or egress to the computed times of internal contact.

For these times the value of p and q are computed by the formulæ and tables already given. They vary so slowly that their change during a small fraction of a minute is nearly insensible.

The values of $c \sin \omega$ and $c \cos \omega$ for the adopted Greenwich mean time are taken from the tables already given, and the value of c_1 is then computed. If the adopted time were perfectly correct we should have $c_1 = r_1$. The small difference, $c_1 - r_1$, is given in the next column. Dividing it by the variation of c_1 in one second, we have the corrections to the adopted mean time given in the last column:

Computation of tabular times of contacts.

1761, I; (INGRESS: EXTERIOR CONTACT).

Place.	Greenwich mean time of computation. h. m. s.	p ″	q ″	c_1 ″	$c_1 - r_1$ ″	δt s.
Madras	14 7 0	−7.74	+1.40	388.47	+0.58	+27
Petersburg	14 7 0	−3.03	+8.08	387.51	−0.38	−18
Cajaneborg	14 7 0	−2.41	+8.26	387.06	−0.83	−40
Stockholm	14 7 0	−2.54	+8.31	387.19	−0.70	−34
Upsala	14 7 0	−2.46	+8.33	387.13	−0.76	−36
Calmar	14 7 0	−2.72	+8.26	387.40	−0.49	−24

1761, II; (INGRESS: INTERIOR CONTACT).

Pekin	14 23 17	−2.74	+3.15	364.49	−0.06	− 3.0
Madras	14 25 42.5	−7.48	+1.13	364.54	−0.02	− 1.0
Calcutta	14 25 18	−6.41	+1.98	364.55	−0.01	− 0.5
Tobolsk	14 25 28	−3.74	+6.79	364.54	−0.03	− 1.5
Petersburg	14 25 35	−3.19	+7.98	364.56	−0.02	− 1.0
Cajaneborg	14 25 14.5	−2.56	+8.17	364.53	−0.05	− 2.5
Tornea	14 25 3.5	−2.24	+8.27	364.55	−0.03	− 1.5
Abo	14 25 25.5	−2.83	+8.17	364.55	−0.03	− 1.5
Stockholm	14 25 24	−2.77	+8.22	364.55	−0.03	− 1.5
Hernosand	14 25 11	−2.39	+8.32	364.54	−0.04	− 2.0
Upsala	14 25 22	−2.69	+8.25	364.55	−0.03	− 1.5
Calmar	14 25 31.5	−2.98	+8.16	364.56	−0.02	− 1.0

1761, III; (EGRESS: INTERIOR CONTACT).

Pekin	20 12 46	+6.18	+3.20	364.52	−0.04	+ 2
Selenginsk	20 12 51	+4.71	+4.40	364.48	−0.08	+ 4
Calcutta	20 15 37.5	+4.37	−0.02	364.43	−0.12	+ 6
Madras	20 17 8	+3.44	−1.54	364.58	+0.03	− 1.5
Tobolsk	20 14 15.5	+1.52	+4.94	364.47	−0.08	+ 4
Rodrigues	20 21 6.5	+0.68	−5.92	364.52	−0.04	+ 2
Petersburg	20 15 23.5	−1.29	+5.57	364.46	−0.09	+ 4.5
Cajaneborg	20 15 1	−1.15	+6.07	364.47	−0.09	+ 4.5
Tornea	20 14 56	−1.16	+6.30	364.52	−0.04	+ 2
Abo	20 15 32.5	−1.76	+5.80	364.50	−0.06	+ 3
Cape Town	20 24 23.5	−4.85	−6.27	364.56	−0.01	+ 0.5

Computation of tabular times of contact—Continued.

1761, III; (EGRESS: INTERNAL CONTACT)—Continued.

Place.	Greenwich mean time of computation.			p	q	c_1	$c_1 - r_1$	δt
	h.	m.	s.	"	"	"	"	s.
Stockholm	20	15	44	−2.11	+5.78	364.48	−0.08	+4
Hernosand	20	15	22	−1.82	+6.12	364.48	−0.08	+4
Upsala	20	15	42	−2.09	+5.84	364.50	−0.06	+3
Tyrnau	20	16	59.5	−3.08	+4.56	364.48	−0.07	+3.5
Calmar	20	16	6	−2.45	+5.54	364.53	−0.03	+1.5
Carlscrona	20	16	8	−2.55	+5.51	364.46	−0.10	+5
Wetzlas	20	17	1	−3.23	+4.66	364.48	−0.07	+3.5
Laibach	20	17	20	−3.50	+4.38	364.48	−0.07	+3.5
Lund	20	16	15	−2.74	+5.53	364.51	−0.05	+2.5
Copenhagen	20	16	13	−2.78	+5.55	364.50	−0.06	+3
Leipzig	20	16	45.5	−3.20	+5.08	364.49	−0.07	+3.5
Munich	20	17	8.5	−3.55	+4.74	364.48	−0.07	+3.5
Ingolstadt	20	17	4	−3.49	+4.81	364.48	−0.07	+3.5
Bologna	20	17	36	−3.87	+4.31	364.50	−0.05	+2.5
Florence	20	17	41	−3.94	+4.22	364.49	−0.06	+3
Dillingen	20	17	6.5	−3.57	+4.82	364.48	−0.07	+3.5
Drontheim	20	15	20.5	−2.11	+6.36	364.45	−0.11	+5.5
Göttingen	20	16	46	−3.34	+5.18	364.48	−0.07	+3.5
Würzburg	20	16	58.5	−3.50	+4.98	364.48	−0.07	+3.5
Schwetzingen	20	17	3	−3.63	+4.98	364.49	−0.06	+3
Lyons	20	17	32.5	−4.21	+4.71	364.50	−0.05	+2.5
Leiden	20	16	46.5	−3.60	+5.43	364.50	−0.05	+2.5
Montpellier	20	17	48.5	−4.47	+4.49	364.49	−0.06	+3
Beziers	20	17	50	−4.54	+4.49	364.49	−0.06	+3
Vincennes	20	17	10.5	−4.05	+5.15	364.50	−0.05	+2.5
Conflans-Sous-Carrière	20	17	10.5	−4.06	+5.14	364.49	−0.06	+3
Paris	20	17	10.5	−4.06	+5.15	364.48	−0.07	+3.5
St. Hubert	20	17	10.5	−4.08	+5.17	364.48	−0.07	+3.5
Rouen	20	17	10	−4.07	+5.26	364.55	−0.01	+0.5
Greenwich	20	16	53	−3.91	+5.91	364.50	−0.06	+3
London	20	16	53	−3.91	+5.52	364.51	−0.05	+2.5
Bayeux	20	17	9	−4.18	+5.31	364.49	−0.06	+3
Shirburn Castle	20	16	52	−3.94	+5.57	364.52	−0.04	+2
Madrid	20	18	15	−5.23	+4.43	364.48	−0.08	+4
Oporto	20	18	14	−5.41	+4.74	364.56	0.00	0
Lisbon	20	18	28	−5.68	+4.48	364.48	−0.08	+4
St. John's	20	16	25	−4.86	+7.17	364.55	−0.01	+0.5

***Computation of tabular times of contact*—Continued.**

1761, IV; (EGRESS: EXTERIOR CONTACT).

Place.	Greenwich mean time of computation.			p	q	c_1	$c_1 - r_1$	δt
	h.	m.	s.	"	"	"	"	s.
Pekin	20	32	30	+6.47	+3.37	389.72	+1.84	− 85
Selenginsk	20	32	30	+5.01	+4.51	389.38	+1.50	− 70
Calcutta	20	35	0	+4.94	+0.07	389.55	+1.67	− 78
Madras	20	36	0	+4.07	−1.48	389.15	+1.27	− 59
Tobolsk	20	34	0	+1.90	+4.93	389.57	+1.69	− 80
Petersburg	20	34	30	−0.96	+5.48	388.52	+0.64	− 30
Cajaneborg	20	34	30	−0.86	+5.99	388.95	+1.07	− 50
Tornea	20	34	0	−0.96	+6.21	388.41	+0.53	− 24
Abo	20	34	30	−1.47	+5.69	388.30	+0.42	− 19
Cape Town	20	42	30	−4.38	−6.44	388.07	+0.19	− 9
Stockholm	20	34	30	−1.82	+5.67	388.05	+0.17	− 8
Hernosand	20	34	30	−1.56	+6.01	388.53	+0.65	− 30
Upsala	20	34	0	−1.81	+5.74	387.46	−0.42	+ 19
Tyrnau	20	36	0	−2.70	+4.42	388.45	+0.57	− 26
Vienna	20	36	0	−2.81	+4.43	388.38	+0.50	− 23
Calmar	20	35	0	−2.15	+5.42	388.27	+0.39	− 19
Carlscrona	20	35	0	−2.24	+5.38	388.18	+0.30	− 14
Wetzlas	20	36	0	−2.86	+4.51	388.41	+0.53	− 24
Laibach	20	36	0	−3.12	+4.23	388.03	+0.15	− 7
Lund	20	35	0	−2.44	+5.40	388.05	+0.17	− 8
Rome	20	36	0	−3.62	+3.77	387.34	−0.54	+ 25
Landscrona	20	35	0	−2.36	+5.53	388.20	+0.32	− 15
Copenhagen	20	35	0	−2.48	+5.41	388.05	+0.17	− 8
Munich	20	36	0	−3.19	+4.57	388.20	+0.32	− 15
Ingolstadt	20	36	0	−3.15	+4.66	388.33	+0.45	− 21
Bologna	20	36	0	−3.51	+4.14	387.67	−0.21	+ 10
Florence	20	36	0	−3.58	+4.05	387.56	−0.32	+ 15
Dillingen	20	36	0	−3.24	+4.66	388.20	+0.32	− 15
Drontheim	20	36	0	−1.86	+6.24	390.34	+2.46	−115
Göttingen	20	35	0	−3.03	+5.03	387.38	−0.50	+ 23
Würzburg	20	36	0	−3.17	+4.82	388.40	+0.52	− 24
Schwetzingen	20	36	0	−3.31	+4.82	388.31	+0.43	− 20
Lyons	20	36	0	−3.90	+4.53	387.67	−0.21	+ 10
Leiden	20	36	0	−3.32	+5.26	388.61	+0.73	− 34
Montpellier	20	36	30	−4.15	+4.30	387.99	+0.11	− 5
Beziers	20	36	0	−4.23	+4.30	387.22	−0.66	+ 31
Vincennes	20	36	0	−3.77	+4.97	388.08	+0.20	− 9
Conflans-Sous-Carrière	20	36	0	−3.77	+4.97	388.08	+0.20	− 9
Paris	20	36	0	−3.78	+4.98	388.09	+0.21	− 10
St. Hubert	20	36	0	−3.80	+5.00	388.10	+0.22	− 10
Rouen	20	36	0	−3.79	+5.09	388.16	+0.28	− 13
Greenwich	20	35	30	−3.66	+5.35	387.81	−0.07	+ 3
London	20	35	30	−3.66	+5.36	387.82	−0.06	+ 3
Chelsea	20	35	30	−3.67	+5.36	387.81	−0.07	+ 3

***Computation of tabular times of contact*—Continued.**

1761, IV; (EGRESS: EXTERIOR CONTACT)—Continued.

Place.	Greenwich mean time of computation.			p	q	c_1	$c_1 - r_1$	δt
	h.	m.	s.	"	"	"	"	s.
Bayeux	20	36	0	−3.91	+5.17	388.13	+0.25	− 12
Shirburn Castle	20	35	30	−3.70	+5.40	387.82	−0.06	+ 3
Madrid	20	37	30	−4.94	+4.22	388.65	+0.77	− 36
Oporto	20	37	30	−5.17	+4.52	388.71	+0.83	− 39
Lisbon	20	37	30	−5.43	+4.26	388.34	+0.46	− 22
St. John's	20	34	30	−5.01	+7.01	386.77	−1.11	+ 52

1769, I; (INGRESS: EXTERIOR CONTACT).

Ponoi	7	7	30	+2.81	+8.22	389.71	+1.79	+ 84
Kola	7	7	30	+3.03	+8.10	389.64	+1.72	+ 81
Wardhus	7	7	30	+2.96	+8.10	389.68	+1.76	+ 83
Stockholm	7	7	0	+4.58	+7.37	389.71	+1.79	+ 84
Hernosand	7	6	30	+4.23	+7.52	390.48	+2.56	+121
Upsala	7	7	30	+4.54	+7.39	389.07	+1.15	+ 54
Lund	7	7	0	+5.14	+7.00	389.59	+1.67	+ 79
Greifswald	7	8	20	+5.28	+6.90	387.86	−0.06	− 3
Greenwich	7	6	30	+6.00	+6.19	390.20	+2.28	+108
London	7	6	30	+6.00	+6.19	390.20	+2.28	+108
Kew	7	6	30	+6.01	+6.18	390.19	+2.27	+108
Caen	7	6	0	+6.25	+5.97	390.82	+2.90	+138
Windsor	7	6	30	+6.02	+6.17	390.19	+2.27	+108
Shirburn Castle	7	6	30	+6.00	+6.17	390.21	+2.29	+109
Leicester	7	7	0	+5.91	+6.25	389.59	+1.67	+ 79
Oxford	7	6	30	+6.01	+6.17	390.20	+2.28	+108
Hawkhill	7	8	20	+5.57	+6.47	387.97	+0.05	+ 2
Gibraltar	7	8	0	+7.49	+4.40	388.58	+0.66	+ 31
Glasgow	7	8	20	+5.60	+6.43	387.98	+0.06	+ 3
Cavan	7	7	0	+5.77	+6.22	389.72	+1.80	+ 86
Newbury	7	11	10	+4.30	+3.00	387.82	−0.10	− 5
Cambridge	7	10	0	+4.27	+2.93	389.34	+1.42	+ 68
Providence	7	11	0	+4.29	+2.85	388.15	+0.23	+ 11
Cape Haïtien	7	12	50	+4.87	−0.41	387.85	−0.07	− 3
Baskenridge	7	9	30	+4.00	+2.64	390.38	+2.46	+118
Lewes	7	10	0	+4.02	+2.36	389.94	+2.02	+ 97
Philadelphia	7	10	0	+3.98	+2.53	389.85	+1.93	+ 93
Norristown	7	10	0	+3.95	+2.56	389.85	+1.93	+ 93
Hudson's Bay	7	11	20	+1.74	+5.01	387.91	−0.01	0
San José	7	15	10	−0.04	+0.08	387.98	+0.06	+ 3
Tahiti	7	21	0	−5.76	−4.20	387.73	−0.19	− 9

***Computation of tabular times of contact*—Continued.**

1769, II; (INGRESS: INTERIOR CONTACT).

Place.	Greenwich mean time of computation.	p	q	c_1	$c_1 - r_1$	δt
	h. m. s.	″	″	″	″	s.
Jakutsk	7 30 40	−2.66	+8.19	364.60	−0.01	−0.5
Ponoi	7 27 46.5	+2.56	+8.30	364.58	−0.03	−1.5
Kola	7 27 42.5	+2.82	+8.19	364.57	−0.04	−2
Wardhus	7 27 44.5	+2.77	+8.18	364.57	−0.04	−2
Cajaneborg	7 27 30.5	+3.44	+7.98	364.58	−0.03	−1.5
North Cape	7 27 40	+2.96	+8.10	364.60	−0.01	−0.5
Wanhallinna	7 27 21.5	+4.04	+7.69	364.57	−0.04	−2
Stockholm	7 27 17	+4.35	+7.52	364.58	−0.03	−1.5
Hernosand	7 27 22	+4.04	+7.68	364.56	−0.05	−2.5
Upsala	7 27 18	+4.32	+7.54	364.57	−0.04	−2
Lund	7 27 11.5	+4.93	+7.16	364.58	−0.03	−1.5
Greifswald	7 27 19	+5.86	+6.13	364.59	−0.02	−1
Saron	7 27 9	+6.01	+6.28	364.58	−0.03	−1.5
Paris	7 27 9.5	+6.04	+6.25	364.58	−0.03	−1.5
St. Hubert	7 27 9	+5.98	+6.31	364.58	−0.03	−1.5
Toulouse	7 27 13.5	+6.65	+5.61	364.59	−0.02	−1
Rouen	7 27 9	+6.03	+6.26	364.58	−0.03	−1.5
Havre	7 27 9.5	+6.06	+6.21	364.59	−0.02	−1
Greenwich	7 27 11	+5.86	+6.39	364.56	−0.05	−2.5
London	7 27 11	+5.87	+6.37	364.57	−0.04	−2
Caen	7 27 9.5	+6.10	+6.18	364.59	−0.02	−1
Bordeaux	7 27 12.5	+6.53	+5.75	364.58	−0.03	−1.5
Windsor	7 27 11	+5.88	+6.33	364.60	−0.01	−0.5
Leicester	7 27 11	+5.78	+6.44	364.58	−0.03	−1.5
Oxford	7 27 11	+5.87	+6.36	364.58	−0.03	−1.5
Colombe	7 27 9	+6.03	+6.26	364.58	−0.03	−1.5
Hawkhill	7 27 13.5	+5.48	+6.60	364.58	−0.03	−1.5
Glasgow	7 27 13.5	+5.52	+6.59	364.57	−0.04	−2
Brest	7 27 12.5	+6.30	+5.94	364.59	−0.02	−1
Gibraltar	7 27 26	+7.36	+4.64	364.58	−0.03	−1.5
Cadiz	7 27 26.5	+7.36	+4.64	364.57	−0.04	−2
Cavan	7 27 16	+5.69	+6.40	364.58	−0.03	−1.5
Agromonte	7 27 21.5	+7.06	+5.04	364.59	−0.02	−1
Isle Coudre	7 29 36.5	+4.49	+3.72	364.56	−0.03	−1.5
Newbury	7 29 53.5	+4.72	+3.08	364.56	−0.02	−1
Cambridge	7 29 55.5	+4.72	+3.01	364.58	−0.00	0
Providence	7 29 59	+4.72	+2.93	364.57	−0.01	−0.5
Cape Haïtien	7 31 40.5	+5.41	−0.31	364.59	+0.01	+0.5
Baskenridge	7 30 15	+4.49	+2.72	364.56	−0.02	−1
Lewes	7 30 25	+4.53	+2.43	364.55	−0.03	−1.5
Philadelphia	7 30 20	+4.47	+2.60	364.56	−0.02	−1
Norristown	7 30 20	+4.44	+2.63	364.55	−0.03	−1.5
Wilmington	7 30 20.5	+4.44	+2.56	364.59	+0.01	+0.5
Hudson's Bay	7 30 7	+2.10	+5.00	364.54	−0.04	−2
San José	7 34 3	+0.62	+0.01	364.58	0.00	0
Tahiti	7 40 2.5	−5.22	−4.43	364.62	+0.03	+1.5

***Computation of tabular times of contact*—Continued.**

1769, III; (EGRESS: INTERIOR CONTACT).

Place.	Greenwich mean time of computation.	p	q	c_1	$c_1 - r_1$	δt
	h. m. s.	"	"	"	"	s.
Jakutsk	13 19 40	−1.32	+5.93	364.53	−0.06	+3
Manilla	13 19 54	−5.28	+0.14	364.58	+0.02	−1
Pekin	13 20 43	−4.12	+3.79	364.58	+0.01	−0.5
Batavia	13 20 15	−7.18	−2.01	364.59	+0.02	−1
Orsk	13 22 31.5	−4.48	+7.31	364.60	+0.01	+0.5
Orenburg	13 22 29	−4.39	+7.37	364.59	0.00	0
Gurief	13 22 42.5	−4.85	+7.17	364.58	−0.01	−0.5
Kola	13 20 55	−1.52	+8.44	364.58	−0.01	−0.5
Wardhus	13 20 46	−1.29	+8.47	364.57	−0.02	+1
Petersburg	13 21 30	−2.42	+8.34	364.57	−0.02	+1
Hudson's Bay	13 15 29	+5.23	+6.73	364.60	+0.01	−0.5
San José	13 11 14.5	+8.33	+2.06	364.61	+0.03	−1.5
Tahiti	13 9 45	+5.63	−5.23	364.67	+0.09	−4.5

1769, IV; (EGRESS: EXTERIOR CONTACT).

Jakutsk	13 40 0	−1.01	+5.83	390.06	+2.16	−102
Manilla	13 38 0	−4.76	−0.08	387.22	−0.68	+ 34
Pekin	13 39 0	−3.74	+3.61	387.51	−0.39	+ 19
Batavia	13 39 0	−6.75	−2.27	387.84	−0.06	+ 3
Orsk	13 40 0	−4.62	+7.07	386.31	−1.59	+ 75
Orenburg	13 40 30	−4.49	+7.23	386.98	−0.92	+ 43
Gurief	13 41 0	−4.99	+7.02	387.28	−0.62	+ 29
Wardhus	13 39 30	−1.44	+8.41	387.81	−0.09	+ 4
Petersburg	13 40 30	−2.64	+8.26	388.10	+0.20	− 9
Cajaneborg	13 40 0	−2.22	+8.33	387.80	−0.10	+ 5
Hudson's Bay	13 34 0	+5.13	+6.87	387.33	−0.57	+ 26
San José	13 30 0	+8.35	+2.31	387.39	−0.51	+ 24
Tahiti	13 28 30	+6.08	−5.09	387.37	−0.53	+ 26

CHAPTER VI.

TABULAR SUMMARY OF THE OBSERVATIONS AND THEIR COMPARISONS WITH THE TABULAR TIMES.

In the following exhibit I present the particulars of each observation in a condensed form. The first five columns do not appear to need any explanation, being derived, without change, from material given in the four preceding chapters.

In the columns "phase" and "class" I have endeavored to indicate in the briefest manner the kind of observation made. The classification of the phase observed has been founded upon the long recognized distinction between the formation or rupture of the thread of light and the time when the outlines of Venus and the Sun appeared to be tangent to each other. But, as pointed out in the introductory chapter, the latter phase is liable to much indefiniteness, owing to the varying brightness of the sky around the Sun, the atmospheric conditions, and the quality of the telescope.

In the column "phase" the symbol L signifies that the observer expressly or impliedly states that he observed the formation of the thread of light at ingress or its rupture at egress, or that historic statements exist indicating that he made such an observation. When the description seems to imply that the phase caught at ingress was that of the earliest formation of the thread of light, or, as we might say, the moment when the thread of light was being formed, the phase is called L_1. When the description seems to imply that the phase was appreciably later than the actual formation of the thread it is called L_2.

G indicates that the observer noted some phase earlier than L at ingress or later than L at egress. The terms in which this phase is described are so varied that it is not easy to classify them, and no classification would have the slightest value.

N signifies that the observer gives no description from which the phase could be inferred.

In the column "class" the quality of the observations, so far as it can be inferred from all the circumstances, is indicated.

In Class I are placed those observations of which the description is so precise and so accordant with the known character of the phenomena as to inspire confidence in the absolute correctness of the observation, and where nothing is indicated to impair that confidence.

In Class II are placed those observations which are defective in some of the requisites which would entitle them to be placed in Class I, but connected with which

there are positive circumstances which inspire confidence. Such circumstances are the known reputation of the observer, the good quality of the air or instrument, the minuteness of the description, and favorable conditions generally.

In Class III are placed observations which there are no data for judging. Commonly, in observations of this class, the observer made no statement or remarks from which the quality of his observation could be inferred.

In Class IV are placed those observations connected with which are positive circumstances calculated to inspire distrust. In some cases, however, this distrust may permit of being removed by comparison and discussion.

A characteristic of this classification is that it is made without any reference to the magnitude of the residual correction, and, in fact, so far as practicable, without any knowledge of this correction.

In the next column I have sought to give a brief indication of the quality of the instrument with which the observation was made:

A signifies an achromatic refracting telescope.

N signifies a non-achromatic refracting telescope.

R signifies a reflecting telescope.

The numbers following the above letters indicate the length of the telescope in feet. I may add that this column was taken entirely from ENCKE's discussion.

I have omitted the magnifying power, conceiving it to be entirely without influence on the observations, except in a few cases, where the smallest kind of instrument was used.

The last column contains the computed altitude of the Sun at the time of observation, generally given only to the nearest degree and not corrected for refraction.

A few observations are included in the table for which I did not find data until after the completion of the second and third chapters.

Tabular summary of observations.

1761, I; (INGRESS: EXTERIOR CONTACT).

Station.	Observer.	Local mean time, June 5, 1761.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
		h. m. s.	h. m. s.	h. m. s.	s.				°
Madras	HIRST	19 29 16	14 8 7	14 7 27	+ 40	. . .	III	R, 2	28
Petersburg	BRAUN	16 7 26	6 12	6 42	— 30	. . .	II	N, 8	7
Do	KRASILNICOW	16 8 7	6 53	. . . . .	+ 11	. . .	II	N, 6	. . .
Do	KURGANOFF	16 7 48	6 34	. . . . .	— 8	. . .	II	R, 2½	. . .
Cajaneborg	PLANMANN	15 58 2	7 6	6 20	+ 46	. . .	I	N, 21	8
Stockholm	WARGENTIN	15 19 43	7 27	6 27	+ 60	. . .	I	N, 19	1.4
Upsala	MALLET	15 18 51	8 19	6 24	+115	. . .	I	R, 1½	1.8
Calmar	WICKSTRÖM	15 17 22	11 57	6 37	+320	. . .	II	N, 21	— 0.4

***Tabular summary of observations*—Continued.**

1761, II; (INGRESS: INTERIOR CONTACT).

Station.	Observer.	Local mean time, June 5, 1761.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
		h. m. s.	h. m. s.	h. m. s.	s.				°
Pekin	DOLLIER	22 8 33	14 23 0	14 23 14	− 14	N	III	(?)	61
Calcutta	MAGEE	20 18 54	25 33	25 18	+ 15	N	IV	(?)	39
Madras	HIRST	19 46 2	24 53	25 42	− 49	N	III	R, 2	29
Tobolsk	CHAPPE	18 58 37	25 31	25 27	+ 4	L_1	I	N, 19	27
Petersburg	BRAUN	16 24 26	23 12	25 34	−142	N	II	N, 8	9
Do	KRASILNIKOW	16 24 46	23 32	. . . . .	−122	N	II	N, 6	. . .
Do	KURGANOFF	16 24 47	23 33	. . . . .	−121	N	II	R, 2½	. . .
Cajaneborg	PLANMANN	16 16 11	25 15	25 12	+ 3	L_1	I	N, 21	10
Tornea*	HELLANT	16 2 0	25 25	25 2	+ 23	L	III	N, 20	10
Do	LAGERBOHM	16 2 8	25 33	. . . . .	+ 31	L	III	N, 32	. . .
Abo	JUSTANDER	15 53 56	24 50	25 24	− 34	N	II	N, 20	6
Do	WALLERIUS	15 54 2	24 56	. . . . .	− 28	N	II	(?)	. . .
Stockholm	WARGENTIN	15 37 30	25 14	25 23	− 9	L	I	N, 19	3
Do	KLINGENSTIERNA	15 37 35	25 19	. . . . .	− 4	L	I	A, 10	. . .
Hernosand	GISSLER	15 36 32	24 46	25 9	− 23	G	. . .	N, 21	5
Do	. . do	15 37 29	25 43	. . . . .	+ 34	L_2	III	. . . .	. . .
Do	STRÖM	15 36 42	24 56	. . . . .	− 13	G	III	N, 20	. . .
Upsala	MALLET	15 36 2	25 30	25 21	+ 9	L_1	I	R, 1½	4
Do	STRÖMER	15 36 12	25 40	. . . . .	+ 19	L_1	II	N, 20	. . .
Do	MELANDER	15 35 15	24 43	. . . . .	− 38	G	II	N, 16	. . .
Do	. . do	15 36 8	25 36	. . . . .	+ 15	L_2	II	. . . .	. . .
Do	BERGMANN	15 35 50	25 18	. . . . .	− 3	L_1	I	N, 21	. . .
Calmar	WICKSTRÖM	15 30 52	25 27	25 31	− 4	G	. . .	N, 21	1.0
Do	. . do	15 31 7	25 42	. . . . .	+ 11	L_1	II	. . . .	. . .

1761, III; (EGRESS: INTERIOR CONTACT).

Station.	Observer.	Local mean time.	Greenwich observed.	Greenwich tabular.	Residual correction.	Phase.	Class.	Telescope.	☉'s altitude.
Pekin	DOLLIER	3 58 8	20 12 35	20 12 48	− 13	N	III	(?)	37
Selenginsk	RUMOVSKY	3 19 44	13 19	12 55	+ 24	L	II	N, 15	42
Calcutta	MAGEE	2 9 42	16 21	15 43	+ 38	N	IV	(?)	59
Madras	The JESUITS	1 35 10	14 1	17 7	−186	N	III	(?)	65
Do	HIRST	1 37 47	16 38	. . . . .	− 29	N	III	R, 2	. . .
Tobolsk	CHAPPE	0 47 32	14 26	14 20	+ 6	L	I	N, 19	53
Rodrigues	PINGRÉ	0 34 58	21 16	21 8	+ 8	N	II	N, 18	47
Petersburg	BRAUN	22 17 9	15 55	15 28	+ 27	. . .	. . .	N, 8	49
Do	KRASILNIKOW	22 17 13	15 59	. . . . .	+ 31	. . .	. . .	N, 6	. . .
Do	KURGANOFF	22 17 10	15 56	. . . . .	+ 28	. . .	. . .	R, 2½	. . .
Cajaneborg	PLANMANN	22 7 8	16 12	15 6	+ 66	L	II	N, 21	44
Do	FROSTERUS	22 6 28	15 32	. . . . .	+ 26	L	IV	(?)	. . .
Tornea*	HELLANT	21 52 16	15 41	14 58	+ 43	L	III	N, 20	42
Do	HÄGGMANN	21 52 27	15 52	. . . . .	+ 54	N	III	. . . .	. . .
Do	LAGERBOHM	21 52 29	15 54	. . . . .	+ 56	N	III	N, 32	. . .

*Time suspected to be erroneous.

Tabular summary of observations—Continued.

1761, III; (EGRESS: INTERIOR CONTACT)—Continued.

Station.	Observer.	Local mean time, June 5, 1761.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	⊙'s altitude.
		h. m. s.	h. m. s.	h. m. s.	s.				°
Abo	Justander . . .	21 45 8	20 16 2	20 15 35	+ 27	N	III	N, 20	46
Cape Town	Mason	21 38 4	24 22	24 24	— 2	N	II	R, 2	24
Do	Dixon	21 37 59	24 17	. . . . .	— 7	N	II	. . . .	. . .
Stockholm	Wargentin . . .	21 28 17	16 1	15 48	+ 13	L	I	N, 19	45
Do	Wilken	21 27 49	15 33	. . . . .	— 15	L	IV	. . . .	. . .
Do	Klingenstierna .	21 28 20	16 4	. . . . .	+ 16	L	I	A, 10	. . .
Hernosand	Gissler	21 27 1	15 15	15 26	— 11	(?)	. . .	N, 21	43
Do	. . do	21 27 30	15 44	. . . . .	+ 18	L	III	. . . .	. . .
Upsala	Mallet	21 26 9	15 37	15 45	— 8	L	I	R, 1½	44
Do	Strömer	21 26 9	15 37	. . . . .	— 8	L	II	N, 20	44
Do	. . do	21 26 16	15 44	. . . . .	— 1	G (?)	II	. . . .	. . .
Do	Bergmann . . .	21 26 18	15 46	. . . . .	+ 1	L	II	N, 21	. . .
Tyrnau	Weiss	21 27 18	16 56	17 3	— 7	N	III	R, 4	50
Calmar	Wickström . . .	21 21 49	16 24	16 8	+ 16	L	II	N, 21	46
Do	. . do	21 22 42	17 17	. . . . .	+ 69	G	I	. . . .	. . .
Carlscrona	Bergström . . .	21 18 9	15 48	16 13	— 25	L	II	R, 3	46
Do	Zegollström . .	21 18 15	15 54	. . . . .	— 19	L	II	N, 21	. . .
Wetzlas	Von Schlug . . .	21 18 57	17 19	17 4	+ 15	N	II	R, 4	49
Laibach*	Schöttl	21 16 24	18 21	17 24	+ 57	N	III	N, 16	50
Lund	Schenmark . . .	21 8 53	16 8	16 18	— 10	N	IV	N, 21	45
Landscrona	Bremer	21 7 30	16 10	16 13	— 3	L	IV	N, 10	44
Do	Landberg	21 7 57	16 37	. . . . .	+ 24	L	IV	N, 21	. . .
Copenhagen*	Horrebow . . .	21 3 45	13 26	16 16	—170	N	II	N, 22	44
Rome	Audifredi . . .	21 7 45	17 50	17 56	— 6	N	III	. . . .	50
Leipzig	Heinsius	21 6 11	16 43	16 49	— 6	N	II	. . . .	46
Munich	Anon	21 3 55	17 38	17 12	+ 26	N	II	N, 3½	47
Ingolstadt	Kratz	21 3 8	17 27	17 7	+ 20	N	II	R, 7	47
Do	Anon	21 3 5	17 24	. . . . .	+ 17	N	III	N, 13	. . .
Do	. . do	21 3 6	17 25	. . . . .	+ 18	N	III	N, 11	. . .
Bologna	Zanotti	21 2 43	17 20	17 38	— 18	N	IV	N, 2½	48
Do	Marinus	21 3 7	17 44	. . . . .	+ 6	N	III	N, 10	. . .
Do	Matheucius . . .	21 3 7	17 44	. . . . .	+ 6	N	III	N, 22	. . .
Do	Frisi	21 3 3	17 40	. . . . .	+ 2	N	III	N, 6	. . .
Do	Cassali	21 3 9	17 46	. . . . .	+ 8	N	III	N, 8	48
Do	Canterzani . . .	21 3 5	17 42	. . . . .	+ 4	N	III	N, 11	. . .
Florence	Ximenes	21 2 37	17 35	17 44	+ 9	N	III	R, 4½	48
Dillingen*	Hauser	20 58 29	16 30	17 10	— 40	N	III	N, 18	46
Drontheim*	Bugge	21 1 36	19 59	15 26	+273	N	II	. . . .	40
Göttingen	T. Mayer	20 56 35	16 49	16 49	0	N	I	(?)	45
Würzburg*	Hubert	20 59 21	19 37	17 2	+155	N	III	. . . .	46
Do	. . do	20 59 52	20 8	. . . . .	+186	N	IV	. . . .	. . .
Schwetzingen	Ch. Mayer . . .	20 51 29	17 11	17 6	+ 5	L	II	A, 10	45

*Time suspected to be erroneous.

***Tabular summary of observations*—Continued.**

1761, III; (EGRESS: INTERIOR CONTACT)—Concluded.

Station.	Observer.	Local mean time, June 5, 1761.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
		h. m. s.	h. m. s.	h. m. s.	s.				°
Lyons	BERAUD	20 36 53	17 36	17 35	+ 1	N	III	N, 19	43
Leiden	LULOSS	20 34 59	17 3	16 49	+ 14	N	II	R, 7	42
Montpellier	TANDON	20 33 3	17 32	17 51	− 19	N	III	N, 18	43
Do	ROMIEU	20 33 9	17 38	.....	− 13	N	III	N, 10	...
Do	DE RAITE	20 33 23	17 52	.....	+ 1	N	III	N, 14	...
Beziers	DE MANSE	20 30 54	18 3	17 53	+ 10	N	III	N, 3½	43
Vincennes	PROLANGE	20 26 58	17 13	17 13	0	N	III	....	41
Conflans-Sous-Carrière	LA CAILLE	20 27 3	17 25	17 14	+ 11	N	II	A, 4½	41
Do	TURGOT	20 27 10	17 32	.....	+ 18	N	III	N, 12	...
Paris (Hôtel de Clugny)	MESSIER	20 26 39	17 16	17 14	+ 2	N	II	R, —	41
Do	BANDOUIN	20 26 36	17 13	.....	− 1	N	II	N, 25	...
Do	LIBOUR	20 26 40	17 17	.....	+ 3	N	III	R, 4½	...
Paris (Collège)	MERVILLE	20 26 49	17 26	.....	+ 12	N	III	R, 6	...
Do	CLOUET	20 26 35	17 12	.....	− 2	N	III	R, 2⅓	...
St. Généviève	DE BARROS	20 26 54	17 30	.....	+ 16	N	III	(?)	...
Luxemburg	LALANDE	20 26 35	17 14	17 14	0	L_1	I	N, 18	41
Do	. . do	20 26 39	17 18	.....	+ 4	L_2	I	....	...
Royal Observatory	MARALDI	20 26 51	17 30	17 14	+ 16	N	I	N, 15	41
Do	BELLERI	20 26 23	17 2	.....	− 12	N	III	N, 6	...
Do	ZANONI	20 26 45	17 24	.....	+ 10	N	III	N, 3½	...
Passy	FERNER and NOËL	20 26 24	17 17	17 14	+ 3	N	III	R, 2⅓	41
St. Hubert	LE MONNIER	20 24 32	17 7	17 14	− 7	N	I	N, 18	41
Do	LA CONDAMINE	20 24 58	17 33	.....	+ 19	N	IV	R, 1½	...
Greenwich	BLISS	20 17 9	17 9	16 56	+ 13	N	II	N, 15	39
Do	GREEN	20 17 9	17 9	.....	+ 13	N	II	R, 2	...
Do	BIRD	20 17 9	17 9	.....	+ 13	N	II	R, 1½	...
Hackney	ELLICOT	20 16 54	17 5	16 55	+ 10	N	III	R, 2	39
Spital Square	CANTON	20 16 50	17 7	16 56	+ 11	N	II	R, 1½	39
Clerkenwell	HEBERDEN	20 16 38	17 5	16 56	+ 9	N	III	R, 2	39
Savile House	SHORT	20 16 31	17 1	16 55	+ 6	N	II	R, 2	39
Chelsea	DUNN	20 16 12	16 53	16 55	− 2	L	I	R, 6	39
Bayeux*	OUTHIER	20 15 14	18 3	17 12	+ 51	N	III	N, 6	39
Shirburn Castle*	HORNSBY	20 13 19	17 15	16 54	+ 21	N	II	N, 12	38
Do	PHELPS	20 13 23	17 19	.....	+ 25	N	III	N, 14	...
Madrid*	BENEVENT	20 5 2	19 52	18 19	+ 93	N	III	R, 2½	38
Do	XIMENES	20 5 5	19 55	.....	+ 96	N	III	(?)	...
Do	RIEGER	20 5 5	19 55	.....	+ 96	N	III	N, 8	...
Oporto*	ALMEIDA	19 42 14	16 40	18 14	− 94	N	III	R, 2	34
Lisbon*	CIERA	19 42 35	19 9	18 32	+ 37	...	...	(?)	34
St. John's	WINTHROP	16 45 29	16 19	16 25	− 6	− N	II	(?)	5

*Time suspected to be erroneous.

Tabular summary of observations—Continued.

1761, IV; (EGRESS: EXTERIOR CONTACT).

Station.	Observer.	Local mean time, June 5, 1761.	Greenwich mean time.		Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
			Observed.	Tabular.					
		h. m. s.	h. m. s.	h. m. s.	s.				°
Pekin	DOLLIER	4 16 6	20 30 34	20 31 5	— 31	...	...	....	33
Selenginsk	RUMOVSKY	3 37 50	31 25	31 20	+ 5	...	...	N, 15	39
Calcutta	MAGEE	2 25 47	32 26	33 42	— 76	...	...	....	57
Madras	The JESUITS	1 51 16	30 7	35 1	—294	...	...	....	60
Do	HIRST	1 53 53	32 44	.....	—157	...	...	R, 2	...
Tobolsk	CHAPPE	1 5 51	32 45	32 40	+ 5	...	...	N, 19	52
Petersburg	BRAUN	22 35 13	33 59	34 0	— 1	...	...	N, 8	50
Do	KRASILNIKOW	22 35 9	33 55	.....	— 5	...	...	N, 6	...
Do	KURGANOFF	22 35 11	33 57	.....	— 3	...	...	R, 2½	...
Cajaneborg	PLANMANN	22 24 31	33 35	33 39	— 4	...	...	N, 21	46
Tornea*	HELLANT	22 10 31	33 56	33 36	+ 40	...	...	N, 20	43
Do	LAGERBOHM	22 10 23	33 48	.....	+ 22	...	...	N, 32	...
Do	HÄGGMANN	22 10 7	33 32	.....	+ 6	...	...	....	...
Abo	JUSTANDER	22 2 51	33 45	34 10	— 25	...	...	N, 20	47
Cape Town	MASON	21 55 34	41 52	42 21	— 29	...	...	R, 2	26
Do	DIXON	21 55 32	41 50	.....	— 31	...	...	....	...
Stockholm	WARGENTIN	21 46 18	34 2	34 20	— 20	...	...	N, 19	46
Do	WILKEN	21 46 8	33 52	.....	— 30	...	...	....	...
Do	KLINGENSTIERNA	21 46 17	34 1	.....	— 21	...	...	A, 10	...
Hernosand	GISSLER	21 44 44	32 58	33 59	— 61	...	...	N, 21	44
Do	STRÖM	21 44 56	33 10	.....	— 49	...	...	N, 20	...
Upsala	MALLET	21 44 38	34 6	34 20	— 14	...	...	R, 1½	46
Do	STRÖMER	21 44 22	33 50	.....	— 30	...	...	N, 20	...
Do	MELANDER	21 44 39	34 7	.....	— 13	...	...	N, 16	...
Do	BERGMANN	21 44 39	34 7	.....	— 13	...	...	N, 21	46
Tyrnau	WEISS	21 45 45	35 23	35 33	— 10	...	...	R, 4	53
Vienna	HELL	21 41 19	35 47	35 36	+ 11	...	...	R, 4½	53
Do	HERBERTH	21 40 53	35 21	.....	— 15	...	...	N, 12	...
Do	RAIN	21 40 58	35 26	.....	— 10	...	...	N, 9	...
Do	LYFOGORSKY	21 41 8	35 36	.....	0	...	...	R, 3	...
Do	SCHERFER	21 40 44	35 12	.....	— 24	...	...	R, 4	...
Do	CASSINI	21 40 58	35 26	.....	— 10	...	...	N, 9	...
Do	STEINKELLNER	21 40 23	34 51	.....	— 45	...	...	N, 16	...
Do	LIESGANIGG	21 41 0	35 28	.....	— 8	...	...	N, 11	...
Do	MASTALIER	21 40 22	34 50	.....	— 46	...	...	N, 5½	...
Calmar	WICKSTRÖM	21 39 24	33 59	34 41	— 42	...	...	N, 21	48
Carlscrona	BERGSTRÖM	21 37 25	35 4	34 45	+ 19	...	...	R, 3	48
Do	ZEGOLLSTRÖM	21 37 30	35 9	.....	+ 24	...	...	N, 21	...
Wetzlas	VON SCHLUG	21 36 59	35 21	35 35	— 14	...	...	R, 4	52
Do	VON SCHLUG, jr	21 36 38	35 0	.....	— 35	...	...	....	...
Laibach*	SCHÖTTL	21 34 29	36 26	35 53	+ 33	...	...	N, 16	52
Lund	BURMESTER	21 27 25	34 40	34 52	— 12	...	...	....	46

*Time determination suspected to be erroneous.

***Tabular summary of observations*—Continued.**

1761, IV; (EGRESS: EXTERIOR CONTACT)—Continued.

Station.	Observer.	Local mean time, June 5, 1761.	Greenwich mean time.		Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
			Observed.	Tabular.					
		h. m. s.	h. m. s.	h. m. s.	s.				°
Lund	SCHENMARK . . .	21 27 21	20 34 36	20 34 52	— 16	. . .	. . .	N, 21	. . .
Rome	AUDIFREDI . . .	21 26 16	36 21	36 25	— 4	. . .	. . .	. . . .	51
Landscrona	LANDBERG	21 25 32	34 12	34 45	— 33	. . .	IV	N, 21	46
Copenhagen*	HORREBOW . . .	21 21 12	30 53	34 52	—239	. . .	. . .	N, 22	47
Munich	. . . .	21 21 57	35 40	35 45	— 5	. . .	. . .	N, 3½	50
Ingolstadt	KRATZ	21 21 13	35 32	35 39	— 7	. . .	. . .	R, 7	49
Do	ANON	21 20 31	34 50	. . . . .	— 49	. . .	. . .	N, 13	49
Do	. . do	21 20 38	34 57	. . . . .	— 42	. . .	. . .	N, 11	. . .
Bologna	ZANOTTI	21 20 39	35 16	36 10	— 54	. . .	. . .	N, 2½	51
Do	MARINUS	21 21 9	35 46	. . . . .	— 24	. . .	. . .	N, 10	. . .
Do	MATHEUCIUS . . .	21 21 16	35 53	. . . . .	— 17	. . .	. . .	N, 22	. . .
Do	FRISI	21 21 3	35 40	. . . . .	— 30	. . .	. . .	N, 6	. . .
Do	CASSALI	21 20 59	35 36	. . . . .	— 34	. . .	. . .	N, 8	. . .
Do	CANTERZANI . . .	21 21 8	35 45	. . . . .	— 25	. . .	. . .	N, 11	. . .
Florence	XIMENES	21 21 5	36 3	36 16	— 13	. . .	. . .	R, 4½	50
Dillingen*	HAUSER	21 16 29	34 30	35 45	— 75	. . .	. . .	N, 18	49
Drontheim*	BUGGE	21 18 24	36 47	34 5	+162	. . .	. . .	N, 8	41
Göttingen	T. MAYER	21 15 3	35 17	35 24	— 7	. . .	. . .	N, 6	47
Würzburg*	HUBERT	21 16 58	37 14	35 36	+ 98	. . .	. . .	R, 1½	48
Do	ANON	21 16 9	36 25	. . . . .	+ 49	. . .	. . .	. . . .	. . .
Do	. . do	21 16 44	37 0	. . . . .	+ 84	. . .	. . .	. . . .	. . .
Lyons	BÉRAUD	20 55 5	35 48	36 10	— 22	. . .	. . .	N, 19	45
Montpellier	TANDON	20 50 17	34 46	36 25	— 99	. . .	. . .	N, 18	46
Do	Three observers . .	20 51 30	35 59	. . . . .	— 26	. . .	. . .	. . . .	. . .
Beziers†	CLAUZADE	20 49 14	36 23	36 32	— 9	. . .	. . .	N, 7	46
Vincennes†	PROLANGE	20 45 21	35 36	35 50	— 14	. . .	. . .	N, 19	43
Conflans-Sous-Carrière	LA CAILLE	20 45 15	35 37	35 50	— 13	. . .	. . .	A, 4½	44
Do	TURGOT	20 44 55	35 17	. . . . .	— 33	. . .	. . .	N, 12	. . .
Do	BAILLY	20 45 20	35 42	. . . . .	— 8	. . .	. . .	N, 6	44
Paris (Hôtel de Clugny)	BANDOUIN	20 44 52	35 29	35 50	— 21	. . .	. . .	N, 25	44
Do	MESSIER	20 44 46	35 23	. . . . .	— 27	. . .	. . .	(?)	. . .
Paris (Coll. L. le Grand)	MERVILLE	20 45 13	35 50	. . . . .	0	. . .	. . .	R, 6	. . .
Do	CLOUET	20 45 4	35 41	. . . . .	— 9	. . .	. . .	R, 2½	. . .
Paris (Luxemburg) . .	LALANDE	20 44 59	35 38	. . . . .	— 12	. . .	. . .	N, 18	. . .
Paris (Royal Obs.) . .	MARALDI	20 45 3	35 42	. . . .	— 8	. . .	. . .	N, 15	. . .
Do	BELLÉRI	20 44 49	35 28	. . . . .	— 22	. . .	. . .	N, 6	. . .
Do	ZANNONI	20 45 5	35 44	. . . . .	— 6	. . .	. . .	N, 3½	. . .
Passy	FOUCHY	20 44 35	35 28	. . . . .	— 22	. . .	. . .	R, 4	. . .
Do	FERNER	20 44 36	35 29	. . . . .	— 21	. . .	. . .	R, 2½	. . .
Paris (Ecole Militaire)	JEAURAT	20 44 55	35 42	. . . . .	— 8	. . .	. . .	N, 18	. . .
St. Hubert	LE MONNIER . . .	20 43 1	35 36	35 49	— 13	. . .	. . .	N, 18	44
Do	LA CONDAMINE . .	20 43 2	35 37	. . . . .	— 12	. . .	. . .	R, 1½	. . .

*Time determination suspected to be erroneous.

†For observations see page 314.

***Tabular summary of observations*—Continued.**

1761, IV; (EGRESS: EXTERIOR CONTACT)—Concluded.

Station.	Observer.	Local mean time, June 5, 1761.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
		h. m. s.	h. m. s.	h. m. s.	s.				°
Rouen	Bouin and Dulague	20 39 44	20 35 21	20 35 46	− 25	. . .	. . .	N, 16	43
Greenwich	Bliss	20 35 18	35 18	35 33	− 15	. . .	. . .	N, 15	42
Hackney	Ellicot	20 35 12	35 23	35 33	− 10	. . .	. . .	R, 2	. . .
London (Spital Square)	Canton	20 35 13	35 30	35 33	− 3	. . .	. . .	R, 1½	. . .
London (Savil House)	Short	20 35 14	35 44	35 33	+ 11	. . .	. . .	R, 2	. . .
Do	Blair	20 34 22	34 52	. . . . .	− 41	. . .	. . .	R, 1½	. . .
Chelsea	Dunn	20 34 30	35 11	35 33	− 22	. . .	. . .	R, 6	. . .
Bayeux*	Outhier	20 33 24	36 13	35 48	+ 25	. . .	. . .	N, 6	42
Shirburn Castle* . . .	Hornsby	20 31 34	35 30	35 32	− 2	. . .	. . .	N, 12	. . .
Madrid*	Anon.	20 22 44	37 34	36 54	+ 40	. . .	. . .	. . . .	41
Do	Anon.	20 22 41	37 31	. . . . .	+ 37	. . .	. . .	. . . .	. . .
Do	Anon.	20 22 42	37 32	. . . . .	+ 38	. . .	. . .	. . . .	. . .
Do	Rieger	20 23 2	37 52	. . . . .	+ 58	. . .	. . .	. . . .	. . .
Oporto*	Almeida	20 0 48	35 14	36 50	− 96	. . .	. . .	R, 2	37
Lisbon*	Ciera	20 0 42	37 16	37 8	+ 8	. . .	. . .	. . . .	36
St. John's	Winthrop . . .	17 3 58	34 48	35 23	− 35	. . .	. . .	. . . .	8

1769, I; (INGRESS: EXTERIOR CONTACT).

Station.	Observer.	Local mean time.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
Ponoi	Mallet	9 54 18	7 9 48	7 8 55	+ 53	. . .	I	A, 12	3
Kola	Rumovsky . . .	9 21 59	9 46	8 51	+ 55	. . .	II	A, 12	5
Do	Ochtenski . . .	9 22 5	9 52	. . . . .	+ 61	. . .	II	R, 2	. . .
Wardhus	Borgrewing . .	9 14 16	9 42	8 53	+ 49	. . .	III	N, 8	7
Stockholm	Wargentin . . .	8 21 35	9 19	8 25	+ 54	. . .	I	N, 21	3.1
Do	Wilke	8 21 50	9 34	. . . . .	+ 69	. . .	II	R, 1½	. . .
Hernosand	Gissler	8 20 44	8 58	8 31	+ 27	. . .	II	N, 22	5
Upsala	Prosperin	8 19 56	9 24	8 25	+ 59	. . .	II	N, 16	3.6
Do	Strömer	8 20 48	10 16	. . . . .	+111	. . .	II	R, 3	. . .
Do	Melander . . .	8 19 45	9 13	. . . . .	+ 48	. . .	III	N, 20	. . .
Do	Bergmann . . .	8 20 29	9 57	. . . . .	+ 92	. . .	III	N, 21	. . .
Do	Salenius	8 19 59	9 27	. . . . .	+ 62	. . .	III	N, 12	. . .
Lund	Schenmark . . .	8 1 49	9 4	8 19	+ 45	. . .	I	N, 21	5
Do	Nenzelius . . .	8 1 59	9 14	. . . . .	+ 55	. . .	III	N, 20	. . .
Greifswald	A. Mayer	8 2 19	8 58	8 17	+ 41	. . .	II	A, 7	2
Greenwich	Maskelyne . . .	7 8 42	8 42	8 18	+ 24	. . .	III	R, 2	7
Do	Hitchins	7 8 38	8 38	. . . . .	+ 20	. . .	III	R, 6	. . .
Do	Hirst	7 8 55	8 55	. . . . .	+ 37	. . .	III	R, 2	. . .
Do	Horsley	7 8 28	8 28	. . . . .	+ 10	. . .	III	A, 10	. . .
Do	Dunn	7 8 21	8 21	. . . . .	+ 3	. . .	III	A, 3½	. . .
Do	Dollond	7 9 3	9 3	. . . . .	+ 45	. . .	III	A, 3½	. . .
Do	Nairne	7 9 14	9 14	. . . . .	+ 56	. . .	III	R, 2	. . .
London	Horsfall	7 8 50	9 5	8 18	+ 47	. . .	II	R, —	7
Do	Canton	7 8 28	8 45	. . . . .	+ 27	. . .	III	(?)	. . .
Do	Aubert	7 8 13	8 33	8 18	+ 15	. . .	III	R, 2	7

*Time determination suspected to be erroneous.

Tabular summary of observations—Continued.

1769, I; (INGRESS: EXTERIOR CONTACT)—Concluded.

Station.	Observer.	Local mean time, June 3, 1769.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
		h. m. s.	h. m. s.	h. m. s.	s.				°
Kew	BEVIS	7 7 43	7 8 57	7 8 18	+ 39	...	III	R, 3½	7
Caen	Anon.	7 7 4	8 29	8 19	+ 10	...	I	R, 1½	6
Windsor	HARRIS	7 6 14	8 35	8 18	+ 17	...	III	R, 1½	8
Shirburn Castle	BARTLETT	7 4 48	8 44	8 19	+ 25	...	II	N, 14	8
Leicester	LUDLAM	7 4 46	9 21	8 20	+ 61	...	I	A, 3	8
Oxford	SYKES	7 3 44	8 44	8 19	+ 25	...	III	A, 3½	8
Do	SHUCKBURGH	7 3 52	8 52	.....	+ 33	...	III	(?)	...
Do	NIKITIN	7 4 28	9 28	.....	+ 69	...	III	R, 1	...
Do	WILLIAMSON	7 4 13	9 13	.....	+ 54	...	III	N, 8	...
Hawkhill	ALEMOOR	6 57 33	10 11	8 22	+109	...	III	R, 1½	11
Do	JAMES HOY	6 57 30	10 8	.....	+106	...	III	A, 3½	...
Do	DR. LIND	6 57 41	10 19	.....	+117	...	III	A, 2	...
Gibraltar	Three observers	6 48 49	10 14	8 28	+106	...	III	{N, 7½ R, 2}	3.9
Glasgow	A. WILSON	6 52 15	9 19	8 23	+ 56	...	I	(?)	11
Do	WILLIAMSON	6 52 12	9 16	.....	+ 53	...	I	A, 2½	...
Do	P. WILSON	6 52 4	9 8	.....	+ 45	...	I	R, 1	...
Cavan	MASON	6 39 2	9 13	8 26	+ 47	...	III	R, 2	13
Newbury	WILLIAMSON	2 27 58	11 25	11 5	+ 20	...	I	R, —	52
Cambridge	WINTHROP	2 27 48	12 16	11 8	+ 68	...	III	R, 2	53
Providence	BROWN	2 27 27	13 4	11 11	+113	...	III	R, 3	53
Cape Haïtien	FLEURIEU	2 23 59	12 47	12 47	0	...	III	A, 5	55
Do	FILIÈRE	2 24 1	12 49	.....	+ 2	...	III	A, 3	...
Do	DESTOURES	2 24 5	12 53	.....	+ 6	...	III	A, 2	...
Do	PINGRÉ	2 23 57	12 45	12 47	— 2	...	III	A, 5	55
Baskenridge	STIRLING	2 13 44	11 55	11 28	+ 27	...	III	(?)	55
Lewes	BIDDLE	2 11 52	12 25	11 38	+ 47	...	III	R, —	56
Philadelphia	SHIPPEN	2 11 52	12 8	11 33	+ 35	...	I	R, 2	56
Do	WILLIAMSON	2 11 31	12 7	.....	+ 34	...	I	N, 24	...
Do	PRIOR	2 11 27	12 3	.....	+ 30	...	I	R, 1½	...
Do	PEARSON	2 11 35	12 11	.....	+ 38	...	I	R, 1	...
Do	EWING	2 11 33	12 9	.....	+ 36	...	I	N, 4½	...
Norristown	RITTENHOUSE	2 10 30	11 58	11 33	+ 25	...	I	N, 36	56
Do	LUKENS	2 10 42	12 10	.....	+ 37	...	I	N, 42	...
Do	SMITH	2 10 37	12 5	.....	+ 32	...	I	R, 2	...
Hudson's Bay	DYMOND	0 54 45	11 21	11 20	+ 1	...	III	R, 2	52
Do	WALES	0 54 52	11 28	.....	+ 8	...	III	R, 2	...
San José	CHAPPE	23 57 1	15 50	15 13	+ 37	...	I	A, 3	90
Do	DOZ	23 56 58	15 47	.....	+ 34	...	III	(?)	...
Do	MEDINA	23 57 2	15 51	.....	+ 38	...	III	(?)	...
Tahiti	GREEN	21 23 24	21 20	20 51	+ 29	...	I	R, 2	35
Do	COOK	21 23 29	21 25	.....	+ 34	...	I	R, 2	...
Do	SOLANDER	21 23 50	21 46	.....	+ 55	...	II	R, 3	...

***Tabular summary of observations*—Continued.**

1769, II; (INGRESS: INTERIOR CONTACT).

Station.	Observer.	Local mean time, June 3, 1769.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
		h. m. s.	h. m. s.	h. m. s.	s.				°
Jakutsk	Islenieff	16 11 28	7 32 30	7 30 40	+110	N	IV	A, 10	8
Ponoi	Mallet	10 12 48	28 18	27 45	+ 33	N	II	A, 12	2
Kola	Rumovsky	9 39 46	27 33	27 40	— 7	N	IV	A, 12	5
Do	. . do	9 40 9	27 56	. . . . .	+ 16	L	IV	. . . .	. . .
Do	Ochtensky	9 40 7	27 54	. . . . .	+ 14	L	III	R, 2	. . .
Wardhus	Hell	9 31 37	27 3	27 42	— 39	N	II	A, 10, —	6
Do	. . do	9 31 47	27 13	. . . . .	— 29	L	II	. . . .	. . .
Do	Sajnovics	9 31 37	27 3	. . . . .	— 39	N	II	N, 10, —	. . .
Do	Borgrewing	9 32 9	27 35	. . . . .	— 7	N	III	N, 8	. . .
Cajaneborg	Planmann	9 18 29	27 33	27 29	+ 4	L	II	N, 21	2
North Cape	Bayley	9 11 45	27 45	27 40	+ 5	G	III	R, 2	8
Do	. . do	9 12 40	28 40	. . . . .	+ 60	L_2	III	. . . .	. . .
Wanhallinna	Gadolin	8 58 10	28 39	27 20	+ 79	L_1	III	N, 20	1
Do	. . do	8 58 40	29 9	. . . . .	+109	L	II	. . . .	. . .
Do	Justander	8 58 36	29 5	. . . . .	+105	N	III	N, 3	. . .
Stockholm	Wargentin	8 39 16	27 0	27 16	— 16	G	III	N, 21	1
Do	. . do	8 39 31	27 15	. . . . .	— 1	L	III	. . . .	. . .
Do	Ferner	8 39 32	27 16	. . . . .	0	L	III	A, 10	. . .
Do	Strussenfelt	8 38 57	26 41	. . . . .	— 35	G	III	. . . .	. . .
Do	Wilke	8 39 14	26 58	. . . . .	— 18	G	III	R, 1½	. . .
Do	. . do	8 39 29	27 13	. . . . .	— 3	L	III	. . . .	. . .
Hernosand	Gissler	8 37 56	26 10	27 20	— 70	G	III	N, 22	4
Do	. . do	8 38 51	27 5	. . . . .	— 15	L	III	. . . .	. . .
Upsala	Prosperin	8 37 56	27 24	27 16	+ 8	L	III	N, 16	2
Do	Strömer	8 37 42	27 10	. . . . .	— 6	G	. . .	R, 3	2
Do	. . do	8 38 16	27 44	. . . . .	+ 28	L	III	. . . .	. . .
Do	Melander	8 37 41	27 9	. . . . .	— 7	G	. . .	N, 20	. . .
Do	. . do	8 37 56	27 24	. . . . .	+ 8	L	III	. . . .	. . .
Do	Bergmann	8 37 53	27 21	. . . . .	+ 5	L	III	N, 21	. . .
Do	Salenius	8 37 0	26 28	. . . . .	— 48	L_1	III	N, 12	. . .
Do	. . do	8 37 59	27 27	. . . . .	+ 11	L_2	III	. . . .	. . .
Lund	Schenmark	8 19 51	27 6	27 10	— 4	L (?)	II	N, 21	1
Do	Nenzelius	8 19 44	26 59	. . . . .	— 11	L (?)	II	N, 20	. . .
Greifswald	A. Mayer	8 20 28	27 7	27 18	— 11	L	IV	A, 7	0
Do	Röhl	8 20 32	27 11	. . . . .	— 7	. . .	IV	. . . .	. . .
Saron	De Saron	7 41 44	26 48	27 8	— 20	N	III	A, 3½	1
Paris (Coll. Le Grand)	Messier	7 36 29	27 6	27 8	— 2	N	II	A, 12	2
Do	. . do	7 36 31	27 8	. . . . .	0	L_1	II	. . . .	. . .
Do	Badouin	7 36 35	27 12	. . . . .	+ 4	N	III	A, 3	. . .
Do	Turgot	7 36 34	27 11	. . . . .	+ 3	N	III	R, 1	. . .
Do	Zannoni	7 36 26	27 3	. . . . .	— 5	N	III	R, 3	. . .
Paris (Royal Obs.)	Cassini	7 36 37	27 16	27 8	+ 8	N	II	A, 3½	2
Do	De Chaulnes	7 36 42	27 21	. . . . .	+ 13	N	III	A, 3½	. . .

Tabular summary of observations—Continued.

1769, II; (INGRESS: INTERIOR CONTACT)—Continued.

Station.	Observer.	Local mean time, June 3, 1769.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
		h. m. s.	h. m. s.	h. m. s.	s.				°
Paris (Royal Obs.) . .	Du Séjour . . .	7 36 27	7 27 6	7 27 8	— 2	N	III	(?)	. . .
Do	Maraldi	7 36 34	27 13	. . . . .	+ 5	N	II	A, 3	. . .
Colombes	Bernoulli . . .	7 35 58	26 57	27 8	— 11	N	III	R, 2	2
St. Hubert	Le Monnier . . .	7 32 40	25 15	27 7	—112	N	II	A, 10	3
Do	De Chabert . . .	7 33 17	25 52	. . . . .	— 75	N	III	N, 18	. . .
Toulouse	D'Arquier . . .	7 32 52	27 6	27 12	— 6	N	III	(?)	0
Do	Garipuy	7 33 14	27 28	. . . . .	+ 16	N	III	(?)	. . .
Rouen	Dulague	7 31 24	27 1	27 8	— 7	N	III	(?)	3
Do	Bouin	7 31 30	27 7	. . . . .	— 1	N	III	(?)	. . .
Havre	Diquemar . . .	7 28 34	28 8	27 8	+ 60	N	III	(?)	4
Greenwich	Maskelyne . . .	7 26 15	26 15	27 8	— 53	G	. . .	R, 2	5
Do	. . do	7 27 7	27 7	. . . . .	— 1	L	II	. . . .	. . .
Do	Hitchins	7 26 31	26 31	. . . . .	— 37	G	III	R, 6	. . .
Do	. . do	7 26 41	26 41	. . . . .	— 27	L	III	. . . .	. . .
Do	Hirst	7 27 2	27 2	. . . . .	— 6	L	III	R, 2	. . .
Do	Horsley	7 25 59	25 59	. . . . .	— 69	G	III	A, 10	. . .
Do	. . do	7 27 12	27 12	. . . . .	+ 4	L	III	. . . .	. . .
Do	Dunn	7 27 12	27 12	. . . . .	+ 4	G	. . .	A, 3½	. . .
Do	. . do	7 27 32	27 32	. . . . .	+ 24	L	II	. . . .	. . .
Do	Dollond	7 27 4	27 4	. . . . .	— 4	L	II	A, 3½	. . .
Do	Nairne	7 27 4	27 4	. . . . .	— 4	L	III	R, 2	. . .
London (Mid. Temple)	Horsfall	7 26 34	26 49	27 9	— 20	L	IV	R, —	5
London (Spital Square)	Canton	. . . . .	27 0	27 9	— 9	L	II	(?)	5
London (Austin Friars)	Aubert	7 26 45	27 5	27 9	— 4	G	. . .	R, 2	5
Do	. . do	7 26 51	27 11	. . . . .	+ 2	L	III	. . . .	. . .
Kew	Bevis	7 25 52	27 6	27 9	— 3	G	. . .	R, 3½	5
Do	. . do	7 26 1	27 15	. . . . .	+ 6	L	III	. . . .	. . .
Caen (Mission) . . .	Pigott, sr	7 24 8	25 31	27 8	— 97	G	. . .	A, 6	4
Do	. . do	7 24 12	25 35	. . . . .	— 93	L	II	. . . .	. . .
Do	Pigott, jr	7 24 39	26 2	27 8	— 66	N	III	R, 1½	4
Do	De Rochefort .	7 24 52	26 15	. . . . .	— 53	N	III	A, 3	. . .
Caen	Anon	7 22 57	24 22	27 8	—166	G	. . .	R, 1½	4
Do	. . do	7 25 27	26 52	. . . . .	— 16	L_3	IV	. . . .	. . .
La Trompette	Faugère	7 25 0	27 13	27 11	+ 2	N	II	R, 2½	2
Bordeaux	La Roque	7 24 50	27 6	27 11	— 5	N	III	R, 2	. . .
Windsor	Harris	7 24 22	26 43	27 10	— 27	L	II	R, 1½	6
Shirburn Castle . . .	Macclesfield . .	7 23 13	27 9	27 9	0	L	III	A, 3½	6
Do	Lady Macclesfield	7 23 0	26 56	. . . . .	— 13	N	III	(?)	. . .
Do	Bartlett	7 23 10	27 6	. . . . .	— 3	N	III	N, 14	. . .
Leicester	Ludlam	7 22 42	27 17	27 9	+ 8	G	. . .	A, 3	6
Do	. . do	7 22 54	27 29	. . . . .	+ 20	L	II	. . . .	. . .
Oxford	Hornsby	7 21 57	26 57	27 10	— 13	L	II	A, 7	6
Do	Clare	7 22 12	27 12	. . . . .	+ 2	L	III	A, 6	. . .

***Tabular summary of observations*—Continued.**

1769, II; (INGRESS: INTERIOR CONTACT)—Continued.

Station.	Observer.	Local mean time, June 3, 1769.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	⊙'s altitude.
		h. m. s.	h. m. s.	h. m. s.	s.				°
Oxford	SYKES	7 22 6	7 27 6	7 27 10	— 4	L	III	A, 3½	6
Do	SHUCKBURGH . .	7 21 0	26 0	. . . . .	— 70	G	IV	(?)	. . .
Do	. . do	7 22 9	27 9	. . . . .	— 1	L	II	. . . .	. . .
Do	NIKITIN	7 21 59	26 59	. . . . .	— 11	N	III	R, 1	. . .
Do	WILLIAMSON . . .	7 21 55	26 55	. . . . .	— 15	N	III	N, 8	. . .
Do	HORSLEY	7 22 12	27 12	. . . . .	+ 2	L	II	R, 1½	. . .
Do	JACKSON	7 22 12	27 12	. . . . .	+ 2	L	III	A, 9	. . .
Hawkhill	ALEMOOR	7 14 10	26 48	27 11	— 23	G	III	R, 1½	9
Do	. . do	7 14 32	27 10	. . . . .	— 1	L	III	. . . .	. . .
Do	HOY	7 12 35	25 13	. . . . .	—118	L	III	A, 3½	. . .
Do	LIND	7 14 37	27 15	. . . . .	+ 4	L	III	A, 2	9
Kirknewton	BRICE	7 12 13	25 53	27 11	— 78	L	III	(?)	9
Glasgow	Dr. A. WILSON . .	7 9 41	26 45	27 12	— 27	L	III	(?)	9
Do	P. WILSON	7 10 8	27 12	. . . . .	0	L_3	III	R, 1	. . .
Do	WILLIAMSON . . .	7 10 8	27 12	. . . . .	0	L	II	A, 2½	. . .
Do	REID	7 8 8	25 12	. . . . .	—120	G	III	. . . .	. . .
Brest	VERDUN	7 9 22	27 20	27 12	+ 8	N	III	N, 7	6
Do	FORTIN	7 9 28	27 26	. . . . .	+ 14	N	III	A, 5	. . .
Do	BLONDEAU	7 9 48	27 46	. . . . .	+ 34	N	III	A, 5	. . .
Do	LE ROY	7 9 51	27 49	. . . . .	+ 37	N	III	N, 14	. . .
Gibraltar	Three observers . .	7 6 2	27 27	27 24	+ 3	N	III	{N, 7½ R, 2}	1
Cadiz	TOFINO	7 0 14	25 26	27 24	—118	G	. . .	N, 7	2
Do	. . do	7 1 44	26 56	. . . . .	— 28	L	III	. . . .	. . .
Cavan	C. MASON	6 56 31	26 42	27 14	— 32	G	. . .	R, 2	11
Do	. . do	6 57 9	27 20	. . . . .	+ 6	L	III	. . . .	. . .
Agromonte	DE QUEIROS . . .	6 52 20	26 46	27 20	— 34	N	III	(?)	6
Isle Coudre	WRIGHT	2 48 4	29 43	29 35	+ 8	G	. . .	R, 2	48
Do	. . do	2 48 35	30 14	. . . . .	+ 39	L	III	. . . .	. . .
Newbury	WILLIAMS	2 46 25	29 52	29 52	0	L	II	R,—	50
Cambridge	WINTHROP . . .	2 45 12	29 40	29 56	— 16	N	II	R, 2	50
Do	. . do	2 45 17	29 45	. . . . .	— 11	L_1	II	. . . .	. . .
Providence	BROWN	2 44 20	29 57	29 58	— 1	N	III	R, 3	50
Cape Haïtien	PINGRÉ	2 42 28	31 16	31 41	— 25	L	II	A, 5	52
Do	LA FILIÈRE . . .	2 42 25	31 13	. . . . .	— 28	L	III	A, 3	. . .
Do	DESTOURES . . .	2 42 34	31 22	31 41	— 19	L	III	A, 2	52
Do	FLEURIEU	2 42 30	31 18	. . . . .	— 23	L	II	A, 2½	. . .
Baskenridge	STIRLING	2 31 56	30 7	30 14	— 7	N	III	(?)	53
Lewes	BIDDLE	2 29 52	30 25	30 24	+ 1	G	. . .	R,—	54
Do	. . do	2 29 56	30 29	. . . . .	+ 5	L	I	. . . .	. . .
Do	BAILEY	2 29 52	30 25	. . . . .	+ 1	. . .	I	A, 4½	. . .
Philadelphia	SHIPPEN	2 29 20	29 56	30 19	— 23	L_1	I	R, 2	54
Do	WILLIAMSON . . .	2 29 8	29 44	. . . . .	— 35	T	I	N, 24	. . .

***Tabular summary of observations*—Continued.**

1769, II; (INGRESS: INTERIOR CONTACT)—Concluded.

Station.	Observer.	Local mean time, June 3, 1769.	Greenwich mean time. Observed.	Greenwich mean time. Tabular.	Residual correction.	Phase.	Class.	Telescope, kind and length.	⊙'s altitude.
		h. m. s.	h. m. s.	h. m. s.	s.				°
Philadelphia	WILLIAMSON	2 29 12	7 29 48	7 30 19	− 31	L_1	. . .	. . . .	. . .
Do	THOMSON	2 29 11	29 47	. . . . .	− 32	L_1	I	R, 1	. . .
Do	. . do	2 29 15	29 51	. . . . .	− 28	. . .	. . .	. . . .	. . .
Do	PRIOR	2 29 12	29 48	. . . . .	− 31	. . .	III	R, 1½	. . .
Do	EWING	2 29 10	29 46	. . . . .	− 33	. . .	II	N, 4½	. . .
Norristown	RITTENHOUSE	2 27 39	29 7	30 18	− 71	G	IV	N, 36	54
Do	. . do	2 29 11	30 39	. . . . .	+ 21	L_2	IV	. . . .	. . .
Do	LUKENS	2 27 52	29 20	. . . . .	− 58	L_1	II	N, 42	. . .
Do	. . do	2 28 9	29 37	. . . . .	− 41	L_2	II	. . . .	. . .
Do	SMITH	2 28 0	29 28	. . . . .	− 50	L_1	I	R, 2	. . .
Do	. . do	2 28 12	29 40	. . . . .	− 38	L_2	I	. . . .	. . .
Wilmington	POOLE	2 28 5	30 17	30 21	− 4	N	III	N, 12	54
Hudson's Bay	DYMOND	1 13 5	29 41	30 5	− 24	L_1	II	R, 2	51
Do	WALES	1 13 9	29 45	. . . . .	− 20	L_1	II	R, 2	. . .
San José	CHAPPE	0 15 11	34 0	34 3	− 3	L	II	A, 3	86
Do	DOZ	0 15 9	33 58	. . . . .	− 5	N	III	(?)	. . .
Do	MEDINA	0 15 14	34 3	34 3	0	N	III	(?)	86
Tahiti	GREEN	21 40 59	38 55	40 4	− 69	G	IV	R, 2	38
Do	. . do	21 41 39	39 35	. . . . .	− 29	L_2	IV	. . . .	. . .
Do	COOK	21 40 59	38 55	. . . . .	− 69	G	IV	R, 2	. . .
Do	. . do	21 41 59	39 55	. . . . .	− 9	L_2	IV	. . . .	. . .
Do	SOLANDER	21 41 12	39 8	. . . . .	− 56	L_1	III	R, 3	. . .
Do	. . do	21 41 46	39 42	. . . . .	− 22	L_2	III	. . . .	. . .

***Tabular summary of observations*—Continued.**

1769, III; (EGRESS: INTERIOR CONTACT).

Station.	Observer.	Local mean time, June 3, 1769.	Greenwich mean time.		Residual correction.	Phase.	Class.	Telescope, kind and length.	⊙'s altitude.
			Observed.	Tabular.					
		h. m. s.	h. m. s.	h. m. s.	s.				°
Jakutsk	ISLENIEFF	22 0 22	13 21 24	13 19 43	+101	L	II	A, 10	46
Do	SOCIUS	22 0 23	21 25	.....	+102	N	III	N, 15	...
Manilla	DE RONAS	21 23 32	19 40	19 53	− 13	N	III	(?)	53
Pekin	DOLLIÈRES	21 6 11	20 39	20 43	− 4	L	III	N, 18	50
Do	. . do	21 6 30	20 58	.....	+ 15	G	III	....	...
Do	COLLAS	21 6 36	21 4	.....	+ 21	G	III	N, 14	...
Batavia	MOHR	20 28 0	20 38	20 14	+ 24	N	II	R, 3	31
Orsk	C. EULER	17 15 23	21 8	22 31	− 83	N	III	A, 12	11
Do	. . do	17 16 13	21 58	.....	− 33	N	III	....	...
Orenburg	L. KRAFT	17 2 54	22 30	22 29	+ 1	L	III	A, 12	9
Gurief	LOWITZ	16 50 29	22 47	22 43	+ 4	N	III	A, 12	4
Do	INOCHODSOW	16 50 29	22 47	.....	+ 4	N	III	R, 2	...
Kola	RUMOVSKY	15 33 9	20 56	20 56	0	N	II	A, 12	6
Do	OCHTENSKI	15 33 30	21 17	.....	+ 21	N	IV	R, 2	...
Wardhus	HELL	15 25 13	20 39	20 47	− 8	L	II	A, 10	6
Do	SAJNOVICS	15 25 23	20 49	.....	+ 2	N	II	N, 10	...
Do	BORGREWING	15 25 11	20 37	.....	− 10	N	III	N, 8½	...
Petersburg	MAYER	15 23 30	22 16	21 31	+ 45	L	II	A, 18	1
Do	. . do	15 23 33	22 19	.....	+ 48	L	II	....	...
Do	EULER	15 23 34	22 20	.....	+ 49	L	II	A, 7	...
Do	LEXELL	15 23 27	22 13	.....	+ 42	L	II	R, 2½	...
Do	STAHL	15 23 20	22 6	.....	+ 35	L	II	R, 3½	...
Hudson's Bay	DYMOND	6 58 32	15 8	15 28	− 20	L_1	II	R, 2	10
Do	WALES	6 58 30	15 6	.....	− 22	L_1	II	R, 2	...
Do	. . do	6 58 54	15 30	.....	+ 2	...	II	R, 2	10
San José	CHAPPE	5 52 37	11 26	11 13	+ 13	L	II	A, 3	9
Do	DOZ	5 52 34	11 23	.....	+ 10	N	III	(?)	...
Do	MEDINA	5 52 34	11 23	.....	+ 10	N	III	(?)	...
Do	PAULY	5 52 15	11 4	.....	− 9	N	III	(?)	...
Tahiti	COOK	3 12 0	9 56	9 40	+ 16	L	III	R, 2	20
Do	. . do	3 12 32	10 28	.....	+ 48	G	II	....	...
Do	GREEN	3 11 50	9 46	.....	+ 6	L	II	R, 2	...
Do	. . do	3 12 38	10 34	.....	+ 54	G	II	....	...

Tabular summary of observations—Concluded.

1769, IV; (EGRESS: EXTERIOR CONTACT).

Station.	Observer.	Local mean time, June 3, 1769.	Greenwich mean time.		Residual correction.	Phase.	Class.	Telescope, kind and length.	☉'s altitude.
			Observed.	Tabular.					
		h. m. s.	h. m. s.	h. m. s.	s.				°
Jakutsk	ISLENIEFF	22 16 43	13 37 45	13 38 18	—33	...	...	A, 10	47
Do	Assistant	22 16 43	37 45	.....	—33	...	...	N, 15	...
Manilla	DE RONAS	21 41 13	37 21	38 33	—72	...	...	....	57
Pekin	DOLLIÈRES	21 24 47	39 15	39 19	— 4	...	...	N, 18	54
Do	COLLAS	21 24 41	39 9	.....	—10	...	...	N, 14	...
Batavia	MOHR	20 46 18	38 56	39 3	— 7	...	...	R, 3	34
Orsk	C. EULER	17 34 43	40 28	41 16	—48	...	...	A, 12	14
Orenburg	KRAFT	17 21 20	40 56	41 14	—18	...	...	A, 12	12
Gurief	LOWITZ	17 8 25	40 43	41 29	—46	...	...	A, 12	8
Do	INOCHODSOW	.....	.....	.....	....	...	...	R, 2	...
Wardhus	HELL	15 43 25	38 51	39 34	—43	...	...	A, 10	10
Do	SAJNOVICS	15 43 28	38 54	.....	—40	...	...	N, 10	...
Do	BORGREWING	15 43 21	38 47	.....	—47	...	...	N, 8	...
Petersburg	MAYER	15 41 27	40 13	40 20	— 7	...	...	A, 18	4
Do	EULER	15 41 17	40 3	.....	—17	...	...	A, 7	...
Do	LEXELL	15 41 10	39 56	.....	—24	...	...	R, 2½	...
Do	STAHL	15 41 0	39 46	.....	—34	...	...	R, 3½	...
Cajaneborg	PLANMANN	15 30 14	39 18	40 5	—47	...	...	N, 21	6
Do	UHLWYK	15 30 11	39 15	.....	—50	...	...	A, 3	...
Hudson's Bay	DYMOND	7 17 7	33 43	34 27	—44	...	...	R, 2	9
Do	WALES	7 16 48	33 24	.....	—63	...	...	R, 2	...
San José	CHAPPE	6 11 6	29 55	30 24	—29	...	...	A, 3	4
Do	DOZ	6 10 28	29 17	.....	—67	...	...	....	...
Do	MEDINA	6 10 33	29 22	.....	—62	...	...	....	...
Do	PAULY	6 10 29	29 18	30 24	—66	...	...	A, 3	4
Tahiti	GREEN	3 30 1	27 57	28 55	—58	...	...	R, 2	22
Do	COOK	3 29 49	27 45	.....	—70	...	...	R, 2	...
Do	SOLANDER	3 30 0	27 56	.....	—59	...	...	R, 3	...

As a postscript to the above summary it may be added that the decision to include Contact I was an afterthought, and that, as will be subsequently explained, the classification is different from that of the internal contacts. As no value would attach to any other classification of Contacts IV than that based on the quality of the telescope and the altitude of the Sun, none has been attempted.

Chapter VII.

FORMATION AND SOLUTION OF THE EQUATIONS OF CONDITION.

1. The classification and presentation of the results of the separate observations in the preceding summary have been made without any reference to their accordance with theory, and with the sole view of showing what result would be given by each observation considered separately. The only exception to this rule is in the occasional omission of records so erroneous and incapable of correction that no interest or value can attach to them.

A very slight examination of the residuals shows that we are not dealing with a homogeneous collection of observations, all having the same degree of precision, but with observations made under widely different conditions, and therefore of very different moduli of precision. It is known that the method of least squares, as usually applied, will not give the best results of such a collection of observations. I have discussed this subject in a paper in the American Journal of Mathematics,* and shown that, in combining a system of observations of different but unknown moduli of precision, the weight to be assigned to each separate observation is partly a function of its discordance from the general result of the others, and can not therefore be assigned *a priori.*

We can determine whether a sufficiently large collection of observations is of this heterogeneous class from the law of distribution of the residuals. If the distribution follows approximately the law expressed by the commonly assumed exponential function, then it may be assumed that the observations are homogeneous, and can be combined by the method of least squares. If, on the other hand, the observations are heterogeneous, then, in all usual cases, large residuals will be proportionally more numerous. Our first step will therefore be to obtain a representation of the law of residuals in magnitude.

The conclusions from such a representation will be made easy by representing them to the eye in a graphical form. The differences between the different residuals arising from errors of adopted parallax and other tabular elements can not exceed a very few seconds. Our conclusions respecting the character of the observations may therefore be drawn from a comparison of the residuals given in the preceding chapter nearly as well as from those which result from the final solution of the equations of condition.

* Vol. VIII, p. 343. A generalized theory of the combination of observations so as to obtain the best result.

2. The magnitude of all the residuals which are given for interior contacts in the preceding summary are exhibited in the following table. The residuals are classified as if multiples of 5 seconds, the nearest multiple in all cases being assigned. For example, residuals of 8, 9, 10, 11, and 12 seconds are each regarded as having the value of 10^s. The first column of the table contains the multiples of 5^s from -75^s to $+75^s$. Opposite each multiple is printed as many 1's as there are residuals having a magnitude differing not more than 2 seconds from that multiple. Residuals without the limits of the tables are collected together at the top or bottom:

Residuals of interior contacts.

Value of Residuals.	1761, Ingress (II).	1761, Egress (III).	1769, Ingress (II).	1769, Egress (III).
s. Exc. −77	III	III	IIIII III	I
−75			I	
−70			IIIII I	
−65			I	
−60			I	
−55			III	
−50	I		II	
−45				
−40	I	I	IIII	
−35	I		IIIII	I
−30	I	I	IIIII IIII	
−25	I	I	IIIII III	
−20		III	IIIII I	II
−15	II	III	IIIII II	I
−10	I	IIIII I	IIIII II	III
− 5	III	IIIII II	IIIII IIIII IIIII IIIII II	I
0		IIIII IIIII III	IIIII IIIII IIIII IIIII II	IIII
+ 5	II	IIIII IIII	IIIII IIIII III	III
+10	II	IIIII IIII	IIIII II	II
+15	II	IIIII IIIII I	IIIII	III
+20	I	IIIII I	II	II
+25	I	IIIII II	I	I
+30	I	II	I	
+35	I	I	III	I
+40		I	I	I
+45		I		I
+50		I		III
+55		III		I
+60			II	
+65		I		
+70		I		
+75				
Exc. +77		IIIII I	IIII	II

However heterogeneous may be a system of observations, there is one datum which can always be determined from them, namely, the closest limits within which a majority of the results are contained. In the case of observations of uniform precision these limits are those of probable error, and the same term may be applied to the corresponding limits in the case of heterogeneous results. That is to say, how heterogeneous soever may be a collection of observations, if one-half the results are contained between the limits A and B, we may assign a probable error of $\frac{1}{2}(A - B)$ to them. In the following table I have taken for the limits A and B such values as included at least one-half of the residuals:

	A.	B.	Number of residuals— Between A and B.	Number of residuals— Without A and B.
	s.	s.		
1761, I	−17	+17	12	12
1761, III	− 7	+17	49	48
1769, II	−22	+ 7	77	74
1769, III	−17	+17	17	16
Total	. . . . . .	. . . . . .	155	150

It appears, then, that making no rejection whatever of results on account of discordance, unfavorable conditions, or unskilfulness of observers, the probable error of a single observation is less than $\pm 15^s$, which corresponds in relative heliocentric positions of the Earth and Venus to $\pm 0''.30$ of arc. This quantity will be reduced to $\pm 0''.24$, if we throw out of the count all residuals of $\pm 68^s$ or upwards. We shall then find that 141 residuals are contained between limits distant by 25^s, while 129 are without them. The exclusion of residuals affected by an erroneous time determination will make a yet farther reduction in the probable error.

A study of the table, having in mind the curve of probability in the case of homogeneous observations, will enable us to reach a satisfactory conclusion respecting the treatment on account of discordance, without any reference to the character of the conclusions to be finally drawn.

In the first place, it is evident that all the residuals without the limits -67^s and $+67^s$, are abnormally discordant, and should be wholly rejected. Furthermore, the residuals within the following limits may be considered as possibly normal, and therefore subject to no marked diminution of weight on account of discordance:

	s.	s.
1761, II	−30	+30
1761, III	−30	+35
1769, II	−40	+25
1769, III	−20	+25

It will be seen that, for the most part, these limits are fairly well defined by a sudden dropping off in the density of the residuals. For example, there are no resid-

uals whatever of -45^s in the whole collection, and only two of $+45^s$. We now have left 34 residuals, belonging to what we may consider the doubtful class, not large enough to be left wholly out of consideration, and yet so large as to show that they are affected by some causes of error which do not act in the case of ordinary observations. We can not but suspect that in many of these cases an error of 1 minute was made in the record, yet I do not deem it safe to act on this supposition. The best course seems to be to give a diminishing weight, converging to zero about the point, $\pm 60^s$. In the paper referred to on page 372 I have shown that this scaling of weights is a legitmate result of the theory of probabilities.

The observations lying outside the limits of possible normal error present the noteworthy peculiarity that ingress was commonly observed too soon and egress too late. This is shown as follows:

1761, II	Early,	3	Late,	0
1761, III	Late,	7	Early,	3
1769, II	Early,	15	Late,	4
1769, III	Late,	2	Early,	1
Sums		27		8

This result is the opposite of what we should expect from observations of the thread of light with insufficient optical power. It probably arises from observations of geometric contact with exaggerated black-drop.

In the case of 1769, III, the number of residuals between the limits -25^s and -40^s may well excite inquiry. Some circumstances connected with this grouping will be considered subsequently.

3. *Exterior contacts.*—The foregoing discussion applies exclusively to the observations of interior contact. Those of exterior contact are so much fewer in number, and so differently made, that no valuable results would be obtained by presenting them graphically. The following considerations will, however, assist in this discussion.

It is evident that all real observations of Contact I must in reality be too late, since the planet could not be seen at all until it had impinged upon the Sun sufficiently to become visible. For a similar reason, all real observations of Contact IV will be too early. It does not follow that the actual tabular residuals in the preceding table must exhibit the algebraic sign thus determined, because it is conceivable that the errors of the tabular elements may be as great as the quantity by which the planet must impinge on the Sun in order to be visible. Yet, a tabular error of this magnitude does seem to me somewhat improbable. We may therefore conclude that a residual of Contact I, which is either negative or nearly zero, or one of Contact IV, which is positive or nearly zero, must be at least subject to suspicion.

The classification of Contact I is based on the consideration that no observation of this contact can have any value unless the observer saw the advancing edge of the planet as soon as it became clearly visible. The numeral I, II, or III is assigned according to the greater or less strength of the evidence that this was the case, I indicating the nearest approach to certainty.

Examining the residuals of Contact I from this point of view, we reach the following conclusions:

Of those of 1761, I, only those of PLANMANN, WARGENTIN, and HIRST can be regarded as entirely normal. This is only one-half the entire number. It is therefore somewhat doubtful whether they should be entitled to an appreciable weight.

With 1769, I, we are much more fortunate. Of 67 residuals none whatever fall between the limits $+70^s$ and $+92^s$, and only 7 follow without the last limit. In view of the fact that we have omitted no observation of this class on account of discordance this showing seems favorable, and justifies us in regarding the limit 69^s as probably that of normal error and in rejecting all results exceeding 90^s.

The residuals of Contact IV are too varied to be disposed of easily. We shall therefore consider them in detail when we come to the formation of the equations of condition.

4. *Errors in time determination.*—There is yet another consideration which will enable us to dispose of a number of doubtful results. Among the possible errors with which the results may be affected are those of clock determination. Such errors are especially to be looked for in the case of comparatively unknown observers at isolated stations. They will show themselves by a common error affecting different observations of contact. We shall now go over the cases where more than one contact was observed in the same transit, with a view of seeing whether erroneous clock error was indicated. We thus conclude:

In 1761:

Tornea.—Time nearly a minute fast.	Contacts	II, III,	IV.
Laibach.—Time fast.	"	III,	IV.
Copenhagen.—Time 3 minutes slow.	"	"	"
Dillingen.—Time slow.	"	"	"
Drontheim.—Time fast.	"	"	"
Würzburg.—Time fast.	"	"	"
Bayeux.—Time fast.	"	"	"
Shirburn Castle.—Time probably fast.	"	"	"
Madrid.—Time fast.	"	"	"
Oporto.—Time slow.	"	"	"
Lisbon.—Time probably fast.	"	"	"

To facilitate reference the above stations have been marked with an asterisk in the "Tabular Summary" (pp. 359–364).

Petersburg.—This case is peculiar. BRAUN recorded Contact I half a minute before the tabular time. That the time was too early would seem to be conclusively proved by the error of all observations of internal contact at ingress; but Contact IV indicates that the time was then rather late. Under these circumstances we can give no weight to the apparently good observations of second interior contact, but must conclude that all the observations are to be set aside.

In 1769:

Batavia.—There is ground for at least a strong suspicion that MOHR's time was too fast by about half a minute, but not for a positive conclusion to that effect.

Cape Haïtien.—The comparison of residual corrections given by the two first contacts would seem to indicate that the time was too slow; but it is difficult to suppose that an expedition of which PINGRÉ was a member could have made such an error.

Lewes, Philadelphia, and Norristown.—The observations of Contact I do not confirm a suspicion I at first entertained that the discordance between these stations was due to clock error.

Hudson's Bay.—Contact I was observed at a suspiciously early moment, yet I do not feel justified in concluding, any more than in the case of Cape Haïtien, that the time was really in error.

5. Of the remaining discordances some may be due to errors of 1 or 2 minutes in recording the time. But I can not feel justified in using any observations which require to be thus corrected, unless some other independent reason exists for the correction. The following are the most marked remaining cases of large discordance, in which it is not shown that the errors or discordances are in the time-determinations:

In 1761:

Calcutta.—Although MAGEE's residuals are not excessive, the imperfect way in which his time was determined must lead us to reject his observations or give them very small weight.

Madras.—The observations of the Jesuits are hopelessly in error. The negative residuals of all HIRST's observations might lead us to suspect that his time was fast, but he describes his clock determination as quite exact. The likeliest explanation of his large errors seems to be some imperfection of his telescope, in consequence of which he lost sight of the planet two minutes before Contact IV.

Abo.—Here also the residuals for the interior contacts (II and III) are large and of opposite signs. It may therefore be inferred that the observations were from some cause affected by a larger error than usual; but we can not attribute it to the clock.

Cajaneborg.—ENCKE considers that there is a misprint of 1 minute in PLANMANN's time of Contact III, and that it should be $22^h\ 6^m\ 8^s$. Assuming that he had good authority for this, I shall accept the correction.

In 1769:

Jakutsk.—From ISLENIEFF's description it would seem that Contact II was lost by clouds; III was supposed to be 2 minutes in error.

North Cape.—BAYLEY's observation is at best doubtful, and the uncertainty of his longitude, as well as the large residual of his observation, will lead us to reject the latter entirely.

Upsala.—We may assume that the first observation by SILENIUS was a mistake of some kind.

Havre, Kirknewton, Cadiz, Agromonte, and Isle Coudre.—Nothing, or at least nothing favorable, is known to me respecting the skill of the observers at these stations and the accuracy with which their time was determined.

In the transit of 1769 we have still to consider some embarrassing cases in which large residuals are exhibited by observations which we should have supposed to have been of the best class. In such cases the suspicion of something abnormal is stronger than in that of bad observations.

Wanhalinna.—These observations are evidently affected by some undiscoverable source of error, probably in the clock.

St. Hubert.—How it happened that so well-known an astronomer as LE MONNIER failed in his observation I can not say. I find no explanation in his published account.

Caen.—The large negative residuals at the Mission, at PIGOT'S observatory, and by PIGOT himself, seem to indicate some abnormal and unaccountable error, on account of which the observations should be thrown aside.

Norristown.—I infer from the description of these observations that the first observer who thought he saw the contact called out for time to an assistant in such a way that all the others heard it, and were thus influenced by him. Under these circumstances, their observations, if admitted at all, should go in with a much diminished weight.

Petersburg.—The evidence of something abnormal in these large positive residuals seems conclusive. The descriptions given by some of the observers, if susceptible of any interpretation, must mean that contact was noted too early. ENCKE remarks (page 67) that, according to a subsequent remark of LEXELL, the observations are uncertain; but it is impossible to infer to what observations this remark applies, nor is it stated where LEXELL made the remark in question. Nor do I understand ENCKE'S statement that the thread of light was not seen. The most probable explanation seems to be that there is an error of 1 minute in the recorded times; but it would not be safe to proceed on this hypothesis. The observations must therefore be entirely set aside.

Tahiti.—To these observations is largely due the small value of the parallax found by ENCKE; but I can not see that any change can be made in ENCKE'S interpretation of them. I have placed some of them in Class IV, because the descriptions of the observers were such as, taken by themselves, would lead one to suspect that the observations were too late. As such is not the case, it seems that *a posteriori* they should be placed in Class II.

6. *Introduction of an expression for irradiation and other imperfections of vision.*—The observations on artificial transits of Venus, made under direction of the American commission in 1874 and 1882, seemed to show that there was at each interior contact only one phase which could be distinctly observed. This phase approximates quite closely to that of true tangency of the limbs. The time to be noted might be defined as, at total ingress, the moment when the cusps had so nearly united into a thread of light that on tracing each one towards their point of meeting their light seemed to converge towards zero at this point. It might also be defined as the moment when the observer saw that the thread of light was about to be formed by the union of the

cusps; possibly, also, as the last definable moment when the thread was certainly incomplete. Of course, this phase would precede the actual formation of a distinct thread of light all around the planet. At egress the corresponding moment was that at which the thread was first completely cut off.

These experiments also showed that the so-called geometric contact, in which the apparent outlines of the Sun and of Venus, when continued through the black-drop, were tangent to each other, could not be observed with any approach to accuracy. The conclusion seemed to be that there was no occasion to consider more than a single phase, which phase might be defined as a mean one, likely to be taken by an observer who judged of contact from the general aspect of the phenomena presented to him.

This view seemed to receive confirmation from my discussion of the transits of Mercury. I there showed that there was no tendency among the observations to group themselves around more than a single phase. The result was that, in commencing the present work, it was inferred that each observation of contact should be reduced to one common mean phase. When the observer noted both geometric contact and thread of light this mean phase would be somewhere between the two, but nearer the latter. When he noted no especial phase, it might be assumed as the mean one. If he noted the distinct formation of the thread of light at ingress, without any geometric phase preceding, a correction determined according to some previously determined law would have to be applied.

On attempting to apply this system to the observations given in the preceding chapters it did not appear so satisfactory as I had anticipated. In most of the observations in northern Europe the two separate phases were noted so distinctly that it was difficult to decide upon any system of deducing the mean phase from them. Moreover, where no phase was noted there seemed to be in most cases good reason for considering that it was the thread of light that was observed. It therefore seemed necessary to do as ENCKE did, and consider the formation of the thread of light as the phase solely to be taken account of. On the other hand, to consider all observations of this phase as strictly comparable would obviously lead to systematic error. At ingress, for example, it is evident that when the Sun is low, vision indistinct, and the telescopic image diffused, the thread of light will be seen later than under more favorable conditions. How much later it is impossible to determine *a priori*. Hence, the only course open to us is to introduce a correction depending upon this quantity into the equations of condition. This, again, requires that we have some means of estimating the amount of indistinctness or diffusion of images at the different stations. Such an estimate may be founded on the following data and circumstances:

I. *The altitude of the Sun.*—The nearer the Sun is to the horizon the more the images will be diffused, and the later the thread of light will be first seen at ingress.

II. *The size of the telescope*—The smaller and more imperfect the telescope the later the thread of light will be first seen. I lay little stress on the magnifying power used, believing that in all ordinary cases the magnifying power was as high as could be advantageously used in the telescope.

III. *The magnitude of the black-drop*, which is indicated by the interval between geometric contact and formation of the thread of light. This difference may be taken as an index to the diffusion of images arising from all causes.

It is impossible to lay down any uniform rule, rigorously derived, for estimating the effect of the two causes, I and II, preceding. We may safely assume that the great bulk of the irradiation is due to the atmosphere and the telescope, but the question of its amount in each case must depend on circumstances which it is impossible to estimate. Yet, in order to avoid bias, it is necessary to proceed upon some general rules, which rules are themselves the result of general impressions derived from experience in observation and repeated examination of the whole mass of material rather than of any special investigation. The following are the hypotheses which I have adopted:

I. That the amount of the irradiation produced by the atmosphere is proportional to the refraction. To express it I have assumed the amount due to a refraction of 6′ as the unit. That is to say, I divide the minutes of refraction by 6 and assume that the quotient, taken to the nearest integer, is proportional to the atmospheric irradiation.

II. I have also assumed that the telescopic irradiation produced by the various instruments, measured on the same scale, is as follows:

In the case of a non-achromatic telescope, exceeding 14 feet, I assign the co-efficient	0
Between 7 and 14 feet	1
Less than 7 feet	2
In that of an achromatic telescope:	
Exceeding 3 feet	0
Less than 3 feet	1
In that of a reflecting telescope:	
Exceeding 2½ feet	0
Less than 2½ feet	1

It will be seen that I assume the imperfections of vision produced by the various telescopes to range between the effects of the atmosphere at the zenith and at an altitude of 3°.

Where the observer gives no data for determining the amount of irradiation, I use the square root of the sum of the squares of the two numbers thus found, expressed only to the nearest integer.

But the statements or remarks of the observer may afford independent evidence of the amount of the irradiation. The clearest evidence of this kind is that afforded by the observed interval between geometric contact and the formation of the thread

of light. For different values of this interval I have assumed the following numbers:

For an observed difference of geometric contact and formation of the thread of light—

Less than 12s	1
Between 12s and 25s	2
Between 25s and 37s	3
Between 37s and 55s	4
Greater than 55s	5

I have already remarked upon the extreme uncertainty of the estimate of geometric contact. In fact, in many cases this phase is merely that at which the observer thought the planet to be wholly within the Sun, although the thread of light was not complete. It admits of being described in a great variety of ways, and observers at the same station and with different instruments differ considerably in their estimates. I have therefore, as a rule, taken the mean of all the observations at each station, and at several stations in the same region, as indicative of the number to be adopted.

In a great number of observations, possibly a majority of all, the observer gives no time of geometric phase. This omission may proceed from two causes, the absence of any striking irradiation or the failure of the observer to note the irradiation which actually existed. There seems to be reason to believe that the last was the case in all the French observations, which are silent on the subject of the distortion described by observers in nearly all other regions.

The data for estimating the irradiation in the case of Contact II are exhibited in the following table:

The first column of numbers are integers, approximately one-sixth of the refraction in minutes. The next column shows the corresponding co-efficient for the telescope, according to the rule already enunciated. The amount of the irradiation, so far as it can be determined from pre-existing conditions, would be the sum of the squares of the numbers in these two columns.

The next column gives the actual amount of irradiation, as inferred from the observed interval between the geometric phase and the formation of the thread of light. This last datum and the square root of the sum of the squares of the preceding numbers afford two independent determinations of the same quantity, of which, on general principles, we might take the mean were all the elements complete. But in many cases the observer only noted a single phase, and no data exist for determining what the differences would have been had it been noted. It would clearly be wrong to assume in this case that the difference was evanescent. At Greenwich the two phases were carefully noted by four observers, mostly with good telescopes, with a mean difference of 39s. Four good differences were also noted in other parts of England, with a mean difference of 12s. I have assumed the irradiation indicated by the mean of these results to be applicable to all the observations made in England, on the ground that the atmospheric conditions were probably nearly uniform.

The fourth column of numbers in the tables gives the co-efficients as finally adopted.

Table of estimated numbers expressive of the irradiation at first interior contact.

1761, II.				
Station.	$\frac{1}{6}$ refraction.	Telescope.	Observed phase.	Adopted.
Pekin	0	(?)	. . . .	1
Calcutta	0	(?)	. . . .	1
Madras	0	1	. . . .	1
Tobolsk	0	0	. . . .	0
Cajaneborg	1	0	. . . .	1
Abo	1	0	. . . .	2
Stockholm	2	0	. . . .	3
Hernosand	2	0	4	3
Upsala	2	0	4	3
Calmar	4	0	2	3
1769, II.				
Jakutsk	1	0	. . . .	2
Ponoi	3	0	. . . .	3
Kola	2	1	2	3
Wardhus	1	0	1	2
Cajaneborg	3	0	(?)	3
North Cape	1	1	5	5
Wanhallinna	4	0	. . . .	4
Do	4	2	. . . .	4
Stockholm	4	0	2	4
Hernosand	2	0	5	4
Upsala	3	0	4	4
Lund	4	0	. . . .	4
Greifswald	5	0	. . . .	5
Saron	4	0	. . . .	4
Paris (College)	3	0	. . . .	3
Colombes	3	1	. . . .	3
St. Hubert	2	0	. . . .	3
Toulouse	5	2	. . . .	5
Rouen	2	2	. . . .	3
Havre	2	2	. . . .	3

1769, II—Continued.				
Station.	$\frac{1}{6}$ refraction.	Telescope.	Observed phase.	Adopted.
Greenwich	2	1	3	3
London (Middle Tower)	2	0	. . . .	2
London (Spital Square)	2	0	. . . .	2
London (Austin Friars)	2	1	1	3
Kew	2	0	1	3
Caen (Mission)	2	1	. . . .	4
La Trompette	2	. . .	. . . .	3
Bordeaux	3	. . .	. . . .	3
Windsor	1	1	. . . .	2
Shirburn Castle	1	0	. . . .	2
Leicester	1	0	. . . .	2
Oxford	1	0	3	2
Hawkhill	1	1	2	2
Kirknewton	1	1	. . . .	2
Glasgow	1	1	5	4
Brest	1	0	. . . .	2
Gibraltar	4	1	. . . .	5
Cadiz	3	2	. . . .	4
Cavan	1	1	4	3
Agromonte	1	2	. . . .	3
Isle Coudre	0	1	3	2
Newbury	0	0	0	1
Cambridge	0	1	0	1
Providence	0	0	. . . .	1
Cape Haïtien	0	1	. . . .	0
Baskenridge	0	. . .	. . . .	. . . .
Lewes	0	1	1	1
Philadelphia	0	1	0	1
Norristown	0	1	1	1
Wilmington	0	1	0	1
Hudson's Bay	0	1	. . . .	2
San José	0	1	. . . .	1
Tahiti	0	1	5	1

Similar co-efficients will be needed for all the other phases. In the case of Phase I, as already remarked, the planet must have impinged on the Sun by a certain amount before becoming visible in the telescope. Moreover, this amount will be greater the greater the imperfections of vision, whether these imperfections arise from deficient telescopic power or other circumstances. In Contact III the more powerful the telescope the longer the observer will perceive the thread of light as it is about to vanish.

In Contact IV the more powerful the telescope and the better the vision the later the time at which the observer will lose sight of the planet.

Hence, for each contact we must have a series of co-efficients, obtained on the same system as that on which we have derived co-efficients for Contact II.

The co-efficients in question are employed in the following way: Let us represent by the symbol c the amount by which Venus must impinge upon the Sun in order to be visible under the best conditions, that is, under the conditions when the co-efficient in question is zero. Let $c+k$ represent the thickness of the thread of light corresponding to the co-efficient unity; then $c+2k$ will represent the thickness for co-efficient 2, $c+3k$ for co-efficient 3, etc. This system may be applied through all the contacts.

The co-efficient k will have a special value for each phase. We may, however, reach *a priori* certain probable conclusions respecting the relations of some of these values. In the first place, it is evident that k ought not to differ much for Contact I and for Contact IV. This is not the case, however, with Contacts II and III. In the case of the latter the observer, watching a gradual disappearance of the thread of light, will be likely to follow it, or to think he sees it, as long as any light is there. It may therefore be inferred that for this phase k should be very nearly zero.

As a matter of fact, in preparing to solve the equations, I at first omitted k in the case of Contact III. Afterward I concluded to retain it for the sake of uniformity.

Another consideration is, that the effect of the imperfections of vision on which k depends will always lead to the actual distance of centers of the Sun and Venus being less than the sum or difference of their radii at the observed moment of contact. In the manner in which the equations of condition are afterward formed this should lead to the actual value of k being always positive.

Each contact having its own k, I have distinguished the four values by k_1, k_2, k_3, k_4, corresponding to the Contacts I, II, III, and IV, respectively.

7. *Consideration of the weights of the observations in detail.*—We shall next go over the observations in regular order, deriving the residual correction for each station and assigning a weight to it. The result of this examination is given in connection with the equations of condition. The general system followed in assigning weights is this:

A single unexceptionable observation by a single observer receives the weight unity. Such an observation is one belonging either to Class I or Class II, not without the limits of normal error, and not attended with any circumstances which would throw serious doubt upon the accuracy of the results.

To observations of Class III, in which there is no doubt about the phase, I have assigned the weight 0.7. An examination of the residuals shows that observations of this class are a little less accurate than those of Class I or II.

When an observation of Class III was made in remote regions by unknown observers, I have reduced its weight to 0.5, 0.4, or 0.3, according to the number and force of the doubtful circumstances.

The most difficulty arises in the cases of observations without the limits of normal probability. It must be noted, however, that these limits are wider in the case of poor observations than in those of good ones. The average limits within which diminished weight has been given to such observations, without entirely rejecting them, have already been stated, and it will be shown in the following examination what I have done in each case.

To observations of Class IV I have assigned, as a general rule, the weight 0.2, unless the conditions or results are such that the observation was evidently worthless.

In combining observations at the same station the weights are assigned on the same system; but the combined weight of the whole is not equal to the sum of the weights, because, as already remarked, a very slight examination of the residuals shows that such observations are, as a rule, affected by like errors.

1761, I. To Madras I assign the weight $\frac{1}{2}$, owing to doubts already set forth, and to Upsala $\frac{1}{2}$ on account of discordance.

1761, II. Pekin: By the general rule of formulating, the weight is 0.7.

Calcutta and Madras: The observations may be regarded as two grades below the standard; hence, weight 0.2.

Tobolsk and Cajaneborg: Unexceptional.

Tornea: The clock has been found to be probably fast; it seems best to reject the results.

Abo: The residuals, though not without the limits of possible normal error, lead to a suspicion that geometric contacts were observed. On the question whether the observations should receive full weight, half weight, or no weight at all, my judgment has vacillated with the result that the treatments of the two contacts in the final solution have not been entirely consistent with each other. II has been rejected and III admitted with half weight.

Stockholm: To the mean of the two good observations I assigned the weight 1.5.

Hernosand: A rough examination of all the residuals of II shows that, on the average, observations of the thread of light which are, from the descriptions of the observers, suspected to be a little late, and therefore marked L_2, are too late by about 12^s. Subtracting this from GISSLER'S observation gives $+22^s$. Owing to the uncertainty of this correction and the magnitude of the black-drop, I reduced the weight from 0.7 to 0.5.

Upsala: Correcting MELANDER'S probably late observation by -12^s, the mean of the four good observations is $+7^s$.

Calmar: WICKSTRÖM'S observation seems unexceptional.

1761, III. Selinginsk: The positive residuals of RUMOVSKY'S observation of Contact IV renders it almost certain that either his time or the adopted longitude is in error. I therefore reduce the weight of his observation by two grades, making it 0.2.

1761, III. Cajaneborg: I do not know on what ground ENCKE considers that PLANMANN's printed time of this contact, 8 minutes, should be changed to 7 minutes. I should, however, accept the change without hesitation were it not for the observation of FROSTERUS. The latter is described only as made with much doubt 40 seconds before PLANMANN's time. It can not therefore be inferred whether it is or is not affected by the supposed error in PLANMANN's time. If we subtract 1 minute from both observations the residual of that of FROSTERUS will be -34^s. In view of PLANMANN's precision in description and certainty of the accuracy of his observation, I am inclined to think that, contrary to the general rule adopted in other cases, the correction of -1^m should be accepted. The observation of FROSTERUS has to be entirely rejected.

Abo: JUSTANDER's result would, considered alone, be entitled to weight 0.7, but as both contacts deviate unusually (*vide supra*), under the adopted rule I shall reduce this to 0.4.

Stockholm: WILKINS's observation receives the weight 0.2.

Hernosand: A careful reading of the record shows that only GISSLER's second time should be used.

Upsala: I take the mean of STRÖMER's two times and combine them with those of the other two observers.

Calmar: Owing to the great distortion described by WICKSTRÖM I give his observation only half weight.

Lund and Landscrona: Following the system already adopted I shall give each of these somewhat doubtful observations the weight 0.2.

Munich: The smallness of the residual at Contact IV leads to a strong suspicion that the clock was too fast. With so small a telescope the observations of Contacts III and IV should have both been early rather than late. The observation is therefore to be thrown aside.

Ingolstadt: I assign the same weight to KRATZ's observation as to the two others, thus making the combined weight 1.5.

Bologna: The six observations are made with instruments of such different qualities that they have been separated into three classes, corresponding to the co-efficient of k. ZANOTTI's observation, though made with an insufficient instrument, is included among those for which k has the largest co-efficient.

Dillingen: Although the residual is not without the limit of possible normal error, I think the weight should be diminished on account of discordance, and therefore assigned it 0.4.

Montpellier: From the manner in which these observations are given I doubt if their combination is worth more than a single one of the best class, and shall therefore assign the mean result the weight unity.

Paris and its neighborhood: The observations have been combined on the same system as the preceding ones.

1761, III. Greenwich: The accordance of the three observations is said to have arisen from Bliss giving an exclamation whereby his time became known to the other observers, each of whom immediately adopted it as his own time. We may infer from this that they noticed no marked error in Bliss's time, to which we may assign the weight of one good observation.

St. John's: Owing to the low altitude of the Sun at this station we reduce the weight from unity to 0.7.

1769, I. The observations of this contact seem to support the view I have elsewhere set forth, that they are worthy of discussion, provided that the observer really catches the first visible entry of the planet upon the Sun. As already stated, I have included in the list all observations, except those which the descriptions of the observers themselves lead us to believe were late, and I have classed under III those in which there was no specific statement respecting the character of the observation. An examination of the column of residuals shows the following circumstances: All the residuals, except seven, are less than 70^s; the interval between 70^s and 92^s contains no residuals whatever; the seven residuals greater than 92^s all belong to Class III, except Strömer's, at Upsala, which, from the description of the observer, there is reason to suspect was late. The interval of 22^s between the two groups leads us to cut off the seven large residuals as abnormal.

1769, II. Ponoi: Mallet's residual is outside the limit of normal error and the longitude of his station uncertain. Altogether, it seems that his result should receive the weight of 0.3.

Kola: It will be seen that Rumovsky recorded two times, differing by 23^s, the second of which agrees well with that of his assistant. From the descriptions it may be inferred that Rumovsky was able to see the Sun, through flying clouds, up to his recorded time, when he thought contact was just formed. Then the Sun was completely covered for some 20^s, and on re-appearing the fine thread of light was distinctly seen. It would seem, then, that the actual phenomenon of contact must have taken place somewhere between these two limits of time. The difference of each extreme from the mean being only 12^s, I conclude that the mean of the two times will be a result equivalent in weight to a good observation.

Wardhus: In combining these observations I shall give the mean of Borgrewing and Sajnovics the same weight as Hell's observation of the thread of light.

North Cape and Wanhalinna: Results rejected on account of discordance. Gadolin and Justander make no statement whatever respecting their time, how determined, whether clock, mean, or apparent; whether that of the town or of the hill on which they observed.

1769, II. Upsala: The first observation of the thread of light made by SALENIUS must have been affected by some mistake, while the second appears from the description to have been clearly too late. Independent of all hypothesis the mean of the two seems to be the most probable result, and I shall combine this mean with the four other observations of the thread of light, giving it half weight.

Greifswald: I do not know ENCKE's authority for ROHL's observation. From the figure given by MAYER in the *Philosophical Transactions* it may be inferred that the observations are entitled to but small weight.

Paris: It appears from a statement made by some French astronomer that all the observations made by the Paris astronomers referred to the formation of the thread of light. I can not give a reference.

Greenwich: There is reason to suppose that the times noted at the Greenwich observatory were quite independent of each other. In combining them I shall assume that the observations of DUNN and DOLLOND, which are not distinctly described, belong to the thread of light.

Caen: We must consider that all these observations are affected by some undiscoverable source of error.

Oxford: I assume that the observations of NIKITIN and WILLIAMSON, which are not specifically described, refer to the thread of light.

Brest: It is difficult to say how far these observations are really independent. In view of the suspicion that they are not, the mean of the four seems entitled to little more than the weight unity.

Gibraltar: From the way in which these observations are made and published, it does not seem that they are entitled to consideration.

Cadiz, Agromonte, and the Isle of Coudre: These observations may be put into the lowest admissible class: the first two on account of the improbability that the observers had the means of accurately determining their time; the last on account of the blundering way in which the observer seems to have done his work.

Cape Haïtien: The early moment at which the first two contacts were noted would lead to the suspicion that either the clock correction or the longitude was in error. If the description given by PINGRÉ is correct there does not appear any possibility of the former, while the latter is placed beyond doubt. We must therefore accept the results with their full weight.

Lewes, Philadelphia, and Norristown: I have already referred to the singular community of errors affecting these seemingly best observations. The likeliest explanation seems to be that the observers being all within hearing of one another were led to adopt a nearly common time, and that each mean is entitled to the weight of a single good observation. In the case of Norristown I have rejected the observation of RITTENHOUSE and adopted the mean of the first observations of the thread of light by LUKEN and SMITH.

1769, II. San José: I suppose the mean of the times given by the other two astronomers to be entitled to the same weight as that of CHAPPE.

Tahiti: It seems best to take for SOLANDER's result the mean of his two recorded times. For uniformity I shall apply the same correction to the seemingly late observations of GREEN and COOK as in the other cases (correction of $L_3 = -12^s$); and, owing to the confused manner in which the phenomena are described, I shall assume that each observer is entitled only to a weight of one-third and the mean of the three to the weight unity.

1769, III and IV. The only observations that call for additional consideration are those of MOHR at Batavia. I have already pointed out the grounds for suspecting an error in the time-determination, but have vacillated in deciding whether the observation should be rejected for that reason. Thus, the equations have been solved both with and without Batavia, and both sets of results are given.

8. *Formation of the equations of condition.*—We have now all the data for forming the equation of condition from each observation. The unknown quantities are:

I. δc, a symbol which, were there no imperfections of vision, and were the radii of the Sun and Venus accurately known, would represent the correction to the tabular distance of centers. As a matter of fact, however, δc is a rather complex combination of quantities, including the sum or difference of errors of the radii of the Sun and Venus, the correction to the distance of centers, and in the case of internal contacts the thickness of the thread of light under conditions where the co-efficient of k is zero, while in the case of external contact the corresponding quantity is the amount by which Venus impinges upon the Sun when first or last seen under the best conditions. We may therefore suppose δc to be of the form

$$\delta c = \delta c_0 + \delta R + \delta R' + k_0$$

II. The correction to the adopted solar parallax, 8″.848, which correction I have represented by $2\pi'$ for convenience in solving the equation of condition.

III. The co-efficient k already defined.

In the following exhibit the first column gives the correction to the tabular time of contact as derived from all the observations at each separate station, except when there were marked differences of telescopic power, in which case the observations are classified according to the co-efficient of k.

The next column gives the change of c_1, the heliocentric distance of centers, in one minute of mean time.

The next column is one-sixtieth the product of the first two, and is therefore the correction to the distance of centers given by the observation itself.

In studying these numbers it is to be especially noted that all quantities given in seconds of arc are heliocentric and not geocentric; that is to say, the errors are such as would be apparent to an observer on the Sun.

9. *Formulæ for the co-efficients of the equations.*—The only elements which can be corrected by the observations are the solar parallax and the longitude and latitude of Venus relative to those of the Earth. From the equations already cited

$$c_1 \cos \omega_1 = l - l' + p$$
$$c_1 \sin \omega_1 = b' - b + q$$

we derive, putting $\delta l' = 0$, $\delta b' = 0$

$$\delta c_1 = \cos \omega_1 (\delta l + \delta p) + \sin \omega_1 (- \delta b + \delta q)$$

From the values of p and q (page 347) we find

$$\frac{dp}{d\Pi} = \frac{p}{\Pi}; \quad \frac{dq}{d\Pi} = \frac{q}{\Pi}$$

We therefore have the following expressions for the partial derivatives of c_1

$$\frac{dc_1}{dl} = \cos \omega_1$$

$$\frac{dc_1}{db} = - \sin \omega_1$$

$$\frac{dc_1}{d\pi_0} = \frac{p \cos \omega_1 + q \sin \omega_1}{\pi_0}$$

π_0 being the Sun's mean equatorial horizontal parallax.

Using these co-efficients the unknown quantities in the equations of condition would be

$$\delta c_1; \ \delta l; \ \delta b; \ \delta \pi_0$$

the first being a function of the corrections to the adopted radii of the Sun and Venus. But the derivatives $\frac{dc_1}{dl}$ and $\frac{dc_1}{db}$ differ so little in different observations of the same contact that we can not derive reliable independent values of δl and δb. We must therefore replace these two quantities by a single linear combination of them. The following are the greatest and least values of $\sin \omega_1$ and $\cos \omega_1$, corresponding to at least three good observations of each contact:

	$\sin \omega_1$			$\cos \omega_1$		
	Greatest.	Least.	Difference.	Least.	Greatest.	Difference.
1761, II	+0.527	+0.510	0.017	—0.850	—0.860	0.010
1761, III	+0.748	+0.719	.029	+0.664	+0.695	.031
1761, IV	+0.713	+0.695	.018	+0.702	+0.719	.017
1769, I	—0.738	—0.719	.019	—0.675	—0.695	.020
1769, II	—0.776	—0.750	.026	—0.630	—0.661	.031
1769, III	—0.567	—0.522	.045	+0.824	+0.853	.029
1769, IV	—0.507	—0.479	.028	+0.862	+0.878	.016

With the possible exception of 1769, II, where the error may amount to $\pm 0''.40$, it may be safely assumed that none of the values of δl or δb can exceed $0''.30$. The differences between the values of $\cos \omega_1 \delta l$ and $\sin \omega_1 \delta b$ for any one contact can therefore scarcely amount to $0''.01$ between the extreme stations, and the differences from the mean value will be yet smaller. We may therefore use a single function of δl and δb for all the observations of each contact. The following values of these functions may be adopted:

		ω_1			
1761,	I	151.0;	$\delta c_1 = -0.87\delta l$	$-0.48\delta b$	$+\delta\rho_1$
	II	148.4;	-0.85	-0.52	$+\delta\rho_2$
	III	47.7;	$+0.67$	-0.74	$+\delta\rho_3$
	IV	44.8;	$+0.71$	-0.71	$+\delta\rho_4$
1769,	I	226.1;	-0.69	$+0.72$	$+\delta\rho_1$
	II	229.2;	-0.65	$+0.76$	$+\delta\rho_2$
	III	327.4;	$+0.84$	$+0.54$	$+\delta\rho_3$
	IV	330.5;	$+0.87$	$+0.49$	$+\delta\rho_4$

Here $\delta\rho$ is put for the correction to the sum or difference of semi-diameters, together with the systematic error of phase, to be more fully considered hereafter.

Equations of condition from the eight observed contacts.

1761, I.

Station.	δt	$\frac{dc}{dt}$	Equations of condition.		Weight.
	s.	$''$	$''$		
Madras	+ 40	−1.28	+0.85 = $\delta c' + 1k_1$		0.5
Cajaneborg	+ 46	−1.26	+0.97	+1	1.0
Stockholm	+ 60	−1.26	+1.26	+4	1.0
Upsala	+115	−1.26	+2.41	+4	0.5

$(\delta c' = \delta c_1 + 0.72\delta\pi_0)$

$$\text{Normal equations}\begin{cases} 3.0\delta c' + \ \ 7.5k_1 = + \ \ 3.86'' \\ 7.5 \qquad + 25.5 \ \ \ = + 11.25 \end{cases}$$

$$\text{Solution}\begin{cases} \delta c' = + 0.69''. & \text{Weight} = 0.8 \\ k_1 = + 0.23. & \text{Weight} = 6.8 \end{cases}$$

1761, II.

	s.				
Pekin	− 14	−1.23	−0.29 = $\delta c' + 1k_2$		0.6
Calcutta	+ 15	−1.23	+0.31	+1	0.2
Madras	− 49	−1.24	−1.01	+1	0.2
Tobolsk	+ 4	−1.22	+0.08	0	1.0
Cajaneborg	+ 3	−1.21	+0.06	+1	1.0
Tornea	+ 27	−1.21	+0.54		0.0
Abo	− 31	−1.21	−0.62		0.0
Stockholm	− 6	−1.21	−0.12	+3	1.5
Hernosand	+ 22	−1.21	+0.44	+3	0.5
Upsala	+ 7	−1.21	+0.14	+3	2.5
Calmar	+ 11	−1.21	+0.22	+3	1.0

$(\delta c' = \delta c_1 + 0.74\delta\pi_0)$

$$\text{Normal equations}\begin{cases} 8.5\delta c' + 18.5k_2 = + 0.44'' \\ 18.5 \qquad + 51.5 \ \ \ = + 1.58 \end{cases}$$

$$\text{Solution}\begin{cases} \delta c' = - 0.071''. & \text{Weight} = \ \ 1.9 \\ k_2 = + 0.056. & \text{Weight} = 11.2 \end{cases}$$

***Equations of condition from the eight observed contacts*—Continued.**

1761, III.

Station.	δt	$\frac{dc}{dt}$	Equations of condition.			Weight.
	s.	"	"			
Pekin	−13	+1.24	$+0.27=\delta c_1$	$+1.5\pi'$	$+1k_3$	0.6
Selenginsk	+24	+1.23	−0.49	+1.5	+1	0.2
Calcutta	+38	+1.25	−0.79	+0.7	+2	0.2
Madras	−29	+1.25	+0.60	+0.3	+1	0.2
Tobolsk	+ 6	+1.23	−0.12	+1.1	0	1.0
Rodrigues	+ 8	+1.26	−0.17	−0.9	0	1.0
Cajaneborg	+ 6	+1.22	−0.12	+0.9	0	1.0
Tornea	+51	+1.22	. . . . . .			0
Abo	+27	+1.22	−0.55	+0.7	0	0.5
Cape Town	− 4	+1.26	+0.08	−1.8	+1	1.5
Stockholm	+12	+1.22	−0.24	+0.7	0	1.7
Hernosand	+18	+1.22	−0.37	+0.8	0	0.7
Upsala	− 4	+1.22	+0.08	+0.7	0	2.0
Tyrnau	− 7	+1.23	+0.14	+0.3	0	0.6
Calmar	+16	+1.22	−0.33	+0.6	0	0.5
Carlscrona	−22	+1.22	+0.45	+0.6	0	1.5
Wetzlas	+15	+1.23	−0.31	+0.3	0	1.0
Lund	−10	+1.22	+0.20	+0.5	0	0.2
Landscrona	+10	+1.22	−0.20	+0.6	+1	0.4
Rome	− 6	+1.23	+0.12	+0.1	+1	0.6
Leipzig	− 6	+1.22	+0.12	+0.4	+1	1.0
Munich	+26	+1.23	−0.53	+0.3	+2	0.2
Ingolstadt	+19	+1.23	−0.39	+0.3	+1	1.3
Bologna	+ 6	+1.23	−0.12	+0.1	0	0.7
Do	+ 6	+1.23	−0.12	+0.1	+1	1.5
Do	− 8	+1.23	+0.17	+0.1	+2	0.5
Florence	− 9	+1.23	+0.18	+0.1	0	0.7
Dillingen	−40	+1.23	+0.81	+0.3	0	0.4
Göttingen	0	+1.22	0.00	+0.4	0	1.0
Schwetzingen	+ 5	+1.22	−0.10	+0.3	0	1.0
Lyons	+ 1	+1.23	−0.02	+0.2	0	0.5
Leiden	+14	+1.22	−0.28	+0.3	0	1.0
Montpellier	−10	+1.23	+0.20	+0.1	+1	1.0
Beziers	+10	+1.23	−0.20	+0.1	+2	0.5
Vincennes	− 1	+1.23	+0.02	+0.3	+1	0.4
Conflans-Sous-Carrière	+13	+1.22	−0.26	+0.3	0	1.3
Paris (Hôtel de Clugny)	+ 1	+1.22	−0.02	+0.3	0	1.5
Paris (Coll. de Louis le Gd.)	+ 5	+1.22	−0.10	+0.3	0	1.0
Paris (St. Généviève)	+16	+1.22	−0.32	+0.3	0	0.7
Paris (Luxemburg)	− 1	+1.22	+0.02	+0.3	0	1.0
Paris (Royal Observatory)	+ 8	+1.22	−0.16	+0.3	+1	1.5
Paris (La Muette)	+ 3	+1.22	−0.06	+0.3	+1	0.7
St. Hubert	− 3	+1.22	+0.06	+0.3	0	1.1
Greenwich	+13	+1.22	−0.26	+0.4	+1	1.0

***Equations of condition from the eight observed contacts*—Continued.**

1761, III—Continued.

Station.	δt	$\frac{dc}{dt}$	Equations of condition.	Weight.
	s.	$''$	$''$	
Hackney	+10	+1.22	$-0.20 = \delta c_1 + 0.3\pi' + 1$	0.7
London (Spital Square) . .	+11	+1.22	$-0.22 \quad +0.4 \quad +1$	0.7
London (Clerkenwell) . .	+ 9	+1.22	$-0.18 \quad +0.4 \quad +1$	0.7
London (Savile House) . .	+ 6	+1.22	$-0.12 \quad +0.4 \quad +1$	1.0
Chelsea	− 2	+1.22	$+0.04 \quad +0.4 \quad 0$	1.0
St. John's	− 6	+1.20	$+0.12 \quad +0.5 \quad +2$	0.7

$$\text{Normal equations}\left\{\begin{array}{llll} 41.7\delta c_1 & + 13.02\pi' & + 19.2k_3 & = -2.81'' \\ 13.0 & + 15.69 & + 3.6 & = -1.30 \\ 19.2 & + 3.55 & + 23.4 & = -1.57 \end{array}\right.$$

$$\text{Solution}\left\{\begin{array}{ll} \delta c_1 = -0.042''. & \text{Weight} = 19.3 \\ \pi' = -0.041. & \text{Weight} = 11.2 \\ k_3 = -0.026. & \text{Weight} = 14.1 \\ \pi = 8.766. & \text{Weight} = 5.6 \end{array}\right.$$

***Equations of condition from the eight observed contacts*—Continued.**

1761, IV.

Station.	δt	$\frac{dc}{dt}$	Equations of condition.			Weight.
	s.	$''$	$''$			
Pekin	−30	+1.29	$+0.64=\delta c_1+1.6\pi'+1k_4$			0.7
Selenginsk	+5	+1.28	−0.11	+1.5	+1	0.2
Calcutta	−76	+1.30	+1.65	+0.8	+2	0.2
Madras	−126	+1.30	+2.73	+0.4	+1	0.2
Tobolsk	+6	+1.28	−0.13	+1.1	0	1.0
Cajaneborg	−4	+1.27	+0.08	+0.8	0	0.5
Abo	−25	+1.27	+0.53	+0.7	0	1.0
Cape Town	−30	+1.31	+0.66	−1.7	+1	1.5
Stockholm	−24	+1.27	+0.51	+0.6	0	2.0
Hernosand	−55	+1.27	+1.16	+0.7	0	0.0
Upsala	−18	+1.27	+0.38	+0.6	0	2.5
Tyrnau	−10	+1.28	+0.21	+0.3	0	0.7
Vienna	−4	+1.28	+0.08	+0.3	0	2.0
Do	−18	+1.28	+0.38	+0.3	+1	2.0
Do	−46	+1.28	+0.98	+0.3	+2	0.7
Calmar	−42	+1.27	+0.89	+0.5	0	0.8
Carlscrona	+22	....	......	......	..	0.
Wetzlas	−18	+1.28	+0.38	+0.3	0	1.0
Laibach	+33	+1.28	+0.70			0.0
Lund	−14	+1.27	+0.30	+0.5	0	0.7
Rome	−4	+1.28	+0.09	0.0	+1	0.7
Landscrona	−32	+1.27	+0.68	+0.6	0	0.2
Munich	−5	+1.28	+0.11	+0.2	+2	0.2
Ingolstadt	−7	+1.28	+0.15	+0.2	0	0.8
Do	−46	+1.28	+0.98	+0.2	+1	1.0
Bologna	−17	+1.28	+0.36	+0.1	0	1.0
Do	−28	+1.28	+0.60	+0.1	+1	2.0
Do	−42	+1.28	+0.90	+0.1	+2	1.0
Florence	−13	+1.28	+0.28	+0.1	0	0.7
Dillingen	−75	+1.28	+1.60			0.0
Göttingen	−7	+1.27	+0.15	+0.3	+2	1.0
Lyons	−22	+1.28	+0.47	+0.1	0	0.8
Montpellier	−26	+1.28	+0.56	0.0	+1	1.5
Beziers	−9	+1.28	+0.19	0.0	+1	0.5
Vincennes	−14	+1.27	+0.30	+0.2	0	0.8
Conflans-Sous-Carrière	−13	+1.27	+0.28	+0.2	0	1.0
Do	−33	+1.27	+0.70	+0.2	+1	0.7
Do	−8	+1.27	+0.17	+0.2	+2	0.5
Paris (Hôtel de Clugny)	−24	+1.27	+0.51	+0.2	0	1.3
Paris (Coll. de L. le Grand)	−4	+1.27	+0.08	+0.2	0	1.5
Paris (Luxemburg)	−12	+1.27	+0.25	+0.2	0	1.0
Paris (Observatory)	−8	+1.27	+0.17	+0.2	0	1.0
Do	−14	+1.27	+0.30	+0.2	+2	1.2
Paris (La Muette)	−22	+1.27	+0.47	+0.2	0	0.7

Equations of condition from the eight observed contacts—Continued.

1761, IV—Continued.

Station.	δt	$\frac{dt}{dc}$	Equations of condition.			Weight.
	s.	″	″			
Paris (La Muette)	− 21	+1.27	+0.45 = δc_1	+0.2π'	+1k_4	0.5
Paris (Ecole Militaire) . .	− 8	+1.27	+0.17	+0.2	0	0.5
St. Hubert	− 13	+1.27	+0.28	+0.2	0	1.0
Rouen	− 25	+1.27	+0.53	+0.2	0	0.7
Greenwich	− 15	+1.27	+0.32	+0.3	0	1.0
Hackney	− 10	+1.27	+0.21	+0.3	+1	1.0
London (Spital Square) . .	− 3	+1.27	+0.06	+0.3	+1	0.7
London (Savile House) . .	− 15	+1.27	+0.32	+0.3	+1	1.2
Chelsea	− 22	+1.27	+0.47	+0.3	0	1.0
Bayeux	+ 25	+1.27	−0.53			0.0
Shirburn Castle	− 2	+1.27	+0.04			0.0
Madrid	+ 43	+1.27	−0.91			0.0
Oporto	− 96	+1.28	+2.04			0.0
Lisbon	+ 8	+1.28	−0.17			0.0
St. John's	− 35	+1.27	+0.74	+0.3	+2	1.0

Normal equations
$$\left\{\begin{array}{llll} 47.4\delta c_1 & +12.30\pi' & +26.0k_4 & =+19.31'' \\ 12.3 & +11.65 & +\ 3.96 & =+\ 4.11 \\ 26.0 & +\ 3.96 & +37.6 & =+13.79 \end{array}\right.$$

Solution
$$\left\{\begin{array}{ll} \delta c_1 = +0.353. & \text{Weight} = 21.3 \\ \pi' = -0.065. & \text{Weight} = \ 8.2 \\ k_4 = +0.130. & \text{Weight} = 22.4 \\ \pi = \ \ 8.718''. & \text{Weight} = \ 4.1 \end{array}\right.$$

***Equations of condition from the eight observed contacts*—Continued.**

1769, I.

Station.	δt	$\frac{dc}{dt}$	Equations of condition.	Weight.
	s.	″	″	
Ponoi	+53	−1.27	$+1.12 = \delta c_1 - 1.8\pi' + 3k_1$	0.3
Kola	+58	−1.27	+1.23 −1.8 +3	1.5
Wardhus	+49	−1.27	+1.04 −1.8 +2	1.0
Stockholm	+59	−1.27	+1.25 −1.9 +4	1.2
Hernosand	+27	−1.27	+0.57 −1.9 +4	0.5
Upsala	+56	−1.27	+1.18 −2.0 +4	2.0
Lund	+50	−1.27	+1.06 −2.0 +4	1.5
Greifswald	+41	−1.27	+0.87 −2.0 +4	0.5
Greenwich	+28	−1.26	+0.59 −2.0 +2	4.0
London	+30	−1.26	+0.63 −2.0 +2	2.0
Kew	+39	−1.26	+0.82 −2.0 +2	1.0
Caen	+10	−1.26	+0.21 −2.0 +4	0.7
Windsor	+17	−1.26	+0.36 −2.0 +2	1.0
Shirburn Castle	+25	−1.26	+0.53 −2.0 +2	1.0
Leicester	+61	−1.26	+1.28 −2.0 +2	0.5
Oxford	+45	−1.26	+0.94 −2.0 +2	2.0
Glasgow	+51	−1.26	+1.07 −1.9 +3	2.0
Cavan	+47	−1.26	+0.99 −1.9 +3	1.0
Newbury	+20	−1.25	+0.42 −1.2 +1	1.0
Cambridge	+68	−1.25	+1.42 −1.2 +1	0.5
Cape Haltien	+ 2	−1.25	+0.04 −0.7 0	2.5
Baskenridge	+27	−1.25	+0.56 −1.1 +1	0.5
Lewes	+47	−1.25	+0.98 −1.0 +1	1.0
Philadelphia	+35	−1.25	+0.73 −1.0 +1	3.0
Norristown	+31	−1.25	+0.65 −1.1 0	2.5
Hudson's Bay	+ 4	−1.26	+0.08 −1.1 +1	1.5
San José	+37	−1.25	+0.77 0.0 +1	2.0
Tahiti	+39	−1.24	+0.81 +1.6 0	2.0

$$\text{Normal equations}\left\{\begin{array}{llll} 40.28\delta c_1 - & 55.7\pi' + & 74.5k_1 = & +30.10'' \\ -55.7 & +108.8 & -134.9 & = -44.12 \\ 74.5 & -134.9 & +205.1 & = +65.23 \end{array}\right.$$

$$\text{Solution}\left\{\begin{array}{ll} \delta c_1 = +0.55''. & \text{Weight} = 10.9 \\ \pi' = +0.124. & \text{Weight} = 16.6 \\ k_1 = +0.200. & \text{Weight} = 35.2 \\ \pi = 9.096. & \text{Weight} = 8.4 \end{array}\right.$$

***Equations of condition from the eight observed contacts*—Continued.**

1769, II.

Station.	δt	$\frac{dc}{dt}$	Equations of condition.			Weight.
	s.	$''$	$''$			
Ponoi	+33	—1.22	+0.67 = δc_1	—1.8π'	+3k_3	0.2
Kola	+ 4	—1.22	+0.08	—1.8	+3	1.0
Wardhus	—26	—1.22	—0.53	—1.8	+2	1.5
Cajaneborg	+ 4	—1.22	+0.08	—1.6	+3	1.0
Stockholm	— 1	—1.22	—0.02	—2.0	+4	1.3
Hernosand	—15	—1.22	—0.30	—1.9	+4	0.7
Upsala	+ 9	—1.22	+0.18	—2.0	+4	1.5
Lund	— 8	—1.22	—0.16	—2.0	+4	1.0
Greifswald	— 9	—1.22	—0.18	—1.9	+5	0.5
Saron	—20	—1.21	—0.40	—2.0	+4	0.5
Paris (Coll. de L. le Grand)	0	—1.21	0.00	—2.0	+3	1.5
Paris (Observatory)	+ 6	—1.21	+0.12	—2.0	+3	1.5
Colombes	—11	—1.21	—0.22	—2.0	+3	0.5
Toulouse	+ 5	—1.21	+0.10	—2.0	+5	0.5
Rouen	— 4	—1.21	—0.08	—2.0	+3	0.8
Greenwich	— 2	—1.22	—0.04	—2.0	+3	3.5
London (Middle Temple)	—20	—1.22	—0.40	—2.0	+2	0.2
London (Spital Square)	— 9	—1.22	—0.18	—2.0	+2	1.0
London (Austin Friars)	+ 2	—1.22	+0.04	—2.0	+3	0.7
Kew	+ 6	—1.22	+0.12	—2.0	+3	0.7
La Trompette	+ 2	—1.21	+0.04	—2.0	+3	0.5
Bordeaux	— 5	—1.21	—0.10	—2.0	+3	0.5
Windsor	—27	—1.22	—0.55	—2.0	+2	0.8
Shirburn Castle	— 2	—1.22	—0.04	—2.0	+2	1.0
Leicester	+20	—1.22	+0.41	—2.0	+2	0.7
Oxford	— 5	—1.22	—0.10	—2.0	+2	3.0
Hawkhill	+ 2	—1.22	+0.04	—2.0	+2	1.0
Glasgow	— 9	—1.22	—0.18	—2.0	+4	1.0
Brest	+23	—1.21	+0.47	—2.0	+2	1.2
Cadiz	—28	—1.20	—0.56	—1.9	+4	0.2
Cavan	+ 6	—1.22	+0.12	—2.0	+3	0.7
Agromonte	—34	—1.20	—0.68	—1.9	+3	0.2
Isle Coudre	+39	—1.20	+0.78	—1.3	+2	0.2
Newbury	0	—1.20	0.00	—1.2	+1	1.0
Cambridge	—11	—1.20	—0.22	—1.2	+1	1.0
Providence	— 1	—1.20	—0.02	—1.2	+1	0.7
Cape Haïtien	—24	—1.19	—0.48	—0.7	0	2.0
Baskenridge	— 7	—1.20	—0.14	—1.1	+1	0.5
Lewes	+ 3	—1.20	+0.06	—1.1	+1	1.0
Philadelphia	—30	—1.20	—0.60	—1.1	+1	1.0
Norristown	—54	—1.20	—1.08	—1.1	+1	1.0
Wilmington	— 4	—1.20	—0.08	—1.1	+1	0.5
Hudson's Bay	—22	—1.21	—0.44	—1.2	+2	1.5
San José	— 3	—1.19	—0.06	—0.1	+1	1.5
Tahiti	—34	—1.18	—0.67	+1.5	+1	2.0

Normal equations
$$\begin{cases} 44.8\delta c_1 - 69.0\pi' + 104.1k_3 = -6.28'' \\ -69.0 \quad +136.2 \quad -186.7 \quad = +4.15 \\ +104.1 \quad -186.7 \quad +302.5 \quad = -8.39 \end{cases}$$

Solution
$$\begin{cases} \delta c_1 = -0.459''. & \text{Weight} = 7.9 \\ \pi' = -0.153. & \text{Weight} = 18.4 \\ k_3 = +0.036. & \text{Weight} = 37.3 \\ \pi = 8.542. & \text{Weight} = 9.2 \end{cases}$$

Equations of condition from the eight observed contacts—Continued.

1769, III.

Station.	δt	$\frac{dc}{dt}$	Equations of condition.			Weight.
	s.	$''$	$''$			
Manilla	−13	+1.20	+0.26 = δc_1	−1.0π'	+2k_3	0.3
Pekin	− 4	+1.22	+0.08	−1.2	0	1.0
Batavia	+24	+1.19	−0.48	−1.1	0	{ 0.0 (?) 1.0 (?)
Orsk	−33	+1.22	+0.67	−1.8	+1	0.7
Orenburg	+ 1	+1.22	−0.02	−1.7	+1	0.7
Gurief	+ 4	+1.22	−0.08	−1.8	+3	1.0
Kola	+ 7	+1.23	−0.14	−1.3	+2	1.2
Wardhus	− 6	+1.22	+0.12	−1.2	+2	2.0
Petersburg	+44	+1.23	−0.86			0.0
Hudson's Bay	−21	+1.22	+0.42	+0.2	+1	1.5
San José	+ 7	+1.20	−0.14	+1.3	+1	1.7
Tahiti	+11	+1.17	−0.22	+1.7	+1	1.5

Normal equations (including Batavia)

$$\left\{\begin{array}{rrrl} 12.6\delta c_1 & -\ 5.75\pi' & +\ 16.1k_3 & = +0.19'' \\ -\ 5.75 & +\ 22.65 & -\ 11.3 & = -1.15 \\ +\ 16.1 & -\ 11.31 & +\ 29.1 & = +0.58 \end{array}\right.$$

Solution

$$\left\{\begin{array}{ll} \delta c_1 = -0.031''. & \text{Weight} = \ \ 3.6 \\ \pi' = -0.050. & \text{Weight} = 18.2 \\ k_3 = +0.018. & \text{Weight} = \ \ 7.7 \\ \pi = \ \ 8.748. & \text{Weight} = \ \ 9.1 \end{array}\right.$$

Normal equations (omitting Batavia)

$$\left\{\begin{array}{rrrl} 11.6\delta c_1 & -\ 4.65\pi' & +\ 16.1k_3 & = +0.67'' \\ -\ 4.65 & +\ 21.44 & -\ 11.3 & = -1.68 \\ +\ 16.1 & -\ 11.31 & +\ 29.1 & = +0.58 \end{array}\right.$$

Solution

$$\left\{\begin{array}{ll} \delta c_1 = +0.196''. & \text{Weight} = \ \ 2.5 \\ \pi' = -0.104. & \text{Weight} = 16.0 \\ k_3 = -0.126. & \text{Weight} = \ \ 5.5 \\ \pi = \ \ 8.640. & \text{Weight} = \ \ 8.0 \end{array}\right.$$

***Equations of condition from the eight observed contacts*—Continued.**

1769, IV.

Station.	δt	$\frac{dc}{dt}$	Equations of condition.	Weight.
	s.	$''$	$''$	
Jakutsk	−33	+1.27	$+0.70 = \delta c_1 - 0.8\pi' + 0k_4$	1.0
Manilla	−72	+1.24	$+1.49 \quad -0.9 \quad +2$	0.6
Pekin	− 7	+1.24	$+0.14 \quad -1.2 \quad 0$	1.5
Batavia	− 7	+1.24	$+0.14 \quad -1.1 \quad 0$	{0.0 1.0
Orsk	−48	+1.26	$+1.00 \quad -1.7 \quad +1$	1.0
Orenburg	−18	+1.27	$+0.38 \quad -1.7 \quad +1$	1.0
Gurief	−46	+1.27	$+0.97 \quad -1.8 \quad +2$	1.0
Wardhus	−43	+1.27	$+0.91 \quad -1.2 \quad +2$	2.0
Petersburg	−40	+1.27	$+0.85$	0.0
Cajaneborg	−48	+1.28	$+1.02 \quad -1.4 \quad +2$	1.3
Hudson's Bay	−54	+1.26	$+1.13 \quad +0.3 \quad +2$	1.5
San José	−29	+1.25	$+0.60 \quad +1.4 \quad +3$	1.0
Do	−65	+1.25	$+1.36 \quad +1.4 \quad +3$	1.0
Tahiti	−62	+1.22	$+1.26 \quad +1.8 \quad +1$	1.5

Normal equations (including Batavia)

$$\left\{\begin{array}{llll} 15.4\delta c_1 - & 7.71\pi' + & 22.3k_4 = & +12.99'' \\ -\ 7.71 & +27.86 & -\ 4.5 & =-\ 3.25 \\ +22.3 & -\ 4.5 & +47.1 & =+22.57 \end{array}\right.$$

Solution

$$\left\{\begin{array}{ll} \delta c_1 = +0.56''. & \text{Weight} = 3.7 \\ \pi' = +0.074. & \text{Weight} = 21.0 \\ k_4 = +0.221. & \text{Weight} = 12.9 \\ \pi = 8.996. & \text{Weight} = 10.5 \end{array}\right.$$

Normal equations (omitting Batavia)

$$\left\{\begin{array}{llll} 14.4\delta c_1 - & 6.61\pi' + & 22.3k_4 = & +12.85'' \\ -\ 6.61 & +26.65 & -\ 4.5 & =-\ 3.10 \\ +22.3 & -\ 4.5 & +47.1 & =+22.57 \end{array}\right.$$

Solution

$$\left\{\begin{array}{ll} \delta c_1 = +0.65''. & \text{Weight} = 3.1 \\ \pi' = +0.076. & \text{Weight} = 21.0 \\ k_4 = +0.178. & \text{Weight} = 11.2 \\ \pi = 9.000. & \text{Weight} = 10.5 \end{array}\right.$$

Chapter VIII.

DISCUSSION OF RESULTS.

1. *Mean errors.*—The weights given with the preceding solution correspond to separate and independent probable errors in the case of each of the four contacts. I have determined the probable errors in each case, not with absolute rigor by the method of least squares, but by taking the square root of the mean of the squares of the residuals found by substituting approximate values of the unknown quantities. In the case of each contact I have only taken that transit in which the greater number of observations were made. The results for the mean error are as follows:

	″
Contact I (1769)	∓0.39
Contact II (1769)	∓0.26
Contact III (1761)	∓0.20
Contact IV (1761)	∓0.29

These results do not support the view which I have expressed on several previous occasions that external contacts are almost as good as internal ones, and that observations of first external contact are nearly as consistent as those of any other contact. The relative values of these mean errors are, I think, about what astronomers in general would have expected. Internal contacts are more accurate than external ones. Observations of Contact III are better than those of Contact II, because the gradual disappearance of the thread of light can be watched with greater precision than the observer can catch its first visible formation.

2. *Values of the co-efficients* k_1, k_2, k_3, *and* k_4.—Since k_1 and k_4 are substantially the same, being expressive of the effect of imperfections of vision upon the visibility of the external contact, we may assume them to be equal. The following are the values of k_1 and k_4, with their probable errors:

		″	″	Weight.
1761, I	$k_1 =$	$+0.23$	± 0.15	2
1769, I	$k_1 =$	$+0.20$	± 0.064	10
1761, IV	$k_4 =$	$+0.13$	± 0.061	11
1769, IV	$k_4 =$	$+0.20$	± 0.083	6
Mean	$k_1 = k_4 =$	$+0.176$	± 0.037	29

The difference between these values of k are no greater than should be expected from their probable errors.

For k_2 we have

		Weight.
1761, II	$k_2 = +0''.056 \pm 0''.08$	1
1769, II	$k_2 = +0.036 \pm 0.04$	4
Mean	$k_2 = +0.040 \pm 0.036$	5

For k_3 we have

		Weight.
1761, III	$k_3 = -0''.026 \pm 0''.05$	13
1769, III	$k_3 = -0.054 \pm 0.08$	5
Mean	$k_3 = -0.034 \pm 0.04$	18

For reasons already given the value of k_3 should be positive and nearly evanescent. We may therefore regard it as zero.

Leaving out a very few of the worst cases, the values of the co-efficients of k in the equations of condition range from 0 to 4. The conclusions from the preceding values may therefore be stated as follows:

I. Under the worst ordinary conditions of vision Venus, at visible external contact, impinged heliocentrically 0''.7 further upon the Sun than it did under the best conditions, telescopic and atmospheric.

II. In the case of second contact the thread of light, when first seen, was heliocentrically 0''.16 thicker under the worst than under the best conditions

III. In the case of third contact no difference is shown in the thickness of the thread of light under different conditions when it vanished from sight.

3. *Results for the equatorial horizontal parallax of the Sun.*—The values of the solar parallax given with each of the preceding sets of normal equations are not to be regarded as the definitive values derived from the contact, because k was determined separately in each case. Having deduced definitive values of k, to be used in all cases, we must suppose these values inserted in the normal equations. The result is exhibited in the clearest manner, by finding the values of π in terms of k, by eliminating δc_1 from each set of normal equations. The results are as follow. The probable errors first given are those derived from the co-efficients of π in each separate solution, and therefore do not include the effect of uncertainty in k. The definitive values of k being inserted in each expression, we have the value of the solar parallax derived from each contact, to which is appended its probable error, including the effect of that of k.

The adopted unit of weight corresponds to the mean error $\mp 0''.20$.

Results for solar parallax.

		Weight.
1761, III . . .	$\pi = 8''.78 + 0.41k_3 \pm 0''.12 = 8''.78 \pm 0''.12$	8
IV . . .	$8.63 + 0.66k_4 \pm 0.20 = 8.75 \pm 0.20$	3
1769, I . . .	$8.69 + 2.00k_1 \pm 0.16 = 9.04 \pm 0.17$	4
II . . .	$8.48 + 1.76k_2 \pm 0.12 = 8.55 \pm 0.13$	7
III . . .	$8.72 + 0.44k_3 \pm 0.09 = 8.72 \pm 0.09$	14
IV . . .	$9.10 - 0.52k_4 \pm 0.12 = 9.01 \pm 0.12$	8
		44

The weighted mean is

$$\pi = 8''.79 \pm 0''.051 \text{ or } \pm 0''.034$$

the first error being mean, the second probable.

Since k_1 or k_4 enters into three of the preceding results it is theoretically possible to determine the value of that quantity from the above equations. The two coefficients of k_3 are so nearly equal that no determination of this quantity can practically be made. It is, however, different in the case of k_1 and k_4. Representing by k simply the common value of these quantities, we have the following three equations for determining π and k:

$$\pi = 8''.63 + 0.66k. \quad \text{Weight} = 3$$
$$\pi = 8.69 + 2.00k. \quad \text{Weight} = 4$$
$$\pi = 9.10 - 0.52k. \quad \text{Weight} = 8$$

The solution of these equations gives

$$k = +0''.17$$
$$\pi = 8''.96$$

Combining these results with that from the three interior contacts the final value of the equatorial horizontal parallax of the Sun again comes out 8''.79. The two results are therefore identical.

4. *Probable errors.*—What is most to be feared in estimating the value of the preceding results is the introduction of systematic errors affecting in a common direction all the values of the solar parallax derived from any one contact. The method of treatment, especially the introduction of the indeterminate quantities k, will greatly diminish this constant error, since in the presence of k itself is to be found the principal source of systematic errors. It will be instructive to notice that by supposing all the k's equal to zero, the first column of figures in the preceding exhibit shows the resulting values of the solar parallax, which vary from 8''.48 to 9''.10. It will also

be noticed in this connection that the weight of 1769, II, is rather below the mean, though it was the result derived from the greatest number of observations. This arises from the fact that the great mass of observations made in Europe were made under similar conditions, and therefore do not serve to determine the solar parallax independently of k_2.

We have now to consider whether the general discordances among the results from each contact are greater than we should expect from the mean errors assigned to the results. Proceeding by the method of simple enumeration it will be seen that three of the results deviate from the mean by less than their respective mean errors and three by more. The one which deviates most in proportion to the mean error is 1769, IV, of which the deviation is nearly twice the mean error, and therefore nearly three times the probable error. I shall presently consider the question whether we should consider this deviation as abnormal. In the first place, we shall determine the mean error from the discordance among the six contacts. The square of each deviation multiplied by the weight is as follows:

		$''$
1761,	III	.0008
	IV	.0048
1769,	I	.2500
	II	.4032
	III	.0686
	IV	.3872
		1.1146

The resulting mean error corresponding to weight unity is $\mp 0''.47$, which is more than $0''.34$, the mean error corresponding to which the unit of weight was assumed. Moreover, it will be seen that this large result arises from the deviations of 1769, II and IV, since, assuming the correctness of the mean error $\pm 0''.34$, the probable value of each product $w\varepsilon^2$ should be $0''.0963$. We therefore meet the question, whether we are to regard the results in question as affected by abnormal systematic errors. An examination of the equations of condition seems to show that the deviations are purely fortuitous, since there is no reason to suppose that any systematic difference could have arisen from the methods or circumstances of observation. I hold that by a fortuitous combination of circumstances the actual errors of the parallax derived from the observations of these contacts happen to be twice as large as the probable errors to which they would be subject.

We are to look upon this result in the same light as we look upon the extraordinary uniformity and consistency among the values of k, where the deviations are less than the probable error. For these reasons I regard the results of 1769, II and IV, as completely normal, and I consider that the probable error obtained *a priori* is entitled to equal weight with that derived from the discordance of the results of the six contacts. Making all reasonable allowances I conceive that $\pm 0''.04$ is a well-balanced estimate of the absolute probable error of the final result and $\pm 0''.06$ of the mean error.

This opinion, it must be remembered, is founded solely on the fact that the largest source of systematic error has been eliminated by the introduction of quantity k into the equations of condition.

5. *Final results for the position of the node of Venus.*—The conditions under which the observations were made would enable us to obtain a very accurate value of the position of the node and of the heliocentric longitude of Venus were it certain that the corrections were the same in the case of the two transits.

When the discussion of the observations of Venus is so far advanced that we can determine the correction of its heliocentric longitude at the corresponding points of its orbit at some epoch near the present, we shall be able to determine the difference between the tabular errors in 1761 and 1769, and then reduce both errors to one standard.

I shall, however, provisionally determine the corrections on the supposition that they are the same for 1761 and 1769, using different symbols, so far as possible, in order to facilitate the final decision when the requisite data shall have been obtained. I use the notation

$x =$ correction of $(l - l')$;
$y =$ correction of $(b' - b)$;
$z =$ constant error peculiar to each contact arising from errors in the adopted semi-diameters and from thickness of the thread of light or impingement of Venus upon the Sun under the best conditions of vision.

We shall then have a separate value of z for each contact. But the value for Contact IV will probably be not very different from that for Contact I, and will be less by an amount of which we can form an approximate estimate. We shall therefore put

$$z_4 = pz_1$$

p being a fraction certainly less than unity, but probably greater than one-half. I shall also accent the quantities which refer to the transit of 1769.

We now have the following data and results from the eight observed contacts, all which are collected from the preceding paper.

	ω	sin ω	cos ω	Equations.
	°			$''$
1761, I	152 38	+0.460	−0.888	$+.46y - .89x + z_1 = +0.89$
II	149 57	+0.501	−0.866	$+.50 \quad -.87 \quad + z_2 \quad -0.07$
III	47 1	+0.732	+0.682	$+.73 \quad +.68 \quad + z_3 \quad -0.06$
IV	44 20	+0.699	+0.715	$+.70 \quad +.72 \quad + pz_1 \quad +0.32$
1769, I	227 12	−0.734	−0.679	$-.73y' - .68x' + z_1 = +0.39$
II	230 12	−0.768	−0.640	$-.77 \quad -.64 \quad + z_2 \quad -0.27$
III	326 47	−0.548	+0.837	$-.55 \quad +.84 \quad + z_3 \quad +0.02$
IV	329 47	−0.503	+0.864	$-.50 \quad +.86 \quad + pz_1 \quad +0.59$

$$1761, \text{ II and III give } x = +0''.02 + 0.75z_2 - 0.51z_3;$$
$$y = -0.10 - 0.70z_2 - 0.89z_3$$
$$1769, \text{ II and III give } x' = +0.16 + 0.55z_2 - 0.77z_3;$$
$$y' = +0.24 + 0.84z_2 + 0.64z_3$$

$$1761, \text{ I and IV give } x = -0''.49 + (0.7 - 0.5p)z_1;$$
$$y = +0.96 - (0.7 + 0.9p)z_1$$
$$1769, \text{ I and IV give } x' = +0.25 + (0.5 - 0.7p)z_1;$$
$$y' = -0.76 + (0.9 + 0.7p)z_1$$

The value of y, which I provisionally derive from these equations, assuming $y' = y$ is $+0''.085$. But a correction, $-2''.00$, has been applied to the value of that quantity from LE VERRIER's tables. We therefore conclude:

Correction to LE VERRIER's latitude of Venus at descending node for mean of 1761 and 1769 . . . . $+1''.915$

and hence, dividing by sin i

Correction to LE VERRIER's longitude of the node, 1765.5 . . $+32''.4$

The further discussion of the results must be postponed for incorporation with the theories of the motion of the Earth and Venus.

DISCUSSION OF THE NORTH POLAR DISTANCES

OBSERVED WITH

THE GREENWICH AND WASHINGTON TRANSIT CIRCLES

WITH

DETERMINATIONS OF THE CONSTANT OF NUTATION.

BY

SIMON NEWCOMB.

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CHAPTER I.

DISCUSSION OF THE NORTH POLAR DISTANCES OF THE GREENWICH TRANSIT CIRCLE, 1851-1887.

§ 1. *Introductory.*

The published observations made with the Greenwich transit circle now afford a mass of material which is unique in several points. The period covered by the observations, 1851-1887, includes substantially two revolutions of the Moon's node. During this period the instrument has remained nearly unchanged. A nearly uniform system of reduction and observation has also been kept up throughout the entire period without interruption or break. Especial interest now attaches to the question, what general conclusions respecting the declinations of the stars and planets and what values of fundamental constants are deducible from so long continued and uniform a series of observations.

Various discussions on this subject have appeared, in addition to those which are given in the annual volumes. As far back as 1861 Mr. A MARTH published an extended paper on the Greenwich declinations in the *Astronomische Nachrichten.** The principal result of this paper is a catalogue of the north polar distance of 239 stars, derived from the observations in question by the application of a number of minor corrections and the reduction of the results to uniform instrumental constants. The object aimed at by the author seems to have been a general discussion of the Greenwich method of determining declinations, as well as of other proposed methods, rather than a deduction of special definitive results from the observations. The views and suggestions which it contains seem to me of value, but do not especially pertain to the subject of the present paper.

In 1880 Mr. W H. M. CHRISTIE published a very exhaustive discussion of the corrections required by the declinations of the transit circle from 1851 to 1879 and of the two mural circles from 1836 to 1850.† The main object of this investigation is to determine what values of the latitude of the observatory and of the obliquity of the ecliptic would have been derived from the observations had uniform constants of reduction been applied. The paper also includes an extended discussion of the corrections to BESSEL'S tables of refraction, of the values of the flexure correction and of their relation to the discordance between observations made directly and by reflexion. The observations of circumpolar stars from 1857 to 1865 are shown to indicate a diminution of BESSEL'S constant of refraction; but the author remarks that this conclusion is not borne out by previous observations, and is contradicted by the results for previous years with the TROUGHTON circle. No discussion of the declinations of stars is given.

* Volume LIII, Nos. 1260-1263.
† Memoirs R. A. S., Vol. XLV, page 151.

When the work on the present paper was nearly completed Mr. W. S THACKERAY published a paper with a somewhat similar object.* The author sums up the results of his paper as follows:

1. The position of the thermometer used in the reductions of the observations of the Sun and Pole star affects the ecliptic investigation and the assumed value of the colatitude with a correction which is tabulated in the paper.

2. An annual variation in the observations of the Sun and Pole star appears to be independent of the position of the thermometer

3. The discordance in the observations of the Sun and Pole star, arranged by months, appears to follow the monthly curves of the temperature corresponding with their times of transit

4 The discordance in the observations of the Pole star, arranged in groups, depending on temperature alone, appear to be systematic, and would be satisfied by a secondary temperature correction.

In this paper, as in those before cited, attention is called to the uncertainty which affects the temperature of the air as determined by the thermometer. From the comparison of the ordinary exterior thermometer used in the computation of the refraction with the meteorological standard, it would seem that the readings of the former were too high by about one-half a degree, except during the warmest portion of the day, when the readings of the exterior thermometer are too low. At 1 o'clock p. m., in March, the correction is $+2^{\circ}.6$.

This correction is founded on comparisons made at intervals of 12 hours, namely, at the upper and lower transits of Polaris, and therefore not at the same hours of the day throughout the year. The general conclusion seems, however, to be well established.

The object and method of the present paper has in common with the two last cited only the determination of results founded on uniform data of reduction. In the author's work on the planets it is necessary to reduce all the observed declinations to a uniform system, defined by the mean declinations of Boss's catalogue. In most cases it is deemed sufficient to determine the necessary correction by a simple comparison of observed declinations of stars with Boss's declinations. But the uniformity of system found in the 37 years of observation with the Greenwich transit circle seem to render the instrument and the results worthy of a more careful discussion.

§ 2. *Outline of the questions to be investigated.*

The first step in our investigation will be a general study of the peculiarities of the instrument as deduced from the investigations given in the annual volumes of observations. Notwithstanding the general uniformity of system in making the observations, there has been no uniformity in the constants of reduction. The adopted flexure correction has been several times changed. The correction for discordance between observations made directly and by reflection has generally been determined separately for each year. The tables of refraction have twice been changed. In discussing the observations the first question in natural order is, what would have been the results had the observations all been reduced with uniform elements, and, especially, what discrepancies would then have remained between the results of different years' work and different methods of observation?

*A discussion of Greenwich observations of north polar distances with reference to the position of the ecliptic and the annual variation in the value of the colatitude. (Memoirs R. A. S., Vol. XLIX, page 239.)

Assuming that the instrument, the surrounding conditions, and the habits of the observers remained absolutely unchanged, the general result of such a reduction should be a constant latitude of the instrument, derivable from the observations of each year, and a set of north polar distances of stars varying uniformly with the time. This uniformity of result is not, however, to be expected, and we have to consider the various possible sources of discordance.

In the first place, it must be remembered that what is obtained directly from observation is the angle between the nadir (zenith) point and a star at the moment of culmination; in other words, the zenith distance of the star. The determination of the latitude and north polar distance is a separate process. Assuming that the nadir point was always determined with unvarying accuracy, or at least with only a constant error extending through the entire period of observation, then the zenith distance of each star, after correction for nutation and aberration, should appear to vary from year to year in accordance with the laws of precession and proper motion.

But we must always admit the possibility of systematic errors in the determination of the zenith point, which may vary from year to year. When this point is determined by the reflected image of the micrometer thread such errors may arise from the special division of the circle which is used in the observation, from personal error in the setting of the thread, or from any cause which acts on the instrument during the observation.

If a constant latitude be used variations in the error of zenith point will cause corresponding changes in the deduced polar distances of stars observed on one side of the pole only. In the absence of any other source of error these changes should correspond to the apparent changes of latitude derived from observations of circumpolar stars

But no such errors need systematically affect the polar distances of the objects observed, because they may be eliminated through the latitude. If the latter be so chosen that the polar distances of circumpolar stars shall in the general mean come out equal at upper and lower culmination, we may conceive that the polar distances of all stars are in effect measured from the pole itself. Varying systematic errors in the zenith point will then appear as apparent variations of latitude. In other words, the measured polar distances, after reduction to the apparent latitude of each year, will be unaffected by systematic error of zenith point, and any discordances which remain are due to some other cause than this.

Definable possible causes are:

α. Changes in the habits of the observers, leading them to place the micrometer thread above or below the center of the image of a star.

β. Change in the flexure of the instrument.

The reflexion observations afford an independent test of the action of these causes. Their results may be so obtained as to be entirely independent of the direct observations. We have only to use the value of the latitude as it results from them alone and we may thus, as in the case of direct observations, eliminate all systematic error of the zenith point from the polar distances.

The causes, α and β, above cited will each result in a discordance between the polar distances observed directly and by reflexion. This discordance may be made evident either by a systematic difference between the zenith points, as derived from direct and reflex observations of stars of different zenith distances, or by discordance between the polar distances measured directly and by reflexion.

In the case of cause β (flexure) the discordance will vary as the sine of the ZD. In the case of α it will be constant on each side of the zenith, but change *per saltum* at the zenith, it being assumed that the observer always faces in the direction from the zenith to the star.

§ 3. *Investigation of flexure and R — D correction.*

I now propose to inquire what conclusions respecting the stability and flexure of the instrument are deducible from the comparison of observations made directly and by reflexion.

In order to decide this question we must inquire what would have been the R — D correction had the same flexure been applied throughout. If we suppose the discordance to vary as the sine of the zenith distance it will be identical in its law with the correction for flexure and will be combined with it.

From and after 1879 the flexure has been assumed zero. Our best course seems to be to show what results for R — D would have been reached had the same system been adopted from the beginning. The reduction would be easy had the R — D correction always been assumed to vary as the sine of the zenith distance. This was the case from 1851 to 1861 and has been the case from 1882 onwards. But from 1862 to 1881, inclusive, the variable part of the discordance was assumed to be of the form

$$c \sin Z \cos^2 Z$$

Since Mr. CHRISTIE has shown that the quantity actually does vary as the sine of the zenith distance we should reduce the correction to this form before employing it.

It is also necessary to correct the coefficient c for the curvature of the level reflecting surface, which was not done before 1882.

Our first step will be to reduce the coefficient of sin Z cos² Z to that of sin Z. The ratio of the two coefficients will be the mean value of cos² Z extended over that portion of the meridian within which reflected observations were made, and weighted according to the weight of the observations at different zenith distances. Presumably, the changes in this mean from year to year were so small that one value of the ratio of the coefficients may be used throughout. To determine it I have made a rough computation of the actual value of the coefficient of sin Z from time to time from the tables of mean results given in the annual volumes, preferring those years in which its value was largest. The result is shown in the following table:

Comparison of certain coefficients of sin Z, as computed, with the coefficients of sin Z cos² Z as deduced in the annual volumes.

Year.	Coefficient of sin Z cos² Z.	Coefficient of sin Z.	Ratio.
	″	″	″
1862	—0.55	—0.45	0.82
1863	—0.62	—0.44	0 71
1865	—0.77	—0.61	0.79
1866	+0 80	+0.55	0.69
1868	+0.91	+0.83	0.91
1869	+0.83	+0.79	0.95
1873	+0.61	+0.33	0.54
1878	+0.64	+0.44	0.69
Mean			0.76

The second column gives the coefficient actually derived in the introductions to the respective volumes of Greenwich observations.

The third column gives the coefficient of sin Z, derived in a somewhat summary way from the same data. Generally I used only the three extreme groups on each side of the zenith, as on this the result should mainly depend. The ratio of the two values is shown in the last column. The ratio varies more widely than I expected it to.

I have assumed that in the other years in question the coefficient required would be found by multiplying that given in the annual volumes by 0.76; and as the above computations are not rigorous I have, for the years of the table, taken the mean between the coefficient actually found and AIRY'S coefficient multiplied by 0.76.

In the following table is found—

1. The flexure coefficient actually used in the reductions.

2. The additional coefficient of sin Z, derived from the comparison of direct and reflex observations, as just explained. Before 1882 this coefficient requires the correction $+0''.08$, on account of curvature of the level reflecting surface.

3. The flexure coefficient as it would have been found from the comparison of the direct and reflex observations had no flexure been applied in the original reductions.

TABLE I.—*Coefficients of flexure of the Greenwich transit circle resulting from the comparison of the direct and reflex observations of each year.*

Year.	Flexure used in reduction.	½(R — D)	Resulting flexure.	Year.	Flexure used in reduction.	½(R — D)	Resulting flexure.
	"	"	"		"	"	"
1851	+0.73	—0.24	+0.57	1870	—0.37	+0.55	+0.26
1852	+0.73	—0.24	+0.57	1871	—0.12	+0.55	+0.51
1853	+0.50	—0.29	+0.29	1872	—0.12	+0.41	+0.37
1854	+0.50	—0.33	+0.25	1873	—0.12	+0.24	+0.20
1855	+0.50	—0.26	+0.32	1874	—0.12	+0.42	+0.38
1856	+0.50	—0.37	+0.21	1875	—0.12	+0.26	+0.22
1857	+0.56	—0.42	+0.22	1876	—0.12	+0.29	+0.25
1858	+0.56	—0.28	+0.36	1877	—0.12	+0.21	+0.17
1859	+0.56	—0.56	+0.08	1878	—0.12	+0.44	+0.40
1860	+0.56	—0.28	+0.36	1879	0.00	+0.39	+0.47
1861	+0.56	—0.54	+0.10	1880	0.00	+0.54	+0.62
1862	+0.56	—0.43	+0.21	1881	0.00	+0.55	+0.63
1863	+0.56	—0.45	+0.19	1882	0.00	+0.44	+0.44
1864	+0.56	—0.35	+0.29	1883	0.00	+0.66	+0.66
1865	+0.76	—0.60	+0.24	1884	0.00	+0.66	+0.66
1866	—0.37	+0.58	+0.29	1885	0.00	+0.69	+0.69
1867	—0.37	+0.44	+0.15	1886	0.00	+0.73	+0.73
1868	—0.37	+0.81	+0.52	1887	0.00	+0.68	+0.68
1869	—0.37	+0.71	+0.42				

The most striking feature of this table is the sudden change in the flexure coefficient found from the opposing collimators at the end of 1865, after piercing the central cube so that the collimators

could be set on each other without raising the telescope from its V's. The evidence that this supposed change is unreal seems to me conclusive. In the first place, it seems very unlikely that a piercing of the central cube would have produced an unequal flexure of the two ends of the telescope. In the second place, how is it possible that a change in the astronomical flexure should have occurred without any change in the zenith distances measured with the instrument? The last column of the table shows that if no flexure or an unchanged flexure had been employed in the reductions there would have been no marked change in the results of either the direct or the reflex observations. The change in R — D, shown in the second column of the table, is not a result of observation, but of a change in the data of reduction. That two opposite causes, one of them inscrutable and the other improbable, should have so operated as to neutralize each other's effects on the measured zenith distances seems to me too improbable an hypothesis for acceptation.

If my view be accepted, it follows that one series of determinations of flexure, either those made before piercing the cube or those made afterward, were wrong. I see no difficulty in accepting this conclusion, and believe that the error is mainly in the later determinations. In the earlier series the pencil of rays from the objective of each collimator passed in its entirety through the objective of the other. In the later series only those portions of the pencil can pass which enter the triangular openings around the central axis. In the possible non-symmetry of these passing pencils as to the axes of one or both objectives, the possible non-coincidence of the foci of those parts of the collimator objectives through which alone the rays pass, with the foci of the entire objective, and the diffusion of the image of one collimator thread in the focus of the other caused by the diffraction of the passing ray, we have, it seems to me, a quite possible cause for the difference of $0''.7$ between the later and the earlier determinations. The approach of the value of the flexure found since 1878 from the reflex observations to that found before 1866 from the opposing collimators is in this connection quite worthy of consideration.

My conclusion is that R — D has from the beginning been due mainly to flexure. If due wholly to this cause we must admit that the flexure is greater now than in the earlier years, as shown by the following table of mean values:

1851–1867, mean	F = +0.27″
1868–1877, mean	F = +0.33
1878–1882, mean	F = +0.51
1883–1887, mean	F = +0.68

Such a result is not impossible. A sagging of the micrometer spider line under the influence of gravity, and the elasticity of the parts of the micrometer causing them to yield to the same influence would both enter into the flexure, and their effect might vary from year to year.

But flexure is not the only possible cause of R — D. Saying nothing of a possible local refraction affecting the two classes of observations unequally, there may be in the instrument itself a looseness of the objective in its fastenings, and in the observer a habit of setting the micrometer thread above or below the true center of the image of a star. Either of the last-named causes would produce a discordance, changing suddenly at the zenith, but constant on each side of it. To determine the flexure independently of this cause it should be obtained separately from the observations on each side of the zenith. With a view of deciding whether any such cause affected the early observations with the Greenwich transit circle, I have derived a summary mean result from the observed values of R — D during the 12 years, 1853–1864, by taking separately the observations on each side of the

zenith. Taking the groups as given in the annual volumes, I divided them into four on each side of the zenith and found the following mean results. In obtaining them numerical precision has not been arrived at, as the only object is to decide whether a change of flexure is or is not indicated:

North.		South.	
$\sin Z = -0.62$;	$R - D = +0''.53$	$\sin Z = +0.62$;	$R - D = -0''.34$
-0.47	$+0.22$	$+0.47$	-0.47
-0.30	$+0.32$	$+0.30$	-0.12
-0.20	$+0.26$	$+0.20$	-0.17

Then, assuming

North . . . . $R - D = a' + b \sin Z$

South . . . . $R - D = a'' + b \sin Z$

we have

North . . . . $R - D = +0''.11 - 0''.57 \sin Z$

South . . . . $R - D = -0.05 - 0.57 \sin Z$

This method of treatment therefore indicates a break of 0''.16 at the zenith, which would be represented by a habit of placing the micrometer thread 0.''04 above the center of the image of a star. Correcting for curvature of level we then have

$$\tfrac{1}{2}(R - D) = -0''.20 \sin Z$$

Applied flexure . . . . $+0.54 \sin Z$

Apparent flexure . . . . $+0.34 \sin Z$

This is much nearer the mean of the results already found for the years in question than it is to the values of 1885–1887.

To complete the comparison I have inquired what flexures would be given by the values of R — D in the last 3 years were north and south differences treated separately. A rough computation from the printed residuals of the formulæ derived annually in the printed volume gives

North . . . . $R - D = -0''.03 + 1''.31 \sin Z$

South . . . . $R - D = -0.23 + 1.75 \sin Z$

We have the anomalous result of a much smaller apparent flexure in the north than in the south. I think this difference should be regarded as only the result of an accidental accumulation of errors, but both coefficients of sin Z show that the flexure must be regarded as greater during the years in question than it was before 1870.

On the whole, the evidence that the value of R — D is appreciably continuous through the zenith is so strong that I shall adopt that conclusion and use the mean values of the flexure already found, but take a general mean for the whole period, 1851–1877.

In order to reduce all measures to the absolute zenith the nadir point, as found by reflection of the micrometer thread, should be reduced to the mean of the direct and reflex observations. This correction for each year is the constant term of $\frac{1}{2}(R-D)$. It is found in the reductions for each year, and is applied to all the printed results as a constant term of the $R-D$ correction.

§ 4. *Constant of refraction and possible periodic error of graduation.*

The refractions used in the reduction of all the observations under discussion have been substantially those of BESSEL's *Tabulæ Regimontanæ*, with the exception of the years 1868–1877, when the refractions of those tables were diminished by the factor 0.00531.

The latest and, presumably, the most authoritative tables are those of Pulkowa,* founded on GYLDÉN's investigation of the law of density in the atmosphere. In these tables BESSEL's constant is diminished by the factor 0.002852. The question now arises, what value of the constant of refraction is indicated by the Greenwich observations of circumpolar stars?

This question is not an easy one to decide. In the first place, there is a large measure of indetermination in the very definition of the term "constant of refraction," because this constant depends upon the wave length of light, and every celestial object is seen by light of all wave lengths. Strictly speaking, the image of each star is by atmospheric refraction spread out into a spectrum. If it is not always seen as such, this is only because the spectrum is so confused by the aberrations of all kinds to which the light from a celestial body is subjected before it reaches the eye, and by the imperfections of the eye itself, that its separate parts can not be discriminated. In fact, under good conditions of vision, the spectrum is always seen at considerable zenith distances. The refraction will then depend on what part of the spectrum the observer sets his micrometer thread upon, and this part admits of no exact definition. It must vary with the color of the star, the sensibility of the eye to the various spectral colors, and the habits of the observer.

In the next place, there is no easy way of making a determination of this constant which shall not be greatly affected by instrumental errors. As we near the horizon, where the refraction is greatest, the law of refraction becomes uncertain, and the absorption of light of the shorter wave lengths by the atmosphere causes a diminished refraction of the visible image.

On the whole, the data afforded by the Greenwich transit circle ought, it would seem, to afford about as good a value of the constant as is now available, at least from meridian observations. When the stars for the present investigation (given in § 8) were originally selected it was supposed that they would be well adapted to this purpose. Group III was selected to terminate at the zenith, so that the altitude of the lowest star, γ *Draconis*, at lower culmination was 13°. I thought it best not to go beyond this, because of a possible break in the continuity of the errors at the zenith.

But before entering upon the subject the Greenwich Ten-Year Catalogue appeared, and was found to contain an exhibit of the observations of circumpolar stars during the years 1877–1886, so complete and convenient in form that its results should supersede any that could be derived from so limited a number of stars as I had selected. Moreover, the star places were already reduced to a system so nearly the same as that which I finally adopted that no corrections for reduction to that system are necessary. The data from which the amount of refraction is to be derived are the apparent latitudes given by stars of different north polar distances. When the adopted refraction is too small the north polar distances, derived from lower culminations, will be greater than those derived from upper culminations by an amount which should continually increase from the pole to the horizon.

* *Tabulæ Refractionum in usum Speculæ Pulkovensis congestæ.* Petropoli, 1870.

I start from the numbers given by the Astronomer ROYAL, on pages 13 and 46–55 of the Ten-Year Catalogue. These are (1) the excess of north polar distances given by observations above the pole in the case of each one of the 688 stars observed on both sides of the pole; (2) the weighted means for groups of 10 stars each; (3) the weighted means for groups of 50 stars each, extending to 50° of polar distance. The quantities which enter into the discussion are shown in Table II.

The first three columns of this table give the north polar distance and zenith distance for the means of groups of 50 stars, except in the case of the last five groups, which include, respectively, 30, 20, 20, 10, and 8 stars.

The refractions for the mean of the group are expressed in minutes of arc, in order to afford convenient coefficients for correction.

The columns $2\Delta\varphi$ show the mean excess of north polar distance above the pole over that below the pole (1) as given in the Ten-Year Catalogue, and therefore as resulting from the use of BESSEL'S refractions; (2) as it would have been had the Pulkowa refractions been used.

In assigning weights regard has been had to the fact that, owing to the systematic errors which we may suppose probably to affect each part of the circle, we must make the result for refraction depend on a more equal division of the weights over the arc than that which would result from the weights of the totality of observations in each portion of the arc. On the other hand, as we approach the horizon the weights must diminish, owing to the large accidental errors of the observations and the uncertainty of the law of refraction. The weight should converge to 0 at zenith distance 86°.

TABLE II.—*Data for correction to the constant of refraction given by Greenwich observation of circumpolar stars.*

(Ten-Year Catalogue, pages 13 and 55.)

North polar distances.	Zenith distances.		Mean refraction.		Σ refraction.	$2\Delta\varphi$		Weight.
	U. C.	S. P.	U. C.	S. P.		Tab. Reg.	Pulkowa.	
°	°	°	′	′	′	″	″	
5.8	−32.7	−44.3	+0.62	+ 0.94	1.56	−0.15	+0.10	6
10.8	−27.7	−49.3	+0.50	+ 1.12	1.62	−0.12	+0.14	5
15.1	−23.4	−53.6	+0.41	+ 1.30	1.71	+0.22	+0.49	6
18.4	−20.1	−56.9	+0.35	+ 1.47	1.82	+0.12	+0.41	5
21.3	−17.2	−59.8	+0.30	+ 1.65	1.95	+0.25	+0.56	5
23.4	−15.1	−61.9	+0.26	+ 1.80	2.06	+0.15	+0.48	5
25.2	−13.3	−63.7	+0.23	+ 1.93	2.16	+0.10	+0.45	5
27.2	−11.3	−65.7	+0.19	+ 2.12	2.31	−0.24	+0.13	4
28.8	− 9.7	−67.3	+0.17	+ 2.28	2.45	−0.20	+0.20	4
30.7	− 7.8	−69.2	+0.13	+ 2.52	2.65	−0.66	−0.23	4
33.6	− 4.9	−72.1	+0.08	+ 2.95	3.03	−0.37	+0.09	4
36.6	− 1.9	−75.1	+0.03	+ 3.55	3.58	−0.51	0 00	4
40.1	+ 1.6	−78.6	−0.03	+ 4.64	4.61	−0.29	+0.32	3
42.2	+ 3.7	−80.7	−0.06	+ 5.64	5.58	−0.55	+0.11	2
44.0	+ 5.4	−82.5	−0.09	+ 6.90	6.81	−0.92	−0.10	2
45.6	+ 7.1	−84.3	−0.12	+ 8.77	8.65	−2.28	−1.10	1
47.6	+ 9.1	−86.3	−0.15	+12.30	12.15	−4.94	. . . .	0

An examination of either of the columns $2\Delta\varphi$ shows that the constant of refraction, as it would result from the measures, continually increases from the pole to near the horizon. Stars about 20°

from the pole give a latitude about 0″.15 greater than those within 10° of it when BESSEL'S refractions are used, while the difference is 0″.20 when the Pulkowa tables are used. This difference, small though it is, is well marked, and must be attributed to some error, varying between the points of the circle corresponding to these results. Whether the outstanding errors in the graduations can amount to this it is impossible to say, but if it does, they should repeat themselves at intervals of 60°. I therefore at first formed the equations of condition necessary to determine an error of the form $\frac{1}{2}e$ cos 6 N. P. D. But this solution gave $e = 0''.06$, and so indicated a periodic error, whose coefficient was only 0″.03 An examination of the residuals showed that the actual period should be regarded as 30°, the coefficient of cos 12 N. P. D. being larger than that of cos 6 N. P. D.

Allowing for this periodic error, it will be seen that the Pulkowa refractions are well represented down to about 77° zenith distance, when a yet farther diminution seems to be indicated. This points not only to a diminution of the constant, but to a change of the law, and suggests a comparison of the laws of BESSEL and GYLDÉN.

In the introduction to his exhaustive memoir* GYLDÉN remarks that the actual diminution of temperature with the height above the earth's surface is much more rapid than that assumed by BESSEL, and that, in consequence, his refractions are too great at great zenith distances. But the following comparison of GYLDÉN'S law with that of BESSEL shows that this statement does not fully express the difference between the two laws. With a given constant the refraction will be the same under every admissible law until we reach a zenith distance of more than 65°. From that point a comparison of the two laws is shown in the following table:

TABLE III.—*Comparison of Gyldén's law of refraction with that of the Tabulæ Regimontanæ with corrections to reduce the Greenwich to the Pulkowa refractions.*

Zenith distance.	μ		GYLDÉN *minus* BESSEL.		Pulkowa *minus* Greenwich, 1851–'67, 1877–'88.	Pulkowa *minus* Greenwich, 1868–'76.
	BESSEL.	GYLDÉN.	$\Delta\mu$	Δ Ref.	Δ Ref.	ΔR
°				″	″	″
65	1.75919	1.75925	+ 6	+ 0.02	− 0.33	+0.32
70	1.75771	1.75781	+ 10	+ 0.04	− 0.41	+0.43
75	1.75457	1.75478	+ 21	+ 0.10	− 0.51	+0.61
80	1.74623	1.74657	+ 34	+ 0.25	− 0.65	+1.04
81	1.74288	1.74328	+ 40	+ 0.32	− 0.68	+1.17
82	1.73845	1.73884	+ 39	+ 0.35	− 0.77	+1.29
83	1.73229	1.73265	+ 36	+ 0.37	− 0.89	+1.43
84	1.72346	1.72375	+ 29	+ 0.34	− 1.11	+1.56
85	1.71020	1.71037	+ 17	+ 0.23	− 1.46	+1.65
86	1.68908	1.68894	− 14	− 0.22	− 2.24	
87	1.65114	1.65179	+ 65	+ 0.28	− 1.20	
88	1.57994	1.57955	− 39	− 0.98	− 4.18	
89	1.40764	1.40332	−432	−14.6	−19.37	

Here μ is the logarithm of the factor by which the tangent of the zenith distance must be multiplied to obtain the mean refraction in seconds. In forming the values of μ GYLDÉN used BESSEL'S

* Untersuchungen über die Constitution der Atmosphäre und die Strahlenbrechung in derselben. Mémoires de Pétersbourg, Série VII, Tome X, 1866.

constant, so that the difference between the values of μ arises wholly from the difference between the assumed laws of atmospheric density. It will be seen that GYLDÉN's positive correction reaches a maximum between 81° and 82° of zenith distance and then diminishes as far as 85°. Below 85° the comparison is not with the same theory, because BESSEL stopped computing μ at 85°, and below that point his refractions, as found in the *Tabulæ Regiomontanæ*, are those determined by ARGELANDER from observation.

The column (GYLDÉN *minus* BESSEL Δ. Ref.) shows GYLDÉN's corrections to BESSEL's theory when the same constant is used, and the last two columns show (1) the correction after the constant of GYLDÉN's theory is diminished by the factor .00285, as in the Pulkowa tables, and (2) the correction to the diminished refractions of 1868–1876.

Now, the result of the Greenwich observations is to justify a diminution of the constant to the Pulkowa value, but it does not enable us to decide between the two laws. The complete investigation of the subject would be foreign to the object of the present paper. I shall therefore only remark that it may be questioned whether GYLDÉN's law of temperature holds good for the upper regions of the atmosphere. According to this law the absolute zero is reached at a height of about 102 kilometers, and for many kilometers below this point the rise is very small. Now, abundant observations on meteors show that at heights between 150 and 180 kilometers the atmosphere not only exists, but is of sufficient density to render incandescent a body moving through it with planetary velocity. It may also be worthy of note that the height at which the atmosphere ceases to reflect the sun's light, usually stated at about 65 kilometers, has never been noticed to vary in different latitudes, and was found by SCHMIDT, at Athens, to be greater in winter than in summer.* These facts suggest the possibility that wide variations of temperature may be mostly confined to comparatively small heights, and that above a certain limit we have a region in which they are slight. What the minimum temperature may be one can not even guess, but I see no reason for supposing that it ever approaches the absolute zero.

But these considerations are by no means conclusive against GYLDÉN's law. The greater part of the refraction takes place at altitudes far below those at which the law can be proved defective, and the fact that SCHMIDT's twilight observations, if interpreted in the usual way, would make the atmosphere higher in winter than in summer shows that nothing bearing on refraction can be inferred from them. At the same time the curious difference between the two laws, as shown by the existence of a maximum at zenith distance 82°, shows that the subject is not yet exhausted.

To exhibit the relative effects of adopting the one or the other law, and of using the Pulkowa tables unchanged, I have solved the equations of condition derived from Table II on all three hypotheses. The quantities to be determined are:

$2\Delta\varphi$, the correction to the latitude;

r, 60 times the factor by which the tabular refraction must be corrected;

e, double the coefficient of a supposed periodic correction of the form $\frac{1}{2}e$ cos 12 N. P. D.

To facilitate the solution I take as unknown quantities

$$x = 2\Delta\varphi + 3r$$
$$y = 6r$$
$$z = e$$

*Astronomische Nachrichten, No. 1496, Vol. LXIII, page 116.

The equations thus formed from the data of Table II are:

	Bessel.	Pulkowa.	B′	P′	Weight.
	″	″	″	″	
$1x - 0.24y + 0.45z =$	-0.15	$+0.10$	-0.17	-0.15	6
$1 \quad -0.23 \quad -0.65 =$	-0.12	$+0.14$	-0.30	-0.30	5
$1 \quad -0.22 \quad -1.00 =$	$+0.22$	$+0.49$	0.00	-0.02	6
$1 \quad -0.20 \quad -0.77 =$	$+0.12$	$+0.41$	-0.05	-0.05	5
$1 \quad -0.18 \quad -0.24 =$	$+0.25$	$+0.56$	$+0.21$	$+0.20$	5
$1 \quad -0.16 \quad +0.19 =$	$+0.15$	$+0.48$	$+0.22$	$+0.20$	5
$1 \quad -0.14 \quad +0.54 =$	$+0.10$	$+0.45$	$+0.24$	$+0.24$	5
$1 \quad -0.12 \quad +0.83 =$	-0.24	$+0.13$	-0.02	-0.02	4
$1 \quad -0.09 \quad +0.97 =$	-0.20	$+0.20$	$+0.08$	$+0.08$	4
$1 \quad -0.06 \quad +0.99 =$	-0.66	-0.23	-0.33	-0.33	4
$1 \quad +0.01 \quad +0.73 =$	-0.37	$+0.09$	$+0.02$	$+0.01$	4
$1 \quad +0.10 \quad +0.19 =$	-0.51	0.00	-0.07	-0.10	4
$1 \quad +0.27 \quad -0.51 =$	-0.29	$+0.32$	$+0.29$	$+0.32$	3
$1 \quad +0.43 \quad -0.83 =$	-0.55	$+0.11$	$+0.21$	$+0.27$	2
$1 \quad +0.64 \quad -0.98 =$	-0.92	-0.10	$+0.12$	$+0.16$	2
$1 \quad +0.94 \quad -0.99 =$	-2.28	-1.10	-0.80	-0.90	1

The resulting normal equations are:

	Bessel.	Pulkowa.
	″	″
$65.00x - 4.06y + 0.75z =$	-11.09	$+14.38$
$-4.06 \quad +3.92 \quad -2.30 =$	-4.35	-3.43
$0.75 \quad -2.30 \quad +31.99 =$	-1.94	-2.52

The results of the three solutions are:

	Adopting Bessel's law.	Adopting Gyldén's law.	Using Pulkowa refractions.
	″	″	″
Value of x	$-$ 0.26	$+$ 0.17	$+$ 0.22
Value of y	$-$ 1.47	$-$ 0.77	0.00
Value of z	$-$ 0.16	$-$ 0.14	$-$ 0.06
$2\Delta\varphi$	$+$ 0.47	$+$ 0.56	$+$ 0.22
Correction of colatitude	$-$ 0.24	$-$ 0.28	$-$ 0.11
Colatitude, 1877–1886	38° 31′ 21.66	21.62	21.79
Factor to correct refraction	$-$ .00408	$-$ .00213	.00000
Correction of log. refraction	$-$ .00177	$-$ .00092	.00000
Value of μ near zenith	1.75979	1.75940	1.76032
	″	″	″
Mean refraction at zenith distance 45°,	57.448	57.396	57.518
Coefficient of periodic term	$-$ 0.08	$-$ 0.07	$-$ 0.03

Of the first two columns of results the first is obtained by using the refractions of the *Tabulæ Regimontanæ;* the second by using the Pulkowa tables. The values of x, y, z, the factor to correct refraction, and the correction of the logarithm of the refraction each refer to the respective tables. But all the quantities are those which result from the observations used in the discussion, assuming the constant of refraction to be that given by the observations, and the two sets would therefore be identical but for the difference between the two laws of refraction. With a given constant GYLDÉN's law gives a refraction at zenith distance 81° larger by 0''.40 than BESSEL's law does; consequently, to represent a given set of observations the constant will be smaller under the former law than under the latter one.

The third column shows the results of using the Pulkowa refractions unchanged. The equations in x and z are solved, using $y = 0$ in advance, and taking the second set of residuals.

The assumed periodic term

$$-0''.07 \cos 12 \text{ N. P. D.}$$

comes out too small to completely remove the anomaly which suggested its possibility But this value is not the best that can be derived from the data, because, for the especial purpose of determining so minute a quantity, the weights should be more nearly proportional to the number of observations than I have assumed them to be. From the totality of the observations within 31° of the pole, using the 10-star groups, I find

$$e = -0''.22$$

Granting that the term is periodic and due to non-correspondence of the mean errors of all the graduations with those determined for every degree, the probable value of the largest term of the correction seems to be

$$+0''.10 \cos 12 \text{ N. P. D.}$$

But a more complete discussion might be expected to give terms having as factors the cosines of 6, 12, and 18 times the north polar distance. Terms depending on the sines of the same angle may really exist, but are not shown in the comparisons of upper and lower culminations.

The preceding discussion shows clearly that the observations of circumpolar stars during the 10 years, 1877–1886, are better represented by the constant of the Pulkowa refractions than by that of BESSEL; indeed, that the latter is entirely inconsistent with the observations, which point to a constant yet smaller than that of Pulkowa.

But the observations of the Sun during the same period do not support this conclusion. In Appendix III to the Greenwich Observations for 1887 is found a carefully corrected reinvestigation of the position of the ecliptic from the observations of the 10 years in question. Here we have for each year a quantity, z, by which the mean of the reduced observed polar distances of the Sun are too small. The result is

$$\text{Mean value of } z = +0''.11$$

which may be expressed in the form

$$\textit{Mean measured north polar distance of the Equator} = 89^\circ\ 59'\ 59''.89$$

To reduce this result to the elements of the present paper and the Pulkowa refractions we have

	° ′ ″	
Greenwich adopted colatitude	38 31 21.90	
Colatitude, using Pulkowa refractions	38 31 21.79	
Supplementary correction to adopted colatitude . .	−0.11	
Greenwich adopted flexure (R − D)	+0.692	sin Z. D.
Mean flexure of the present paper	+0.59	sin Z. D.
Supplementary correction for flexure	−0.10	sin Z. D.
Reduction to Pulkowa refractions, about	−0.26	
The sum of the corrections will be	−0.45	

giving

Mean measured north polar distance of the Equator $= 89° \ 59' \ 59''.44$

There appears to be a mean outstanding error of $0''.55$ when the Pulkowa refractions are used and about one-half of this quantity with BESSEL'S refractions. But I do not regard this result as outweighing the evidence already adduced in favor of the former. Observations of the Sun are subjected to several possible causes of constant error which may well amount to half a second.

§ 5. *Corrections for reduction to equinox during the years* 1851–1856.

Before applying the general reductions to a uniform system already found it is necessary to apply certain special corrections to the mean results of the earlier years. One of these arises from the fact that the nutation and equinox of the Nautical Almanac during the years 1851–1856 were not the same as those used since 1857. At Greenwich the apparent places of the Nautical Almanac stars have always been reduced to mean place at the beginning of the year by finding the difference between the mean and apparent places of the Nautical Almanac and applying it to the observed apparent place.

During the years in question the following values of the constants of aberration and of nutation, were adopted in the Nautical Almanac:

	″
Aberration constant	20.42
Nutation constant	9.25

But the standard values used from 1857 were $20''.445$ and $9''.224$, respectively. As it is not intended to determine the aberration constant from these observations the small difference in its value is of no importance; it will at most have no other effect than that of changing the mean places of some of the stars by perhaps $0''.01$, and will be without appreciable systematic effect on the constant of nutation derived from the observations.

The difference of nutation constants is, however, to be allowed for. Putting N for this constant, the reduction of the declination to mean place contains the terms

$$- N \cos \Omega \sin \alpha + 0.745 \, N \sin \Omega \cos \alpha$$

Hence, the reduction of the mean north polar distances from the old to the new value of the constant of nutation will be

$$\Delta \text{ north polar distance} = -0''.026 \cos \Omega \sin \alpha + 0''.019 \sin \Omega \cos \alpha$$

or

$$\Delta \text{ north polar distance} = +0''.023 \sin (\Omega - \alpha) - 0''.003 \sin (\Omega + \alpha)$$

A yet more important and troublesome question is the epoch to which the observed places are referred by the reduction to mean place during the six years in question. In the Nautical Almanac no specific statement is found of the sense in which the words "beginning of the year" and "fraction of the year" are to be understood. It is therefore necessary to determine the epoch by induction from the numbers of the ephemeris.

The mean places of the stars are said to be given for January 1 of each year. Were this rigorously the case the annual precessions actually applied would be different, according as the year was one of 365 or of 366 days. But by differencing the mean R. A's of Polaris, as given for several consecutive years, no such changes in the precession are found, but the annual differences proceed with entire regularity. We may therefore regard the mean places as referring to the beginning of the Besselian fictitious year, and if, in the constants of reduction to apparent place, the same epoch is adopted no correction will be necessary.

But an examination of the constant C (the Besselian A) shows that, January 1, Greenwich mean noon is taken as the beginning of each year. Hence, by the use of these numbers the mean place to which a star will be reduced from the apparent place at any epoch will be January 1 of each year.

From the *Introduction* to the Almanac it would also appear that the mean places given for each year were used as if belonging to the epoch January 1. The result is, that all the mean places in the annual volumes of *Greenwich Observations*, 1851 to 1856, are, in effect, for January 1, Greenwich mean noon of each year. They must therefore be reduced to the fictitious year. The following table shows the beginnings of this year, expressed in Greenwich mean time, and the resulting corrections to carry the mean places back to the beginning of the fictitious year:

		From January 1.0.	Precession in north polar distance.
	d.	d.	$''$
1851	January, 0.45	−0.55	$+0.030 \cos \alpha$
1852	0.69	−0.31	$+0.017 \cos \alpha$
1853	−0.07	−1.07	$+0.059 \cos \alpha$
1854	0.17	−0.83	$+0.046 \cos \alpha$
1855	0.42	−0.58	$+0.032 \cos \alpha$
1856	0.66	−0.34	$+0.019 \cos \alpha$

In applying the reduction to PETERS's constant of nutation it will suffice to take for Ω its value at the middle of each year, as shown in the following table, which gives also the total correction on account of epoch and nutation:

	Ω	Correction to mean north polar distance.
	°	″ ″
1851	117.3	$+0.012 \sin\alpha + 0.046 \cos\alpha$
1852	98.0	$+0.004 \sin\alpha + 0.036 \cos\alpha$
1853	78.6	$-0.005 \sin\alpha + 0.077 \cos\alpha$
1854	59.2	$-0.013 \sin\alpha + 0.063 \cos\alpha$
1855	39.8	$-0.019 \sin\alpha + 0.045 \cos\alpha$
1856	20.4	$-0.024 \sin\alpha + 0.025 \cos\alpha$

§ 6. *Errors of graduation of the circle for the four polar stars.*

Another subject requiring attention is the possible correction for errors of division. The Greenwich circle is divided to every 5′, and remains in an invariable position on its axis. Hence, it may be assumed that in the case of a star with but a small precession in declination, δ Ursæ Minoris for example, all the declinations depend on the same set of divisions, or at most on the same pair of adjoining sets. Hence, the absolute declinations will be systematically affected by any outstanding error in the determination of these divisions. A star whose precession in declination is near the maximum, Polaris for example, will, on the average, depend on a new set of divisions every 15 years, and if the circle were always set accurately at a definite pointing for each star the change to new division would occur *per saltum* at the end of this interval. But an examination of the printed observations shows that the microscope micrometer may for any star be set on one system of divisions or another, according to the chance pointing of the telescope. The change will therefore in any case be gradual.

All necessity for a rigorous investigation of the subject is obviated by the precision with which the divisions are cut and the accuracy with which they have been determined for every degree. Moreover, special determinations of the divisions used in the observations of the four polar stars have from time to time been introduced. It thus happens that for the earlier epochs the corrections for the special divisions on which the places of these stars depend were found by interpolation from the corrections for the adjacent entire degrees, while at later periods they were independently determined. The data necessary to reduce all the results to one system I borrow from Mr. STONE's paper on the Constant of Nutation,* in which is found a table of the corrections to the north polar distances of the "Ledger results," which were actually applied in the reductions during the early years, together with the values which, according to the latest investigations then made, should have been applied. In order to compare these results it is necessary to subtract the flexure, which is included with the division errors, both in the original reductions and in Mr. STONE's table.

In the following table the columns D + F give the sum of the corrections for division and flexure, as actually used and applied up to 1865, and then as found by Mr. STONE.

* Memoirs R. A. S., Vol. XXXVII, pages 77–78.

Columns F contain the flexure corrections by the addition of which the preceding columns are obtained.

The corrections for division alone are then to be found by subtracting F from D + F.

Table IV (*a*).—*Corrections for flexure and errors of division applied in the annual published reductions of the Greenwich north polar distances of three polar stars.*

Star.	1851 and 1852.		1853 to 1856.		1857.		1858 to 1864.		1865.		Stone.	
	D + F	F	D + F	F	D + F	F	D + F	F	D + F	F	D + F	F
	"	"	"	"	"	"	"	"	"	"	"	"
Nadir	0.86	0.00	0.63	0.00	0.76	0.00	1.02	0.00	1.20	0.00	0.75	0.00
Polaris	0.69	−0.44	0.60	−0.30	1.07	−0.34	1.09	−0.34	1.14	−0.46	0.64	−0.35
Polaris, S. P	0.92	−0.47	0.84	−0.32	1.18	−0.36	1.40	−0.36	1.45	−0.49	0.97	−0.37
51 Cephei	0.72	−0.43	0.63	−0.29	0.99	−0.33	0.99	−0.33	1.05	−0.44	0.73	−0.34
51 Cephei, S. P	0.79	−0.48	0.72	−0.33	0.92	−0.37	0.92	−0.37	0.96	−0.50	0.45	−0.38
δ Ursæ Minoris	0.81	−0.42	0.71	−0.29	0.94	−0.32	0.94	−0.32	1.00	−0.44	0.64	−0.33
δ Ursæ Minoris, S. P	0.79	−0.49	0.72	−0.33	0.92	−0.37	0.92	−0.37	0.96	−0.51	0.52	−0.39

The values of D, derived from this table, will be the corrections for division alone. But the actual measures are made from the nadir point to the star. To find the effective corrections to the zenith distance of the star we must therefore subtract the correction applied to the nadir reading. On doing this it is found the corrections effectively applied are nearly the same from 1851 to 1856, and that, in order to reduce to Mr. Stone's values, the following supplementary corrections are to be applied to the annual results as reduced:

Table IV (*b*).—*Supplementary corrections to be applied for new determination of errors of division.*

Star.	1851 to 1856.	1857.	1858 to 1865.
	"	"	"
δ Ursæ Minoris S. P	−0.26	−0.37	−0.11
51 Cephei, S. P	−0.33	−0.45	−0.19
Polaris, S. P	+0.06	−0.19	−0.15
Polaris	−0.03	−0.41	−0.17
51 Cephei	+0.03	−0.24	+0.02
δ Ursæ Minoris	−0.15	−0.28	−0.02

When, however, we consider only the polar distance of the star, whatever correction is applied to the nadir reading will be eliminated from the mean results of observations at upper and lower culmination. The corrections which effectively change the north polar distance will therefore be

$$\tfrac{1}{2}(D - D')$$

D being the value of D for upper and D′ for lower culmination. Thus, we have the following system of corrections to the north polar distance of the three stars in question, which have in effect been used in the reductions given in the annual volumes of Greenwich Observations:

Corrections which have in effect been applied to the observed north polar distances of polar stars on account of errors of division.

Star.	1851 to 1856.	1857.	1858 to 1865.	1865 (STONE).	1887.
	″	″	″	″	″
Polaris	−0.13	−0.06	−0.17	−0.17	−0.21
51 Cephei	−0.06	+0.02	+0.02	+0.12	+0.11
δ Ursæ Minoris	−0.02	−0.02	−0.02	+0.03	+0.02

The values in the column 1865 (STONE) were presumably used from about 1866 onward. Those in the last column are derived from the transit circle tables found in the volume for 1887, page cxxx. The change from STONE's values is so small that no account need be taken of it. I shall assume that the corrections given by Mr. STONE are applicable up to 1865, and that those since applied need no modification. Thus, we shall have the following supplementary corrections for errors of division:

Star.	1851 to 1856.	1857.	1858 to 1865.
	″	″	″
Polaris	−0.04	−0.11	−0.01
51 Cephei	+0.18	+0.10	+0.10
δ Ursæ Minoris	+0.05	+0.05	+0.05

The case of λ *Ursæ Minoris* might have been examined in the same way; but it seems from the data of 1877 that the correction now applied does not differ materially from that taken out of the general table of corrections for every degree. The examination has therefore been deemed unnecessary.

§ 7. *Summary and tables of supplementary corrections to the published results.*

We have found in §§ 3 and 4 the following corrections to reduce the concluded mean results of direct observations in each year, as printed in the annual volumes, to a uniform system of instrumental constants:

1. The negative of the variable part of the R — D correction as applied in each year.

2. The negative of the flexure actually applied.

3. The flexure as concluded in § 3 from the comparison of direct and reflex observations. Of the four values of the flexure coefficient given on page 416 the first two differ so slightly that I have used one value for the entire period, 1851–1877. The adopted values are:

	″
1851–1877	+0.30
1878–1882	+0 51
1883–1887	+0.68

4. The reduction to the Pulkowa refractions. In this reduction no allowance has been made for a possible systematic error of perhaps 1° Fahr. in the reading of the external thermometer.

5. The reduction to Bessel's fictitious year and Peters's nutation during the years 1851–1856. This reduction is shown in § 5.

6. The supplementary corrections to the places of the 4 polar stars on account of errors of graduations.

The first four corrections are tabulated individually in Table V for each 5° of zenith distance south. They have the same values for absolute zenith distances toward the north.

Table VI contains the sum of the four corrections.

Table V.—*Corrections necessary to reduce the published Greenwich zenith distances of each year from 1851 to 1887 to the adopted instrumental standard and to the Pulkowa refraction table.*

Zenith distance south.	Flexure. 1851 to 1877.	Flexure. 1878 to 1882.	Flexure. 1883 to 1887.	Refraction. 1851 to 1867.	Refraction. 1868 to 1876.	Refraction. 1877 to 1887.	Negative of flexure applied. 1851 and 1852.	1853 to 1856.	1857 to 1864.	1865.	1866 to 1870.	1871 to 1878.	From 1879.
°	"	"	"	"	"	"	"	"	"	"	"	"	"
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	(*)
5	+0.03	+0.04	+0.06	−0.01	+0.02	−0.01	−0.06	−0.04	−0.05	−0.07	+0.03	+0.01	(*)
10	+0.05	+0.09	+0.12	−0.03	+0.03	−0.03	−0.13	−0.09	−0.10	−0.13	+0.06	+0.02	(*)
15	+0.08	+0.13	+0.18	−0.04	+0.05	−0.04	−0.19	−0.13	−0.14	−0.20	+0.10	+0.03	(*)
20	+0.10	+0.17	+0.23	−0.05	+0.06	−0.05	−0.25	−0.17	−0.19	−0.26	+0.13	+0.04	(*)
25	+0.13	+0.21	+0.29	−0.07	+0.08	−0.07	−0.31	−0.21	0.24	−0.32	+0.16	+0.05	(*)
30	+0.15	+0.25	+0.34	−0.08	+0.09	−0.08	−0.36	−0.25	−0.28	−0.38	+0.18	+0.06	(*)
35	+0.17	+0.29	+0.39	−0.10	+0.11	−0.10	−0.42	−0.29	−0.32	−0.44	+0.21	+0.07	(*)
40	+0.19	+0.33	+0.44	−0.13	+0.13	−0.13	−0.47	−0.32	−0.36	−0.49	+0.24	+0.08	(*)
45	+0.21	+0.36	+0.48	−0.15	+0.15	−0.15	−0.52	−0.35	−0.40	−0.54	+0.26	+0.08	(*)
50	+0.23	+0.39	+0.52	−0.18	+0.18	−0.18	−0.56	−0.38	−0.43	−0.58	+0.28	+0.09	(*)
55	+0.25	+0.42	+0.56	−0.22	+0.22	−0.22	−0.60	−0.41	−0.46	−0.62	+0.30	+0.10	(*)
60	+0.26	+0.44	+0.59	−0.27	+0.26	−0.27	−0.63	−0.43	−0.48	−0.66	+0.32	+0.10	(*)
65	+0.27	+0.46	+0.62	−0.33	+0.32	−0.33	−0.66	−0.45	−0.51	−0.69	+0.34	+0.11	(*)
70	+0.28	+0.48	+0.64	−0.41	+0.42	−0.41	−0.69	−0.47	−0.53	−0.71	+0.35	+0.11	(*)
75	+0.29	+0.49	+0.66	−0.50	+0.62	−0.50	−0.70	−0.48	−0.54	−0.73	+0.36	+0.12	(*)
80	+0.30	+0.50	+0.67	−0.65	+1.02	−0.65	−0.72	−0.49	−0.55	−0.75	+0.36	+0.12	(*)
85	+0.30	+0.51	+0.68	−1.46	+1.65	−1.46	−0.73	−0.50	−0.56	−0.76	+0.37	+0.12	(*)
90	+0.30	+0.51	+0.68	...	...	...	−0.73	−0.50	−0.56	−0.76	+0.37	+0.12	(*)

* No flexure applied.

Note (*a*).—In 1877 the same refractions were used for the observations of the planets as from 1868 to 1876.

" (*b*).—The reductions to the Pulkowa tables for zenith distances exceeding 80° may be found more accurately from Table III, page 420.

TABLE V.—*Corrections necessary to reduce the Greenwich zenith distances, etc.*—Continued.

Negative of variable part of R — D correction annually applied.

Z. D. S.	1851 and 1852.	1853.	1854.	1855.	1856.	1857.	1858.	1859.	1860.	1861.	1862.	1863.
°	"	"	"	"	"	"	"	"	"	"	"	"
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	+0.02	+0.03	+0.03	+0.02	+0.03	+0.04	+0.02	+0.05	+0.02	+0.06	+0.05	+0.05
10	+0.04	+0.05	+0.06	+0.05	+0.06	+0.07	+0.05	+0.10	+0.05	+0.09	+0.09	+0.10
15	+0.06	+0.08	+0.09	+0.07	+0.10	+0.11	+0.07	+0.14	+0.07	+0.14	+0.13	+0.15
20	+0.08	+0.10	+0.11	+0.09	+0.13	+0.14	+0.10	+0.19	+0.10	+0.18	+0.17	+0.19
25	+0.10	+0.12	+0.14	+0.11	+0.16	+0.18	+0.12	+0.24	+0.12	+0.23	+0.19	+0.21
30	+0.12	+0.14	+0.17	+0.13	+0.19	+0.21	+0.14	+0.28	+0.14	+0.27	+0.21	+0.23
35	+0.14	+0.17	+0.19	+0.15	+0.21	+0.24	+0.16	+0.32	+0.16	+0.31	+0.21	+0.24
40	+0.15	+0.19	+0.21	+0.17	+0.24	+0.27	+0.18	+0.36	+0.18	+0.35	+0.21	+0.23
45	+0.17	+0.20	+0.23	+0.18	+0.26	+0.30	+0.20	+0.40	+0.20	+0.38	+0.19	+0.22
50	+0.18	+0.22	+0.25	+0.20	+0.28	+0.32	+0.21	+0.43	+0.21	+0.41	+0.17	+0.20
55	+0.20	+0.24	+0.27	+0.21	+0.30	+0.34	+0.23	+0.46	+0.23	+0.44	+0.15	+0.17
60	+0.21	+0.25	+0.29	+0.23	+0.32	+0.36	+0.24	+0.49	+0.24	+0.47	+0.12	+0.13
65	+0.22	+0.26	+0.30	+0.24	+0.34	+0.38	+0.25	+0.51	+0.25	+0.49	+0.09	+0.10
70	+0.23	+0.27	+0.31	+0.24	+0.35	+0.39	+0.26	+0.53	+0.26	+0.51	+0.06	+0.07
75	+0.23	+0.28	+0.32	+0.25	+0.36	+0.41	+0.27	+0.54	+0.27	+0.52	+0.04	+0.04
80	+0.24	+0.29	+0.33	+0.26	+0.36	+0.41	+0.28	+0.55	+0.28	+0.53	+0.02	+0.02
85	+0.24	+0.29	+0.33	+0.26	+0.37	+0.42	+0.28	+0.56	+0.28	+0.54	0.00	0.00
90	+0.24	+0.29	+0.33	+0.26	+0.37	+0.42	+0.28	+0.56	+0.28	+0.54	0.00	0.00

Z. D. S.	1864.	1865.	1866.	1867.	1868.	1869.	1870 and 1871.	1872.	1873.	1874.	1875.	1876.
°	"	"	"	"	"	"	"	"	"	"	"	"
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	+0.04	+0.07	−0.07	−0.06	−0.08	−0.07	−0.06	−0.05	−0.05	−0.05	−0.03	−0.03
10	+0.08	+0.13	−0.13	−0.12	−0.15	−0.14	−0.12	−0.09	−0.10	−0.09	−0.06	−0.07
15	+0.11	+0.19	−0.19	−0.17	−0.22	−0.20	−0.18	−0.13	−0.15	−0.13	−0.08	−0.09
20	+0.14	+0.23	−0.24	−0.21	−0.27	−0.27	−0.23	−0.17	−0.19	−0.17	−0.10	−0.12
25	+0.16	+0.27	−0.28	−0.24	−0.32	−0.29	−0.25	−0.19	−0.21	−0.19	−0.12	−0.14
30	+0.17	+0.29	−0.30	−0.26	−0.34	−0.31	−0.27	−0.21	−0.23	−0.21	−0.13	−0.15
35	+0.18	+0.30	−0.31	−0.27	−0.35	−0.32	−0.28	−0.21	−0.23	−0.22	−0.13	−0.15
40	+0.17	+0.30	−0.30	−0.26	−0.34	−0.31	−0.28	−0.21	−0.23	−0.21	−0.13	−0.15
45	+0.16	+0.27	−0.28	−0.24	−0.32	−0.29	−0.26	−0.19	−0.22	−0.20	−0.12	−0.14
50	+0.14	+0.24	−0.25	−0.22	−0.29	−0.26	−0.23	−0.18	−0.19	−0.18	−0.11	−0.12
55	+0.12	+0.21	−0.22	−0.19	−0.25	−0.22	−0.20	−0.15	−0.16	−0.15	−0.09	−0.11
60	+0.10	+0.16	−0.17	−0.15	−0.20	−0.18	−0.16	−0.12	−0.13	−0.12	−0.07	−0.08
65	+0.07	+0.12	−0.13	−0.11	−0.15	−0.13	−0.12	−0.09	−0.10	−0.09	−0.05	−0.06
70	+0.05	+0.08	−0.09	−0.08	−0.10	−0.09	−0.08	−0.06	−0.07	−0.06	−0.04	−0.04
75	+0.03	+0.05	−0.05	−0.04	−0.06	−0.06	−0.05	−0.04	−0.04	−0.04	−0.02	−0.03
80	+0.01	+0.02	−0.02	−0.02	−0.03	−0.02	−0.02	−0.02	−0.02	−0.02	−0.01	−0.01
85	0.00	+0.01	−0.01	−0.01	−0.01	−0.01	−0.01	0.00	0.00	0.00	0.00	0.00
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

TABLE V.—*Corrections necessary to reduce the Greenwich zenith distances, etc.*—**Continued.**

Negative of variable part of R D correction annually applied.

Z. D. S.	1877.	1878.	1879.	1880.	1881.	1882.	1883.	1884.	1885.	1886.	1887.
°	"	"	"	"	"	"	"	"	"	"	"
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	-0.02	-0.06	-0.03	-0.05	-0.05	-0.04	0.06	-0.06	-0.06	-0.06	-0.06
10	-0.04	-0.11	-0.07	-0.09	-0.09	-0.08	-0.11	-0.11	-0.12	-0.13	-0.12
15	-0.07	-0.15	-0.09	-0.13	-0.14	-0.11	-0.17	-0.17	-0.18	-0.19	-0.18
20	-0.08	-0.19	-0.12	-0.16	-0.19	-0.15	-0.23	-0.23	-0.24	-0.25	-0.23
25	-0.10	-0.22	-0.14	-0.19	-0.23	-0.18	-0.28	-0.28	-0.29	-0.31	-0.29
30	-0.11	-0.24	-0.15	-0.20	-0.27	-0.22	0.33	-0.33	-0.34	-0.37	-0.34
35	-0.11	-0.25	-0.15	-0.21	-0.32	-0.25	-0.38	-0.38	-0.39	-0.42	-0.39
40	-0.11	-0.24	-0.15	-0.21	-0.35	-0.28	-0.42	-0.42	-0.44	-0.47	-0.44
45	-0.10	-0.23	-0.14	-0.19	-0.39	-0.31	-0.47	-0.47	-0.49	-0.52	-0.48
50	-0.09	-0.20	-0.12	-0.17	-0.42	-0.33	-0.50	-0.50	-0.53	-0.56	-0.52
55	-0.08	-0.17	-0.11	-0.15	-0.45	-0.36	-0.54	-0.54	-0.56	-0.60	-0.56
60	-0.06	-0.14	-0.08	-0.12	-0.48	-0.38	-0.57	-0.57	-0.60	-0.63	-0.59
65	-0.05	-0.10	-0.06	-0.09	-0.50	-0.40	-0.60	-0.60	-0.62	-0.66	-0.62
70	-0.03	-0.07	-0.04	-0.06	-0.52	-0.41	-0.62	-0.62	-0.65	-0.69	-0.64
75	-0.01	-0.04	-0.03	-0.04	-0.53	-0.42	-0.64	-0.64	-0.66	-0.71	-0.66
80	-0.01	-0.02	-0.01	-0.02	-0.54	-0.43	-0.65	-0.65	-0.68	-0.72	-0.67
85	0.00	-0.01	0.00	0.00	-0.55	-0.43	-0.66	-0.66	-0.68	-0.73	-0.68
90	0.00	0.00	0.00	0.00	-0.55	-0.44	-0.67	-0.67	-0.69	-0.73	-0.68

We now form, tor each year, the sum of the four quantities given in this table. We shall thus have the sum total of the supplementary corrections to the zenith distances applicable to the printed results of direct observations as finally reduced. Since, however, in the published volumes the reflex results are reduced to the same system as the direct ones and combined with them, these same corrections are, in the general mean, applicable to the concluded results of both classes of observations.

TABLE VI.—*Sum of the four corrections to the zenith distances.*

Zenith distance.	1851 and 1852.	1853.	1854.	1855.	1856.	1857.	1858.	1859.	1860.	1861.	1862.	1863.	1864.
°	″	″	″	″	″	″	″	″	″	″	″	″	″
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	−0.02	+0.01	+0.01	0.00	+0.01	+0.01	−0.01	+0.02	−0.01	+0.03	+0.02	+0.02	+0.01
10	−0.07	−0.02	−0.01	−0.02	−0.01	−0.01	−0.03	+0.02	−0.03	+0.01	+0.01	+0.02	0.00
15	−0.09	−0.01	0.00	−0.02	+0.01	+0.01	−0.03	+0.04	−0.03	+0.04	+0.03	+0.05	+0.01
20	−0.12	−0.02	−0.01	−0.03	+0.01	0.00	−0.04	+0.05	−0.04	+0.04	+0.03	+0.05	0.00
25	−0.15	−0.03	−0.01	−0.04	+0.01	0.00	−0.06	+0.06	−0.06	+0.05	+0.01	+0.03	−0.02
30	−0.17	−0.04	−0.01	−0.05	+0.01	0.00	−0.07	+0.07	−0.07	+0.06	0.00	+0.02	−0.04
35	−0.21	−0.05	−0.03	−0.07	−0.01	−0.01	−0.09	+0.07	−0.09	+0.06	−0.04	−0.01	−0.07
40	−0.26	−0.07	−0.05	−0.09	−0.02	−0.03	−0.12	+0.06	−0.12	+0.05	−0.09	−0.07	−0.13
45	−0.29	−0.09	−0.06	−0.11	−0.03	−0.04	−0.14	+0.07	−0.14	+0.04	−0.15	−0.12	−0.18
50	−0.33	−0.11	−0.08	−0.13	−0.05	−0.06	−0.17	+0.05	−0.17	+0.04	−0.20	−0.17	−0.24
55	−0.37	−0.14	−0.11	−0.17	−0.08	−0.09	−0.20	+0.03	−0.20	+0.01	−0.28	−0.26	−0.31
60	−0.43	−0.19	−0.15	−0.21	−0.12	−0.13	−0.25	0.00	−0.25	−0.02	−0.37	−0.36	−0.39
65	−0.50	−0.25	−0.21	−0.27	−0.17	−0.19	−0.32	−0.06	−0.32	−0.08	−0.48	−0.47	−0.50
70	−0.59	−0.33	−0.29	−0.36	−0.25	−0.27	−0.40	−0.13	−0.40	−0.15	−0.60	−0.59	−0.61
75	−0.68	−0.41	−0.37	−0.44	−0.33	−0.35	−0.49	−0.22	−0.49	−0.24	−0.72	−0.72	−0.73
80	−0.83	−0.55	−0.51	−0.58	−0.48	−0.49	−0.62	−0.35	−0.62	−0.37	−0.88	−0.88	−0.89
85	−1.65	−1.37	−1.33	−1.40	−1.29	−1.30	−1.44	−1.16	−1.44	−1.18	−1.72	−1.72	−1.72

Zenith distance.	1865.	1866.	1867.	1868.	1869.	1870.	1871.	1872.	1873.	1874.	1875.	1876.
°	″	″	″	″	″	″	″	″	″	″	″	″
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	+0.02	−0.02	−0.01	0.00	+0.01	+0.02	0.00	+0.01	+0.01	+0.02	+0.04	+0.04
10	+0.02	−0.05	−0.04	−0.01	0.00	+0.02	−0.02	+0.01	0.00	+0.02	+0.05	+0.04
15	+0.03	−0.05	−0.03	+0.01	+0.03	+0.05	−0.02	+0.03	+0.01	+0.03	+0.08	+0.07
20	+0.02	−0.06	−0.03	+0.02	+0.02	+0.06	−0.03	+0.03	+0.01	+0.03	+0.10	+0.08
25	+0.01	−0.06	−0.02	+0.05	+0.08	+0.12	+0.01	+0.07	+0.05	+0.07	+0.14	+0.12
30	−0.02	−0.05	−0.01	+0.08	+0.11	+0.15	+0.03	+0.09	+0.07	+0.08	+0.16	+0.14
35	−0.07	−0.03	+0.01	+0.14	+0.17	+0.21	+0.07	+0.14	+0.12	+0.14	+0.23	+0.21
40	−0.13	0.00	+0.04	+0.22	+0.25	+0.28	+0.12	+0.19	+0.17	+0.19	+0.27	+0.25
45	−0.21	+0.04	+0.08	+0.30	+0.33	+0.36	+0.18	+0.25	+0.22	+0.23	+0.31	+0.29
50	−0.29	+0.08	+0.11	+0.40	+0.43	+0.46	+0.27	+0.32	+0.31	+0.32	+0.39	+0.38
55	−0.38	+0.11	+0.14	+0.52	+0.55	+0.57	+0.37	+0.42	+0.41	+0.42	+0.48	+0.46
60	−0.51	+0.14	+0.16	+0.64	+0.66	+0.68	+0.46	+0.50	+0.49	+0.50	+0.55	+0.54
65	−0.63	+0.15	+0.17	+0.78	+0.80	+0.81	+0.58	+0.61	+0.60	+0.61	+0.65	+0.64
70	−0.76	+0.13	+0.14	+0.95	+0.96	+0.97	+0.73	+0.75	+0.74	+0.74	+0.76	+0.76
75	−0.89	+0.10	+0.11	+1.21	+1.21	+1.22	+0.98	+0.99	+0.99	+0.99	+1.01	+1.00
80	−1.08	−0.01	−0.01	+1.65	+1.66	+1.66	+1.42	+1.42	+1.42	+1.42	+1.43	+1.43
85	−1.91	−1.80	−1.80	+2.31	+2.31	+2.31	+2.06	+2.07	+2.07	+2.07	+2.07	+2.07

Table VI.—*Sum of the four corrections to the zenith distances*—Continued.

Zenith distance.	1877 (*a*) (planets).	1877 (*b*) (stars).	1878.	1879.	1880.	1881.	1882.	1883.	1884.	1885.	1886.	1887.
°	″	″	″	″	″	″	″	″	″	″	″	″
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	+0.04	+0.01	−0.02	0.00	−0.02	−0.02	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
10	+0.06	0.00	−0.03	−0.01	−0.03	−0.03	−0.02	−0.02	−0.02	−0.03	−0.04	−0.03
15	+0.09	0.00	−0.03	0.00	−0.04	−0.05	−0.02	−0.03	−0.03	−0.04	−0.05	−0.04
20	+0.12	+0.01	−0.03	0.00	−0.04	−0.07	−0.03	−0.05	−0.05	−0.06	−0.05	−0.05
25	+0.16	+0.01	−0.03	0.00	−0.05	−0.09	−0.04	−0.06	−0.06	−0.07	−0.09	−0.07
30	+0.19	+0.02	−0.01	+0.02	−0.03	−0.10	−0.05	−0.07	−0.07	−0.08	−0.11	−0.08
35	+0.24	+0.03	+0.01	+0.04	−0.02	−0.13	−0.06	−0.09	−0.09	−0.10	−0.13	−0.10
40	+0.29	+0.03	+0.04	+0.05	−0.01	−0.15	−0.08	−0.11	−0.11	−0.13	−0.16	−0.13
45	+0.34	+0.04	+0.06	+0.07	+0.02	−0.18	−0.10	−0.14	−0.14	−0.16	−0.19	−0.15
50	+0.41	+0.05	+0.10	+0.09	+0.04	−0.21	−0.12	−0.16	−0.16	−0.19	−0.22	−0.18
55	+0.49	+0.05	+0.13	+0.09	+0.05	−0.25	−0.16	−0.20	−0.20	−0.22	−0.26	−0.22
60	+0.56	+0.03	+0.13	+0.09	+0.05	−0.31	−0.21	−0.25	−0.25	−0.28	−0.31	−0.27
65	+0.65	0.00	+0.14	+0.07	+0.04	−0.37	−0.27	−0.31	−0.31	−0.33	−0.37	−0.33
70	+0.78	−0.05	+0.11	+0.03	+0.01	−0.45	−0.34	−0.39	−0.39	−0.42	−0.46	−0.41
75	+1.02	−0.10	+0.07	−0.04	−0.05	−0.52	−0.43	−0.48	−0.48	−0.50	−0.55	−0.50
80	+1.43	−0.24	−0.05	−0.16	−0.17	−0.69	−0.58	−0.63	−0.63	−0.66	−0.70	−0.65
85	+2.07	−1.04	−0.84	−0.95	−0.95	−1.50	−1.38	−1.44	−1.44	−1.46	−1.51	−1.46

§ 8. *Application of the preceding corrections to forty-six standard stars most frequently observed.*

The reductions which we have hitherto derived are applicable to the zenith distances of the stars. In order that definitive north polar distances may be derived it is necessary to investigate the corrections to the latitude. In connection with this it is also necessary to examine what conclusions follow from the observations of the standard stars generally. To form a basis for all conclusions of this character a selection has been made of 46 of the standard stars most frequently observed through the whole history of the instrument. Eleven of the stars are circumpolar, so that, in all, they afford 57 different points on the meridian. Besides being most frequently and continuously observed, the following conditions were aimed at:

The stars should include a sufficient range of north polar distance for the determination of the systematic differences between the instrumental results and any standard catalogue.

They should be distributed as equally as possible in right ascension, not only in the whole, but in each zone of considerable breadth.

In selecting the stars the meridian was divided into six parts, three on each side of the zenith, making as many zones and groups of stars.

Group I extends from zenith distance 71° north to zenith distance 52° north, the extreme stars of the group being γ Cephei and γ Draconis.

Group II includes the four polar stars.

Group III is identical with Group I, and includes stars which culminate more than 13° above the pole but north of the zenith. It was determined by the condition that it should not include any stars culminating south of the zenith.

Groups IV to VI include stars culminating between the zenith and 62° zenith distance south It might have been advantageously carried nearer the south horizon, but observations of individual stars south of north polar distance 100° are less numerous than those further north.

The tabular north polar distances adopted for comparison are those of Boss in his well-known standard catalogue of the declinations of 500 stars. They are identical with the places of the American Ephemeris since 1881. The following is a list of the stars, together with the standard north polar distances, for 1875. They are arranged in the order of north polar distance:

TABLE VII.—*Adopted tabular standard north polar distances of stars from Boss's catalogue.*

Group.	Star.	R. A.		North polar distance (1875.0).			Ann. var. (1875.0).	Sec. var. (1875.0).
		h.	m.	°	′	″	″	″
Group II	λ Ursæ Minoris	19	49	1	4	8.52	− 9.2170	+7.8715
	α Ursæ Minoris	1	13	1	21	26.14	−19.0458	+0.9670
	51 (Hev.) Cephei	6	41	2	45	55.65	+ 3.6289	+4.3267
	δ Ursæ Minoris	18	13	3	23	32.42	− 1.1574	+2.8244
Group I and III	γ Cephei	23	34	13	3	55.13	−20.0711	− .0304
	β Ursæ Minoris	14	51	15	20	1.18	+14.7165	+ .0187
	50 Cassiopeæ	1	53	18	11	6.16	−17.6975	+ .3503
	κ Draconis	12	28	19	31	21.15	+19.8980	− .0548
	α Camelopardalis	4	42	23	52	23.27	− 6.7231	+ .8151
	α Ursæ Majoris	10	56	27	34	28.69	+19.3439	+ .1420
	α Cephei	21	16	27	56	37.00	−15.1537	− .1334
	α Cassiopeæ	0	33	34	8	54.88	−19.8006	+ .0800
	γ Draconis	17	54	38	29	44.56	+ 0.5808	− .2037
Group IV	η Ursæ Majoris	13	43	40	3	44.20	+18.1020	− .1570
	α Persei	3	15	40	35	8.57	−13.1625	+ .4725
	α Aurigæ	5	7	44	7	54.44	− 4.1231	+ .6312
	α Cygni	20	37	45	9	56.03	−12.6916	− .2253
	α Canum Venat	12	50	51	0	22.28	+19.5274	− .0970
	α Lyræ	18	33	51	19	53.84	− 3.1246	− .2946
	ι Aurigæ	4	49	57	2	2.72	− 6.1044	+ .5437
	ζ Cygni	21	8	60	17	5.78	−14.5765	− .2477
	β Tauri	5	18	61	30	1.51	− 3.4402	+ .5452
	β Geminorum	7	38	61	40	25.80	+ 8.3472	+ .4790
	α Andromedæ	0	2	61	35	59.21	−19.8866	+ .0124
	β Pegasi	22	58	62	35	41.66	−19.4466	− .1061
	α Coronæ	15	29	62	51	48.45	+12.3498	− .2995
Group V	μ Pegasi	22	44	66	3	28.89	−18.9105	− .1294
	α Arietis	2	0	67	7	46.83	−17.2103	+ .2543
	μ Geminorum	6	15	67	25	28.07	+ 1.4680	+ .5289
	δ Geminorum	7	13	67	47	22.40	+ 6.2680	+ .4948
	β Arietis	1	48	69	48	14.06	−17.7609	+ .2262
	α Bootis	14	10	70	9	57.42	+18.9148	− .2151
	γ Geminorum	6	30	73	29	45.90	+ 2.7078	+ .5009
	α Tauri	4	29	73	44	38.22	− 7.5836	+ .4658
	β Leonis	11	43	74	43	45.38	+20.1161	+ .0248
	α Herculis	17	9	75	27	56.24	+ 4.4039	− .3905
	α Pegasi	22	59	75	28	0.99	−19.2872	− .1071

TABLE VII.—*Adopted standard north polar distances of stars from Boss's catalogue*—Continued.

Group.	Star.	R. A.	North polar distance (1875.0).	Ann. var. (1875.0).	Sec. var. (1875.0).
		h. m.	° ′ ″		
Group VI	γ Pegasi	0 7	75 30 41.33	−20.0269	+ .0220
	α Ophiuchi	17 29	77 20 50.56	+ 2.9299	− .4044
	α Leonis	10 2	77 25 21.74	+17.4430	+ .2217
	γ Aquilæ	19 40	79 41 23.69	− 8.4920	− .3732
	α Aquilæ	19 45	81 27 37.25	− 9.2151	− .3841
	α Orionis	5 48	82 37 5.66	− 1.0210	+ .4735
	α Aquarii	21 59	90 55 34.69	−17.3257	− .2185
	α Hydræ	9 21	98 7 4.31	+15.4132	+ .2676
	β Orionis	5 9	98 20 51.96	− 4.4598	+ .4114
	α Virginis	13 19	100 30 29.68	+18.9239	− .1625

The work of comparing these north polar distances with the Greenwich observations was commenced before the corrections in the preceding paper had been worked out, from which it happens that the work has not been carried on in logical order nor done in the most direct way. Not knowing that definitive elements of reduction so satisfactory could be obtained, the first step actually taken was to reduce all the Greenwich results to the following standard:

No flexure.
No R — D correction.
The colatitude 38° 31′ 21″.90.
The Pulkowa refraction tables.

The sum of the four corrections for reduction to this standard was tabulated for each year as a function of the zenith distance.

The concluded north polar distances, given annually in the catalogue of concluded mean right ascensions and north polar distances, were then compared with the tabular north polar distances, and to the difference was applied the sum of the four corrections just cited In this part of the work direct and reflex results, as well as those above and below the pole, were treated separately. As these separate results are not given in the annual catalogues after 1884 the comparison subsequent to that year was made with the preliminary mean results in the section "Ledgers of Mean Right Ascensions and North Polar Distances," etc.

This preliminary table, as I have just said, included reflex observations. As those made on particular stars are for the most part few and scattered they were subsequently allowed to drop out, so that the following discussion rests solely upon the results of direct observations.

When this part of the work was completed it was found that the true standard of comparison should be that of the preceding section. The additional reductions required are the constant error of nadir point, which had been taken out in connection with the R — D discordance, and the flexure actually derived in the present paper. These two corrections being applied to the first results we have a system of concluded corrections to Boss's standard equivalent to those obtained by applying the following reductions:

I. The sum of the reductions given at the end of § 7, Table VI, for reduction to the zenith.
II. The reduction of each year's results to the constant colatitude 38° 31′ 21″.90.

These reduced corrections to the standard north polar distances are given completely in the following table. From the indirect way in which they have been obtained they will frequently differ by a few hundredths of a second from those which would be found by a direct application of the corrections as tabulated. The largest source of difference is the use of a constant factor for reduction to the Pulkowa refractions, whereby the refraction of γ Draconis, S. P. is in error by 0″.10.

The affixed numbers indicate the weights of the several annual results, which have been determined by the formula given in the introduction to the volumes of the later Greenwich observations

TABLE VIII.—*Corrections given by the Greenwich observations to the standard N. P. D.'s of 47 stars, after applying the reductions of Table VI and the reduction to the colatitude 38° 31′ 21″.90.*

Star.	N. P. D.	Z. D.	1851.	1852.	1853.	1854.	1855.	1856.	1857.	1858.
	°	°	″	″	″	″	″	″	″	″
γ Draconis, S. P . . .	−38.5	−77.0	. . .	. . .	−0.19$_{1}$	−1.76$_{2}$	+0.79$_{2}$	+1.20$_{3}$	−0.93$_{6}$	−0.48$_{4}$
α Cassiopeiæ, S. P . . .	−34.2	−72.7	. . .	−0.06$_{2}$	−1.54$_{1}$	+0.03$_{3}$	−0.68$_{2}$	−0.29$_{2}$	+0.67$_{2}$	+0.90$_{1}$
α Cephei, S. P	−28.0	−66.5	. . .	+0.92$_{2}$	−0.09$_{2}$	. . .	−0.84$_{4}$	+0.44$_{4}$	−0.15$_{3}$	+0.15$_{2}$
α Ursæ Majoris, S. P .	−27.5	−66.0	. . .	+0.81$_{1}$	. . .	+0.23$_{5}$	−0.51$_{2}$	+0.24$_{3}$	−0.58$_{2}$	−0.10$_{4}$
α Camelopardalis, S. P .	−23.9	−62.4	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
κ Draconis, S. P . . .	−19.5	−58.0	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
50 Cassiopeiæ, S. P . . .	−18.2	−56.7	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
β Ursæ Minoris, S. P. .	−15.3	−53.8	−0.89$_{1}$	. . .	+1.40$_{1}$	. . .	+1.33$_{1}$	−0.43$_{2}$	+0.13$_{3}$	+0.39$_{4}$
γ Cephei, S. P	−13.1	−51.6	−0.05$_{1}$	+0.66$_{1}$	+0.80$_{1}$	−1.53$_{1}$	+0.23$_{3}$	−0.05$_{1}$	+0.25$_{3}$	+0.28$_{1}$
Mean	. . .	. . .	−0.47$_{2}$	+0.53$_{6}$	+0.05$_{6}$	−0.35$_{11}$	−0.15$_{14}$	+0.31$_{15}$	−0.25$_{19}$	+0.04$_{16}$
δ Ursæ Minoris, S. P. .	− 3.4	−41.9	−0.34$_{6}$	−0.10$_{6}$	−0.58$_{4}$	−0.57$_{6}$	−0.14$_{4}$	−0.12$_{3}$	−0.55$_{6}$	+0.01$_{6}$
51 (Hev.) Cephei, S. P .	− 2.8	−41.3	. . .	+0.24$_{3}$	+0.27$_{4}$	+0.08$_{7}$	−0.18$_{6}$	−0.19$_{7}$	−0.29$_{7}$	+0.02$_{8}$
α Ursæ Minoris, S. P .	− 1.4	−39.9	+0.15$_{9}$	+0.26$_{9}$	−0.10$_{8}$	−0.12$_{9}$	−0.16$_{9}$	+0.08$_{9}$	+0.50$_{9}$	+0.18$_{9}$
λ Ursæ Minoris, S. P .	− 1.1	−39.6	−0.38$_{3}$	+0.35$_{5}$	+0.30$_{4}$	+0.16$_{4}$	−0.78$_{2}$	+0.39$_{3}$	−0.79$_{2}$	−0.01$_{6}$
λ Ursæ Minoris . . .	+ 1.1	−37.4	+0.21$_{5}$	−0.43$_{5}$	+0.14$_{5}$	+0.08$_{5}$	0.00$_{6}$	−0.11$_{6}$	−0.50$_{4}$	+0.49$_{5}$
α Ursæ Minoris . . .	+ 1.4	−37.1	+0.21$_{9}$	+0.25$_{9}$	−0.28$_{8}$	−0.31$_{9}$	−0.08$_{9}$	+0.16$_{9}$	−0.07$_{9}$	+0.35$_{9}$
51 (Hev.) Cephei . . .	+ 2.8	−35.7	. . .	−0.70$_{4}$	−1.06$_{3}$	−0.90$_{6}$	−0.60$_{4}$	−0.19$_{4}$	+0.08$_{4}$	+0.02$_{6}$
δ Ursæ Minoris . . .	+ 3.4	−35.1	+0.03$_{7}$	+0.08$_{7}$	−0.15$_{6}$	+0.05$_{7}$	+0.40$_{6}$	−0.11$_{7}$	−0.04$_{7}$	+0.24$_{9}$
Mean	. . .	. . .	+0.03$_{39}$	+0.04$_{48}$	−0.15$_{42}$	−0.20$_{53}$	−0.12$_{46}$	−0.01$_{48}$	−0.10$_{48}$	+0.17$_{58}$
γ Cephei	+13.1	−25.4	+0.84$_{2}$	+0.61$_{4}$	+0.57$_{5}$	+0.34$_{3}$	+0.89$_{4}$	+0.32$_{2}$	−0.81$_{1}$	. . .
β Ursæ Majoris . . .	+15.3	−23.2	+0.30$_{6}$	+0.18$_{5}$	+0.24$_{5}$	+0.48$_{4}$	+0.64$_{4}$	−0.05$_{4}$	−0.05$_{4}$	+0.04$_{4}$
50 Cassiopeiæ	+18.2	−20.3	. . .	. . .	. .	. . .	. . .	. . .	. . .	. . .
κ Draconis	+19.5	−19.0	. . .	+0.78$_{1}$	+0.09$_{4}$	−0.14$_{4}$	+1.10$_{2}$	. . .	+0.23$_{4}$	+0.65$_{2}$
α Camelopardalis . . .	+23.9	−14.6	−0.64$_{2}$	−0.80$_{1}$	−0.03$_{4}$	+0.17$_{4}$	−0.49$_{2}$	−0.47$_{3}$	+0.19$_{3}$	+0.10$_{2}$
α Ursæ Majoris . . .	+27.5	−11.0	−0.61$_{1}$	+0.33$_{6}$	+0.08$_{4}$	+0.15$_{6}$	+1.05$_{5}$	+0.11$_{4}$	−0.31$_{2}$	+0.38$_{4}$
α Cephei	+28.0	−10.5	−0.11$_{6}$	−0.10$_{5}$	−0.18$_{6}$	+0.39$_{5}$	+0.17$_{3}$	−0.51$_{4}$	+0.40$_{4}$	+0.95$_{4}$
α Cassiopeiæ	+34.2	− 4.3	+0.85$_{1}$	+0.31$_{5}$	−0.13$_{2}$	+0.07$_{2}$	−0.10$_{3}$	−0.49$_{2}$	. . .	−0.71$_{2}$
γ Draconis	+38.5	0.0	+0.18$_{6}$	−0.04$_{5}$	0.00$_{4}$	. . .	+0.38$_{1}$	+0.14$_{2}$	−0.27$_{4}$	−0.06$_{3}$
Mean	. . .	. . .	+0.12$_{24}$	+0.19$_{32}$	+0.10$_{34}$	+0.22$_{28}$	+0.55$_{24}$	−0.16$_{21}$	+0.02$_{22}$	+0.26$_{21}$

TABLE VIII.—*Corrections given by the Greenwich observations to the standard N. P. D.'s of 47 stars, etc.*—**Continued.**

Star.	N. P. D.	Z. D.	1851.	1852.	1853.	1854.	1855.	1856.	1857.	1858.
	°	°	"	"	"	"	"	"	"	"
η Ursæ Majoris	+ 40.0	+ 1.5	-0.17_{5}	-0.14_{3}	$+1.51_{1}$	$+0.70_{3}$	. . .	. . .	-0.23_{2}	-0.08_{3}
α Persei	+ 40.6	+ 2.1	-0.13_{3}	$+0.49_{3}$	$+0.17_{4}$	$+0.19_{3}$	$+0.29_{4}$	-0.06_{4}	. . .	$+0.07_{4}$
α Aurigæ	+ 44.1	+ 5.6	-0.55_{3}	$+0.31_{5}$	-0.43_{3}	$+0.46_{2}$	$+0.65_{5}$	$+0.60_{5}$	$+0.56_{4}$	$+0.38_{3}$
α Cygni	+ 45.2	+ 6.7	$+0.07_{5}$	$+0.34_{7}$	0.00_{4}	-0.03_{5}	$+0.25_{5}$	-0.12_{4}	-0.13_{2}	$+0.27_{4}$
α Canum Ven.	+ 51.0	+12.5	-0.05_{2}	$+0.32_{4}$	-0.94_{1}	-0.20_{3}	$+0.64_{2}$	. . .	$+0.36_{4}$	-0.67_{2}
α Lyræ	+ 51.3	+12.8	-0.53_{7}	$+0.12_{7}$	$+0.35_{7}$	$+0.33_{7}$	$+0.50_{7}$	$+0.46_{6}$	$+0.36_{5}$	$+0.40_{4}$
ι Aurigæ	+ 57.0	+18.5	-0.63_{4}	$+0.66_{3}$	-0.39_{5}	$+0.19_{6}$	-0.15_{5}	-0.35_{3}	$+0.50_{3}$	$+0.32_{3}$
ζ Cygni	+ 60.3	+21.8	0.00_{6}	$+0.08_{6}$	-0.23_{6}	-0.29_{7}	-0.36_{6}	-0.20_{3}	$+0.43_{5}$	$+0.65_{5}$
β Tauri	+ 61.5	+23.0	-0.06_{6}	$+0.05_{6}$	-0.06_{6}	-0.08_{6}	$+0.44_{5}$	$+0.08_{5}$	$+0.63_{4}$	$+0.35_{5}$
β Geminorum	+ 61.6	+23.1	-0.44_{7}	-0.49_{7}	-0.64_{4}	-0.07_{8}	$+0.03_{7}$	$+0.16_{7}$	$+0.36_{6}$	$+0.55_{7}$
α Andromedæ	+ 61.6	+23.1	-0.45_{7}	-0.12_{6}	$+0.08_{5}$	$+0.02_{4}$	-0.20_{4}	-0.50_{2}	$+0.49_{4}$	-0.20_{2}
β Pegasi	+ 62.6	+24.1	-0.52_{2}	-0.16_{3}	. . .	. . .	. . .	. . .	. . .	. . .
α Coronæ	+ 62.8	+24.3	-0.09_{7}	-0.01_{8}	-0.31_{7}	-0.03_{6}	-0.34_{6}	-0.24_{5}	$+0.71_{4}$	$+0.36_{6}$
Mean	. . .	. . .	-0.27_{64}	$+0.08_{68}$	-0.11_{53}	$+0.05_{60}$	$+0.13_{56}$	$+0.06_{44}$	$+0.42_{43}$	$+0.29_{48}$
μ Pegasi	+ 66.1	+27.6	-0.54_{2}	$+0.68_{3}$	$+0.77_{5}$	$+1.14_{5}$	$+0.12_{4}$	$+0.58_{2}$	$+0.10_{4}$	$+0.53_{4}$
α Arietis	+ 67.2	+28.7	-0.55_{6}	$+0.27_{7}$	$+0.47_{6}$	$+0.06_{7}$	$+0.07_{5}$	$+0.64_{4}$	$+0.73_{4}$	$+0.46_{5}$
μ Geminorum	+ 67.4	+28.9	-0.08_{6}	$+0.35_{5}$	$+0.17_{4}$	$+0.44_{5}$	$+0.38_{4}$	$+0.73_{5}$	-0.70_{3}	-0.40_{2}
δ Geminorum	+ 67.8	+29.3	-0.37_{6}	$+0.08_{7}$	$+0.45_{5}$	$+0.30_{5}$	$+0.53_{4}$	$+0.53_{4}$	$+0.29_{2}$	$+0.37_{5}$
β Arietis	+ 69.8	+31.3	. . .	-0.82_{4}	$+0.98_{2}$	$+0.43_{6}$	$+0.38_{3}$	$+0.05_{2}$	-0.65_{3}	$+0.71_{3}$
α Bootis	+ 70.1	+31.6	$+0.22_{8}$	-0.05_{8}	-0.01_{7}	$+0.37_{7}$	-0.28_{7}	$+0.06_{6}$	$+0.16_{5}$	$+0.24_{7}$
γ Geminorum	+ 73.5	+35.0	-0.72_{2}	$+0.57_{2}$	-0.03_{2}	$+0.36_{3}$	$+0.96_{2}$	. . .	-0.64_{1}	$+0.69_{2}$
α Tauri	+ 73.8	+35.3	-0.19_{6}	-0.21_{7}	-0.09_{6}	$+0.65_{7}$	$+0.46_{6}$	$+0.33_{6}$	$+0.31_{5}$	$+0.26_{4}$
β Leonis	+ 74.7	+36.2	$+0.20_{7}$	$+0.28_{7}$	$+0.50_{5}$	$+0.68_{6}$	$+0.21_{5}$	$+0.80_{2}$	$+0.80_{3}$	-1.39_{1}
α¹ Herculis	+ 75.5	+37.0	$+0.19_{7}$	$+0.87_{6}$	$+0.05_{5}$	$+0.49_{6}$	$+0.99_{3}$	$+0.86_{3}$	$+0.29_{3}$	$+0.46_{4}$
α Pegasi	+ 75.5	+37.0	$+0.23_{7}$	$+0.35_{6}$	$+0.52_{5}$	$+0.52_{5}$	$+0.07_{5}$	$+0.34_{2}$	-0.07_{2}	$+0.84_{3}$
Mean	. . .	. . .	-0.06_{57}	$+0.18_{62}$	$+0.31_{52}$	$+0.49_{62}$	$+0.27_{48}$	$+0.47_{36}$	$+0.13_{35}$	$+0.37_{40}$
γ Pegasi	+ 75.5	+37.0	$+0.40_{7}$	$+0.43_{6}$	$+0.59_{5}$	$+0.73_{4}$	$+0.55_{5}$	$+1.30_{2}$	$+0.62_{4}$	$+0.12_{3}$
α Ophiuchi	+ 77.3	+38.8	$+0.03_{8}$	$+0.36_{8}$	$+0.16_{6}$	$+0.15_{7}$	$+0.12_{6}$	$+0.34_{6}$	$+0.16_{5}$	$+0.53_{5}$
α Leonis	+ 77.4	+38.9	-0.08_{7}	$+0.18_{7}$	$+0.17_{6}$	$+0.30_{8}$	$+0.35_{6}$	-0.22_{6}	$+0.11_{4}$	$+0.12_{7}$
γ Aquilæ	+ 79.7	+41.2	$+0.20_{6}$	$+0.42_{7}$	$+0.21_{6}$	$+0.44_{7}$	$+0.33_{4}$	$+0.89_{3}$	$+0.23_{4}$	$+0.27_{6}$
α Aquilæ	+ 81.5	+43.0	-0.34_{6}	$+0.48_{7}$	$+0.74_{5}$	$+0.47_{8}$	$+0.89_{6}$	-0.10_{4}	$+0.09_{6}$	$+0.04_{6}$
α Orionis	+ 82.6	+44.1	-0.29_{7}	-0.17_{7}	$+0.13_{4}$	$+0.07_{6}$	$+0.24_{5}$	$+0.75_{5}$	$+0.55_{4}$	$+0.67_{3}$
α Aquarii	+ 91.0	+52.5	-0.69_{7}	-0.49_{6}	$+0.28_{5}$	-0.51_{5}	-0.15_{6}	. . .	$+0.37_{3}$	$+0.04_{2}$
α Hydræ	+ 98.1	+59.6	-0.24_{7}	-0.20_{5}	-0.28_{4}	$+0.47_{3}$	-0.05_{4}	-0.33_{4}	$+2.66_{1}$	$+0.64_{2}$
β Orionis	+ 98.4	+59.9	-0.33_{6}	-0.03_{5}	-0.14_{5}	$+0.35_{7}$	$+0.18_{5}$	-0.01_{5}	-0.34_{4}	$+0.87_{4}$
α Virginis	+100.5	+62.0	-0.82_{8}	-0.57_{8}	-0.27_{7}	-0.67_{8}	-0.12_{7}	-0.93_{7}	-1.03_{7}	$+0.04_{7}$
Mean	. . .	. . .	-0.22_{69}	$+0.05_{66}$	$+0.15_{53}$	$+0.15_{63}$	$+0.23_{54}$	$+0.03_{42}$	$+0.06_{42}$	$+0.29_{45}$

TABLE VIII.—*Corrections given by the Greenwich observations to the standard N. P. D.'s of 47 stars, etc.*—**Continued.**

Star.	1859.	1860.	1861.	1862.	1863.	1864.	1865.	1866.	1867.	1868.
	"	"	"	"	"	"	"	"	"	"
γ Draconis, S. P . . .	-0.18_{4}	-0.75_{4}	-0.23_{2}	. . .	-0.42_{3}	-0.72_{2}	. . .	$+1.59_{1}$	. . .	-1.61_{1}
α Cassiopeæ, S. P . . .	-0.07_{2}	. . .	. . .	. . .	. . .	$+0.11_{2}$	. . .	. . .	. . .	$+0.03_{4}$
α Cephei, S. P	-0.65_{3}	$+0.04_{2}$	-0.55_{2}	$+0.44_{1}$	-0.39_{2}	$+0.36_{2}$	. . .	$+0.94_{1}$	. . .	-0.25_{4}
α Ursæ Majoris, S. P .	$+0.25_{4}$	$+0.19_{2}$	$+0.17_{3}$	. . .	. . .	. . .	$+0.24_{2}$	$+0.62_{2}$	-0.14_{4}	-0.26_{5}
α Camelopardalis, S. P .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
κ Draconis, S. P . . .	$+0.68_{3}$	$+0.99_{1}$	-0.06_{2}	. . .	-0.36_{1}	. . .	. . .	$+0.20_{2}$	$+0.18_{3}$	$+0.29_{4}$
50 Cassiopeæ, S. P . . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
β Ursæ Minoris, S. P .	$+1.07_{4}$	$+0.46_{1}$	-0.01_{2}	-0.01_{2}	. . .	. . .	. . .	$+1.22_{3}$	. . .	$+0.78_{2}$
γ Cephei, S. P	-0.71_{1}	-0.80_{2}	-0.02_{2}	. . .	-0.53_{1}	$+0.70_{1}$	. . .	$+0.85_{3}$	-0.03_{1}	$+0.76_{3}$
Mean	$+0.18_{21}$	0.22_{12}	0.09_{13}	$+0.14_{3}$	-0.42_{7}	$+0.03_{7}$	$+0.24_{2}$	$+0.86_{12}$	-0.01_{8}	$+0.05_{23}$
δ Ursæ Minoris, S. P .	$+1.01_{6}$	-0.19_{6}	-0.56_{2}	$+0.64_{2}$	-0.41_{6}	-0.34_{6}	-0.61_{5}	-0.62_{6}	. . .	-0.03_{5}
51 (Hev.) Cephei, S. P .	$+0.16_{8}$	-0.11_{5}	$+0.08_{7}$	-0.18_{7}	$+0.04_{8}$	$+0.22_{8}$	-0.22_{6}	-0.19_{5}	-0.52_{4}	-0.60_{7}
α Ursæ Minoris, S. P .	$+0.18_{9}$	-0.02_{9}	$+0.32_{9}$	$+0.22_{9}$	$+0.21_{9}$	$+0.20_{9}$	-0.16_{9}	0.00_{8}	$+0.05_{8}$	$+0.25_{9}$
λ Ursæ Minoris, S. P .	-0.22_{5}	-0.08_{2}	-0.23_{2}	. .	-0.39_{7}	-1.42_{1}	-1.06_{2}	-2.40_{2}	$+0.52_{1}$	$+0.07_{1}$
λ Ursæ Minoris	$+0.03_{4}$	-0.38_{4}	0.00_{5}	-0.04_{5}	$+0.15_{6}$	-0.20_{3}	-0.26_{3}	-0.75_{1}	-0.57_{1}	$+0.29_{3}$
α Ursæ Minoris	$+0.42_{9}$	$+0.11_{9}$	$+0.14_{9}$	$+0.42_{8}$	$+0.39_{9}$	$+0.13_{9}$	$+0.31_{9}$	$+0.43_{8}$	-0.05_{8}	-0.08_{9}
51 (Hev.) Cephei . . .	-0.10_{7}	-0.44_{7}	-0.39_{4}	-0.13_{2}	-0.38_{7}	-0.23_{6}	-0.52_{5}	-0.58_{5}	$+1.05_{2}$	0.00_{5}
δ Ursæ Minoris	$+0.56_{8}$	$+0.07_{6}$	-0.18_{7}	-0.50_{6}	-0.20_{7}	$+0.39_{7}$	$+0.24_{6}$	$+0.15_{5}$	-0.18_{4}	-0.16_{7}
Mean	$+0.28_{56}$	-0.11_{48}	$+0.01_{45}$	$+0.05_{39}$	-0.04_{59}	$+0.04_{49}$	-0.16_{45}	-0.22_{40}	-0.03_{28}	-0.07_{46}
γ Cephei	$+0.60_{4}$	. . .	$+0.95_{2}$	. . .	-0.40_{2}	$+0.07_{2}$	. . .	$+0.86_{1}$	-0.63_{3}	-0.17_{3}
β Ursæ Majoris	-0.38_{4}	$+0.20_{2}$	-0.03_{4}	0.00_{2}	-0.28_{2}	-0.59_{3}	-0.66_{4}	$+0.03_{5}$	$+0.28_{5}$	-0.24_{5}
50 Cassiopeæ	. . .	$+0.85_{2}$	. . .	. . .	. . .	-0.06_{3}	. . .	-0.46_{2}	-0.04_{4}	-0.32_{3}
κ Draconis	-0.46_{1}	. . .	-0.62_{1}	. . .	$+0.91_{1}$	-1.03_{1}	$+0.03_{1}$	. . .	. . .	$+0.22_{4}$
α Camelopardalis . . .	$+0.74_{2}$	. . .	. . .	. . .	-0.69_{2}	-0.20_{2}	. . .	-0.19_{2}	-0.98_{1}	$+0.44_{2}$
α Ursæ Majoris	$+1.28_{1}$	$+0.13_{1}$	$+0.33_{2}$	$+0.05_{2}$	. . .	-0.19_{3}	$+0.11_{4}$	-0.09_{4}	$+0.23_{3}$	-0.04_{5}
α Cephei	$+1.22_{5}$	-0.28_{3}	$+0.57_{4}$	. . .	$+0.38_{3}$	. . .	$+0.74_{2}$	. . .	$+0.34_{4}$	$+0.09_{5}$
α Cassiopeæ	$+0.40_{2}$	$+0.30_{2}$	-0.73_{1}	. . .	-0.72_{2}	$+0.38_{4}$	. . .	-0.34_{2}	-0.35_{4}	-0.05_{4}
γ Draconis	$+0.45_{5}$	-0.06_{5}	-0.01_{3}	. . .	$+0.31_{1}$	. .	$+0.20_{2}$	. . .	-0.82_{2}	$+0.03_{2}$
Mean	$+0.51_{24}$	$+0.11_{15}$	$+0.20_{17}$	$+0.02_{4}$	-0.14_{13}	-0.13_{18}	-0.02_{13}	-0.08_{16}	-0.10_{26}	-0.02[illegible]

TABLE VIII.—*Corrections given by the Greenwich observations to the standard N. P. D.'s of 47 stars, etc.*—**Continued.**

Star.	1859.	1860.	1861.	1862.	1863.	1864.	1865.	1866.	1867.	1868.
	"	"	"	"	"	"	"	"	"	"
η Ursæ Majoris	-0.01_{1}	-0.60_{1}	$+0.07_{4}$	. . .	-1.01_{1}	. . .	-0.39_{2}	$+0.03_{4}$	$+0.73_{4}$	$+0.34_{5}$
α Persei	-0.12_{2}	$+0.58_{1}$	-0.32_{2}	. . .	. . .	$+2.55_{1}$	-0.44_{2}	-0.08_{2}	$+0.89_{1}$	-0.01_{3}
α Aurigæ	$+0.52_{5}$	$+0.25_{4}$	$+0.04_{5}$	$+0.74_{2}$	. . .	-0.20_{4}	-0.43_{2}	$+0.57_{5}$	-0.14_{3}	$+0.38_{5}$
α Cygni	$+0.40_{6}$	$+0.55_{2}$	$+0.41_{2}$	$+0.60_{1}$	$+0.11_{2}$	$+0.64_{1}$	$+0.25_{2}$	-0.44_{2}	-0.37_{3}	-0.25_{4}
α Canum Ven.	$+0.19_{2}$	-0.57_{1}	-0.54_{3}	$+0.28_{1}$	$+0.44_{1}$	-0.16_{1}	$+0.07_{3}$	-0.87_{2}	$+0.10_{3}$	$+0.44_{5}$
α Lyræ	$+0.61_{4}$	$+0.73_{5}$	$+0.21_{7}$	$+0.63_{2}$	$+0.35_{5}$	$+0.09_{7}$	$+0.50_{6}$	-0.01_{6}	-0.15_{5}	-0.13_{6}
ι Aurigæ	$+0.88_{4}$	$+0.52_{4}$	$+0.50_{2}$	$+0.46_{2}$	-0.66_{2}	-0.56_{2}	$+0.09_{2}$	$+0.54_{2}$	-1.16_{2}	$+0.91_{2}$
ζ Cygni	$+0.82_{6}$	$+1.11_{1}$	$+0.20_{4}$	-0.86_{1}	$+1.33_{2}$	$+0.47_{2}$	$+0.48_{5}$	-0.53_{2}	$+0.32_{2}$	-0.31_{4}
β Tauri	$+0.67_{6}$	$+0.57_{4}$	$+0.51_{4}$	$+0.22_{5}$	-0.02_{3}	$+0.14_{5}$	-0.51_{4}	$+0.71_{3}$	$+1.04_{2}$	$+0.25_{5}$
β Geminorum	$+0.27_{5}$	$+0.03_{5}$	-0.32_{3}	$+0.43_{3}$	$+0.42_{2}$	-0.01_{5}	-0.53_{6}	$+0.05_{2}$	-0.44_{3}	$+0.08_{5}$
α Andromedæ	$+0.96_{6}$	$+0.70_{4}$	$+0.41_{2}$	$+0.45_{3}$	-0.32_{3}	$+0.31_{5}$	$+0.05_{4}$	-0.16_{4}	$+0.18_{3}$	$+0.40_{4}$
β Pegasi	. . .	$+0.24_{2}$	$+0.40_{2}$	$+0.69_{2}$	$+0.30_{3}$	. . .	. . .	. . .	. . .	. . .
α Coronæ	$+0.74_{4}$	$+0.23_{6}$	$+0.17_{5}$	$+0.62_{2}$	$+0.72_{4}$	$+0.47_{6}$	$+0.70_{5}$	$+1.27_{3}$	$+0.43_{5}$	$+0.05_{6}$
Mean	$+0.59_{51}$	$+0.39_{40}$	$+0.13_{45}$	$+0.42_{24}$	$+0.23_{28}$	$+0.20_{39}$	$+0.05_{43}$	$+0.15_{37}$	$+0.10_{36}$	$+0.15_{54}$
μ Pegasi	. . .	$+1.09_{1}$	-0.72_{3}	$+0.12_{4}$	$+0.47_{3}$	-0.36_{2}	. . .	. . .	$+0.23_{3}$	-0.85_{3}
α Arietis	$+0.85_{5}$	$+0.92_{4}$	$+0.49_{4}$	$+0.58_{4}$	$+0.62_{2}$	$+1.47_{3}$	$+0.26_{4}$	$+0.25_{5}$	$+0.24_{4}$	-0.32_{4}
μ Geminorum	$+1.04_{3}$	$+0.70_{5}$	$+0.14_{3}$	$+0.67_{4}$	$+1.43_{4}$	-0.20_{4}	$+0.06_{4}$	-0.73_{2}	-0.48_{4}	-0.28_{2}
δ Geminorum	$+0.80_{2}$	$+0.72_{4}$	$+0.41_{3}$	$+0.70_{3}$	$+0.20_{2}$	-0.33_{3}	$+0.63_{3}$	-0.21_{4}	-0.08_{3}	$+0.05_{2}$
β Arietis	$+0.63_{5}$	$+1.22_{4}$	$+0.58_{5}$	$+0.60_{4}$	-0.05_{2}	$+0.30_{2}$	$+0.94_{2}$	$+0.05_{4}$	$+0.24_{2}$	$+0.63_{3}$
α Bootis	$+0.65_{7}$	$+0.18_{5}$	$+0.29_{5}$	$+0.14_{4}$	$+0.48_{4}$	$+0.76_{6}$	-0.10_{7}	$+0.20_{7}$	$+0.10_{6}$	$+0.54_{6}$
γ Geminorum	$+0.02_{1}$	$+0.27_{3}$	. . .	$+0.08_{3}$	$+0.18_{3}$	$+0.20_{3}$	-0.50_{5}	-0.32_{4}	$+0.56_{3}$	-0.49_{3}
α Tauri	$+1.26_{5}$	$+1.07_{5}$	-0.01_{3}	$+0.31_{5}$	$+0.32_{4}$	$+0.24_{6}$	$+0.27_{7}$	-0.09_{5}	$+0.42_{4}$	$+0.11_{6}$
β Leonis	$+0.76_{2}$	-0.15_{5}	-0.37_{4}	-0.28_{1}	. . .	$+0.57_{3}$	$+0.92_{5}$	-0.04_{5}	$+0.33_{3}$	$+0.22_{4}$
α′ Herculis	$+1.46_{4}$	-0.12_{3}	$+0.25_{3}$	$+0.55_{2}$	-0.20_{2}	$+0.48_{3}$	$+0.77_{4}$	$+0.33_{4}$	. . .	$+0.47_{4}$
α Pegasi	$+0.77_{4}$	. . .	$+1.03_{2}$	$+1.33_{2}$	$+0.45_{1}$	-0.32_{4}	$+0.76_{3}$	$+0.75_{2}$	$+0.14_{2}$	-0.56_{4}
Mean	$+0.88_{38}$	$+0.56_{39}$	$+0.20_{35}$	$+0.44_{36}$	$+0.46_{27}$	$+0.28_{39}$	$+0.31_{44}$	$+0.03_{42}$	$+0.15_{34}$	$+0.01_{41}$
γ Pegasi	$+0.68_{4}$	$+0.49_{4}$	$+0.36_{3}$	-0.30_{1}	$+0.20_{2}$	$+0.16_{4}$	$+0.24_{4}$	-0.79_{2}	-0.11_{2}	-0.35_{5}
α Ophiuchi	$+0.80_{6}$	-0.15_{4}	$+0.05_{4}$	-0.69_{1}	$+0.27_{5}$	$+0.45_{4}$	$+0.44_{6}$	$+0.49_{4}$	-0.09_{4}	$+0.02_{7}$
α Leonis	$+0.67_{6}$	$+0.46_{5}$	-0.03_{5}	$+0.31_{3}$	-0.89_{2}	$+0.56_{4}$	-0.26_{6}	-0.60_{6}	$+0.65_{5}$	$+0.05_{6}$
γ Aquilæ	$+0.39_{5}$	$+0.02_{3}$	$+0.76_{4}$	$+1.20_{2}$	$+0.33_{4}$	-0.45_{3}	-0.19_{3}	-0.13_{4}	. . .	$+0.43_{5}$
α Aquilæ	$+0.52_{6}$	-0.23_{5}	$+0.19_{5}$	$+0.30_{3}$	$+0.18_{5}$	$+0.46_{5}$	-0.13_{6}	$+0.17_{3}$	$+0.50_{3}$	$+0.17_{4}$
α Orionis	$+1.02_{5}$	$+0.01_{5}$	$+0.25_{2}$	$+0.34_{1}$	$+0.28_{2}$	-0.30_{5}	$+0.21_{6}$	-0.56_{6}	-0.78_{4}	-0.09_{5}
α Aquarii	$+0.92_{4}$	$+1.31_{2}$	$+0.27_{2}$	$+0.46_{2}$	$+0.48_{1}$	$+0.78_{1}$	-0.73_{3}	. . .	-0.39_{2}	$+0.23_{2}$
α Hydræ	$+0.74_{4}$	-0.11_{3}	-1.65_{2}	-0.69_{1}	$+0.43_{2}$	$+0.14_{2}$	-0.71_{3}	-0.31_{3}	. . .	-0.28_{4}
β Orionis	$+0.43_{6}$	$+0.39_{6}$	$+1.76_{2}$	$+0.25_{5}$	$+0.22_{2}$	-0.77_{5}	-0.52_{4}	-1.01_{4}	-4.16_{1}	-0.36_{5}
α Virginis	$+0.38_{7}$	$+0.06_{6}$	-0.32_{6}	$+0.06_{5}$	-0.16_{6}	-0.10_{6}	-0.72_{7}	-0.20_{6}	$+0.17_{5}$	$+0.06_{7}$
Mean	$+0.63_{53}$	$+0.18_{43}$	$+0.13_{35}$	$+0.22_{24}$	$+0.12_{31}$	$+0.02_{39}$	-0.20_{48}	-0.34_{38}	-0.12_{26}	-0.02_{50}

TABLE VIII.—*Corrections given by the Greenwich observations to the standard N. P. D.'s of* 47 *stars, etc.*—**Continued.**

	1869.	1870.	1871.	1872.	1873.	1874.	1875.	1876.	1877.	1878.
	"	"	"	"	"	"	"	"	"	"
γ Draconis, S. P . . .	-1.25_{3}	-1.17_{2}	-0.52_{2}	-1.27_{1}	$+0.51_{3}$	$+0.03_{2}$	-1.05_{2}	$+0.69_{3}$	-0.55_{2}	. . .
α Cassiopeiæ, S. P . .	$+0.52_{3}$	$+0.12_{4}$	-0.40_{1}	. . .	$+0.49_{2}$	. . .	$+2.72_{1}$	. . .	-0.04_{2}	. . .
α Cephei, S. P	-2.79_{1}	-0.66_{4}	$+0.72_{1}$	$+1.84_{1}$	-1.14_{3}	. . .	$+0.44_{2}$	$+1.11_{2}$	0.00_{1}	. . .
α Ursæ Majoris, S. P .	. . .	-0.49_{2}	. . .	$+0.18_{4}$	$+0.55_{2}$	. . .	-1.03_{2}	$+1.52_{2}$	. . .	-0.23_{1}
α Camelopardalis, S. P.	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
κ Draconis, S. P . . .	$+1.15_{4}$	-0.78_{1}	-0.76_{1}	$+0.46_{2}$	$+0.01_{2}$	. . .	. . .	. . .	. . .	. . .
50 Cassiopeiæ, S. P . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
β Ursæ Minoris, S. P .	$+0.35_{3}$	$+0.93_{4}$	$+3.43_{1}$	$+1.70_{2}$	$+0.43_{1}$	-0.58_{2}	$+0.87_{2}$	-1.30_{1}	$+0.18_{1}$	. . .
γ Cephei, S. P	0.00_{3}	$+0.88_{2}$	-0.09_{2}	$+0.66_{2}$	. . .	. . .	$+0.45_{3}$	-0.04_{3}	$+0.91_{2}$	$+0.01_{1}$
Mean	$+0.04_{17}$	-0.04_{19}	$+0.22_{8}$	$+0.58_{12}$	$+0.05_{13}$	-0.28_{4}	$+0.21_{12}$	$+0.54_{11}$	$+0.10_{8}$	-0.11_{2}
δ Ursæ Minoris, S. P .	-0.45_{4}	-0.37_{5}	-0.63_{5}	-0.18_{7}	$+0.54_{6}$	$+0.15_{4}$	-0.60_{6}	-0.55_{8}	-0.15_{7}	-0.73_{6}
51 (Hev.) Cephei, S. P .	-0.30_{6}	-0.33_{8}	-0.39_{7}	-0.75_{7}	-0.37_{7}	-0.36_{7}	$+0.16_{6}$	$+0.17_{8}$	$+0.23_{8}$	$+0.36_{7}$
α Ursæ Minoris, S. P .	$+0.16_{9}$	$+0.17_{9}$	$+0.01_{9}$	$+0.08_{9}$	$+0.44_{9}$	$+0.28_{9}$	0.00_{9}	$+0.11_{9}$	$+0.16_{9}$	0.00_{9}
λ Ursæ Minoris, S. P .	-0.20_{3}	. . .	-0.64_{4}	-0.27_{3}	-0.03_{6}	$+0.14_{5}$	$+0.25_{4}$	-0.04_{7}	$+0.12_{6}$	-0.22_{3}
λ Ursæ Minoris	-0.06_{2}	-0.42_{7}	-0.38_{4}	-0.41_{3}	-0.55_{6}	$+0.05_{5}$	-0.27_{4}	$+0.03_{8}$	$+0.11_{7}$	-0.07_{7}
α Ursæ Minoris	-0.09_{8}	-0.06_{9}	$+0.19_{8}$	$+0.67_{9}$	$+0.36_{9}$	$+0.04_{9}$	$+0.11_{9}$	$+0.09_{9}$	$+0.13_{9}$	$+0.04_{8}$
51 (Hev.) Cephei . . .	-0.06_{5}	-0.56_{5}	-1.22_{6}	-0.77_{6}	-0.35_{6}	$+0.34_{7}$	-0.51_{6}	-0.40_{8}	-0.05_{6}	-0.34_{6}
δ Ursæ Minoris	-0.07_{6}	$+0.43_{8}$	-0.04_{7}	-0.20_{7}	-0.01_{8}	$+0.15_{8}$	$+0.24_{8}$	$+0.06_{8}$	$+0.15_{8}$	$+0.24_{8}$
Mean	-0.10_{43}	-0.11_{51}	-0.32_{50}	-0.15_{51}	$+0.04_{57}$	$+0.10_{54}$	-0.06_{52}	-0.06_{65}	$+0.10_{60}$	-0.05_{54}
γ Cephei	-0.09_{3}	-0.15_{1}	-0.26_{2}	-0.90_{1}	$+0.19_{3}$	0.00_{2}	-0.32_{3}	-0.32_{3}	-0.54_{2}	$+0.12_{1}$
β Ursæ Minoris	-0.31_{5}	-0.15_{4}	$+0.33_{1}$	-0.69_{6}	$+0.31_{2}$	-0.19_{4}	-0.17_{5}	-0.02_{5}	$+0.15_{5}$	-0.86_{4}
50 Cassiopeiæ	. . .	. . .	$+0.99_{1}$	$+0.85_{2}$	$+1.26_{2}$	$+1.08_{1}$	. . .	$+0.46_{1}$	$+0.31_{3}$	. . .
κ Draconis	$+1.99_{1}$	-0.45_{1}	. . .	-0.55_{2}	-0.04_{3}	$+0.42_{2}$	$+0.26_{3}$	-1.86_{2}	$+0.33_{2}$	-0.54_{1}
α Camelopardalis . . .	. . .	. . .	$+1.03_{2}$	$+1.20_{1}$	$+0.88_{2}$	. . .	. . .	. . .	. . .	. . .
α Ursæ Majoris	$+0.19_{5}$	$+0.21_{4}$	-0.75_{3}	-0.29_{2}	$+0.96_{4}$	$+0.37_{4}$	$+0.52_{4}$	$+0.52_{2}$	$+0.36_{2}$	$+0.17_{4}$
α Cephei	-0.13_{4}	$+0.29_{4}$	-0.01_{2}	$+0.07_{4}$	$+0.46_{5}$	$+0.52_{4}$	-0.32_{3}	$+0.21_{5}$	$+0.45_{4}$	$+0.24_{3}$
α Cassiopeiæ	$+0.75_{2}$	-0.51_{1}	-0.41_{1}	$+0.19_{1}$	-0.34_{2}	$+0.54_{1}$	-0.89_{2}	. . .	$+1.09_{1}$	. . .
γ Draconis	-0.53_{2}	. . .	-0.32_{4}	-0.17_{6}	$+0.15_{4}$	$+0.41_{2}$	-0.07_{2}	-0.62_{3}	-0.01_{3}	$+0.02_{3}$
Mean	$+0.05_{22}$	$+0.02_{15}$	-0.07_{16}	-0.17_{25}	$+0.42_{27}$	$+0.30_{20}$	-0.08_{22}	-0.19_{21}	$+0.22_{22}$	-0.15_{16}

TABLE VIII.—*Corrections given by the Greenwich observations to the standard N. P. D.'s of 47 stars, etc.*—**Continued.**

	1869.	1870.	1871.	1872.	1873.	1874.	1875.	1876.	1877.	1878.
	"	"	"	"	"	"	"	"	"	"
η Ursæ Majoris	+0.27$_{3}$	+0.25$_{4}$	−0.01$_{4}$	+0.91$_{1}$	0.00$_{4}$	−0.11$_{4}$	−0.22$_{3}$	−0.91$_{4}$	−0.48$_{3}$	−0.11$_{2}$
α Persei	−0.80$_{4}$	+0.19$_{2}$	. . .	−0.20$_{2}$	−0.72$_{1}$	+0.56$_{1}$	−0.13$_{2}$	. . .	−1.87$_{2}$	−1.52$_{1}$
α Aurigæ	+0.37$_{3}$	+0.03$_{4}$	+0.56$_{5}$	−0.09$_{3}$	+0.36$_{1}$	−0.13$_{1}$	+2.34$_{2}$	+0.01$_{3}$	+0.44$_{2}$	−0.24$_{1}$
α Cygni	−0.54$_{3}$	+0.01$_{3}$	−1.39$_{1}$	−0.19$_{3}$	−0.82$_{3}$	+0.14$_{3}$	−0.78$_{3}$	−0.27$_{6}$	+0.05$_{3}$	+0.97$_{2}$
α Canum Ven.	+0.16$_{4}$	−0.21$_{3}$	−0.10$_{2}$	−0.73$_{3}$	−1.32$_{2}$	+0.39$_{2}$	+0.16$_{3}$	+0.01$_{4}$	+0.09$_{4}$	−0.07$_{4}$
α Lyræ	−0.25$_{7}$	−0.22$_{7}$	−0.44$_{6}$	−0.02$_{6}$	−0.20$_{6}$	−0.13$_{6}$	−0.40$_{7}$	−0.67$_{7}$	−0.10$_{7}$	+0.07$_{6}$
ι Aurigæ	−0.30$_{2}$	−0.67$_{3}$	−0.46$_{4}$	. . .	. . .	+0.40$_{2}$	−0.31$_{2}$	+0.59$_{1}$	−1.57$_{1}$	. . .
ζ Cygni	+0.49$_{3}$	+0.12$_{2}$	+0.40$_{4}$	+0.30$_{4}$	−0.27$_{3}$	+0.07$_{4}$	+0.27$_{1}$	−0.05$_{4}$	−0.26$_{1}$	+0.03$_{3}$
β Tauri	−0.30$_{2}$	+0.20$_{4}$	−0.48$_{6}$	−0.45$_{4}$	−0.65$_{3}$	−0.43$_{4}$	−0.14$_{4}$	+0.32$_{3}$	−0.28$_{4}$	+0.86$_{4}$
β Geminorum	−0.10$_{6}$	−0.05$_{5}$	−0.31$_{6}$	−0.06$_{6}$	−0.32$_{6}$	+0.08$_{3}$	+0.29$_{4}$	+0.30$_{5}$	+0.14$_{4}$	−0.95$_{6}$
α Andromedæ	+0.18$_{5}$	+0.35$_{3}$	−0.62$_{4}$	−0.23$_{3}$	+0.19$_{5}$	+0.03$_{3}$	−0.26$_{3}$	+0.55$_{3}$	−0.20$_{2}$	+0.10$_{2}$
β Pegasi	. . .	. . .	+0.27$_{2}$	. . .	+0.85$_{1}$	+0.02$_{2}$	+0.90$_{1}$	−0.32$_{2}$	. . .	. . .
α Coronæ	−0.56$_{6}$	+0.45$_{1}$	+0.55$_{1}$	−0.28$_{5}$	−0.25$_{4}$	+0.18$_{5}$	. . .	+0.22$_{4}$	+0.12$_{4}$	+0.43$_{3}$
Mean	−0.14$_{48}$	−0.01$_{41}$	−0.17$_{45}$	−0.14$_{40}$	−0.27$_{39}$	+0.03$_{40}$	−0.02$_{35}$	−0.11$_{46}$	+0.02$_{37}$	−0.02$_{34}$
μ Pegasi	−0.08$_{1}$	+0.16$_{3}$	−0.29$_{2}$	+0.51$_{3}$	+1.41$_{1}$	+0.36$_{1}$	+0.02$_{1}$	. . .	+0.82$_{2}$	−0.43$_{1}$
α Arietis	−0.33$_{4}$	+0.40$_{6}$	+0.23$_{4}$	+0.36$_{3}$	+0.94$_{3}$	+0.31$_{3}$	+0.68$_{2}$	+0.93$_{1}$	+0.70$_{2}$	+1.36$_{3}$
μ Geminorum	−0.39$_{2}$	−1.55$_{3}$	−0.41$_{2}$	−0.34$_{3}$	−0.67$_{2}$	+0.27$_{1}$	+1.51$_{1}$	. . .	. . .	. . .
δ Geminorum	−0.09$_{3}$	−0.49$_{2}$	−0.26$_{3}$	−0.07$_{4}$	−0.10$_{2}$	+0.24$_{3}$	. . .	−2.14$_{2}$	+0.45$_{2}$	. . .
β Arietis	. . .	+0.52$_{3}$	+0.65$_{3}$	+0.01$_{1}$	−0.64$_{2}$	−0.74$_{1}$	+0.47$_{1}$	−0.67$_{2}$	−0.24$_{2}$	+0.35$_{2}$
α Bootis	+0.49$_{6}$	+0.67$_{6}$	−0.11$_{6}$	−0.31$_{7}$	−0.23$_{6}$	−0.08$_{6}$	+0.25$_{6}$	−0.06$_{6}$	+0.02$_{6}$	+0.27$_{7}$
γ Geminorum	+1.10$_{1}$	−0.11$_{3}$	−0.55$_{3}$	−0.36$_{3}$	+0.25$_{3}$	+0.36$_{2}$	−0.55$_{1}$	+0.14$_{3}$	−0.16$_{2}$	−0.09$_{1}$
α Tauri	−0.51$_{6}$	+0.02$_{5}$	−0.40$_{6}$	−1.44$_{4}$	−0.09$_{3}$	+0.31$_{6}$	+0.27$_{4}$	+0.02$_{4}$	+0.32$_{4}$	−0.10$_{4}$
β Leonis	+0.71$_{4}$	+0.69$_{3}$	+0.76$_{4}$	−0.06$_{4}$	+1.22$_{3}$	−0.58$_{1}$	+0.75$_{4}$	+0.09$_{2}$	+0.71$_{3}$	+0.91$_{4}$
α[1] Herculis	+0.57$_{4}$	+0.27$_{3}$	+0.73$_{2}$	+0.75$_{2}$	+0.20$_{3}$	+0.09$_{1}$	−1.24$_{3}$	. . .	+0.11$_{2}$	+0.50$_{4}$
α Pegasi	+0.64$_{3}$	−0.36$_{3}$	−2.97$_{1}$	+0.44$_{4}$	+0.39$_{3}$	+0.19$_{2}$	. . .	+0.35$_{3}$	+0.40$_{1}$	+0.50$_{2}$
Mean	+0.16$_{34}$	+0.11$_{40}$	−0.07$_{36}$	−0.12$_{38}$	+0.19$_{31}$	+0.13$_{27}$	+0.20$_{23}$	−0.14$_{23}$	+0.28$_{26}$	+0.44$_{28}$
γ Pegasi	+0.72$_{5}$	−0.16$_{2}$	+0.02$_{4}$	−0.67$_{2}$	+0.70$_{3}$	. . .	. . .	. . .	−0.15$_{1}$	−0.58$_{1}$
α Ophiuchi	−0.06$_{6}$	−0.20$_{6}$	−0.40$_{5}$	−0.27$_{6}$	+0.14$_{4}$	0.00$_{3}$	+0.35$_{5}$	+0.30$_{4}$	+0.99$_{2}$	+0.39$_{5}$
α Leonis	+0.30$_{3}$	+0.35$_{7}$	+0.18$_{6}$	−0.19$_{4}$	+0.20$_{6}$	−0.27$_{3}$	−0.19$_{5}$	−0.28$_{4}$	−0.11$_{5}$	+0.65$_{6}$
γ Aquilæ	+0.82$_{4}$	+0.66$_{3}$	−0.06$_{4}$	−0.01$_{3}$	+0.34$_{4}$	+0.83$_{3}$	+0.52$_{2}$	−0.02$_{2}$	+0.66$_{2}$	+1.39$_{2}$
α Aquilæ	+0.19$_{5}$	+0.04$_{6}$	+0.12$_{3}$	+0.59$_{6}$	+0.26$_{5}$	+0.36$_{6}$	−0.22$_{3}$	−0.95$_{4}$	+0.56$_{4}$	+0.35$_{3}$
α Orionis	−0.40$_{2}$	+0.59$_{6}$	−0.38$_{6}$	+0.17$_{4}$	+0.33$_{4}$	−1.10$_{3}$	−1.13$_{2}$	−0.17$_{5}$	+0.05$_{3}$	−0.21$_{3}$
α Aquarii	. . .	−1.19$_{3}$	+0.83$_{1}$	−0.96$_{4}$	0.00$_{4}$	−1.09$_{1}$	. . .	−1.27$_{2}$	+1.04$_{1}$	. . .
α Hydræ	−0.67$_{2}$	−0.44$_{4}$	−0.67$_{4}$	+0.76$_{2}$	−0.59$_{5}$	. . .	−0.39$_{2}$	+0.23$_{2}$	−0.42$_{2}$	−0.29$_{3}$
β Orionis	−0.38$_{3}$	+0.26$_{3}$	−0.72$_{5}$	−0.32$_{4}$	+0.06$_{4}$	+0.70$_{3}$	−0.50$_{3}$	−0.42$_{3}$	−0.71$_{2}$	+0.25$_{1}$
α Virginis	+0.23$_{6}$	+0.31$_{6}$	−0.08$_{6}$	+0.60$_{6}$	−0.63$_{6}$	+0.02$_{4}$	−0.85$_{5}$	−0.68$_{5}$	−0.23$_{6}$	−0.11$_{6}$
Mean	+0.18$_{36}$	+0.09$_{46}$	−0.20$_{44}$	+0.01$_{41}$	+0.03$_{45}$	+0.06$_{26}$	−0.28$_{27}$	−0.37$_{31}$	+0.09$_{28}$	+0.24$_{30}$

TABLE VIII.—*Corrections given by the Greenwich observations to the standard N. P. D.'s of 47 stars, etc.*—**Continued.**

	1879.	1880.	1881.	1882.	1883.	1884.	1885.	1886.	1887.
	"	"	"	"	"	"	"	"	"
γ Draconis, S. P. . . .	. . .	−0.20$_{1}$	−0.66$_{2}$	−0.46$_{2}$	+1.34$_{2}$	. . .	. . .	. . .	. . .
α Cassiopeiæ, S. P. . .	+0.30$_{1}$	+0.37$_{1}$	−0.39$_{3}$	−0.46$_{2}$	−0.22$_{1}$	. . .	. . .	. . .	. . .
α Cephei, S. P.	. . .	−0.24$_{2}$	+0.13$_{2}$	−0.33$_{2}$	+0.08$_{2}$	−0.88$_{1}$	−0.41$_{2}$	. . .	. . .
α Ursæ Majoris, S. P. .	−0.69$_{1}$	−0.39$_{3}$	+0.45$_{2}$	−0.04$_{1}$	. . .	−0.71$_{2}$	+0.25$_{2}$	−0.41$_{3}$	. . .
α Camelopardalis, S. P.	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
κ Draconis, S. P. . . .	. . .	+0.39$_{2}$	. . .	−0.44$_{1}$	. . .	+2.29$_{2}$	+0.33$_{4}$	+0.54$_{3}$	. . .
50 Cassiopeiæ, S. P. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
β Ursæ Minoris, S. P. .	+1.07$_{2}$	+0.38$_{3}$	+1.02$_{3}$	. . .	+0.14$_{3}$	+0.56$_{2}$	+0.87$_{2}$	+1.90$_{2}$	. . .
γ Cephei, S. P.	−0.08$_{1}$	+0.74$_{1}$	−0.06$_{1}$	. . .	+0.60$_{2}$	+0.96$_{2}$	−0.12$_{4}$	+0.35$_{2}$	+0.31$_{2}$
Mean	+0.33$_{5}$	+0.09$_{13}$	+0.13$_{13}$	−0.37$_{8}$	+0.42$_{10}$	+0.59$_{9}$	+0.16$_{14}$	+0.49$_{10}$	+0.31$_{2}$
δ Ursæ Minoris, S. P. .	−0.37$_{5}$	−0.60$_{7}$	−0.40$_{7}$	+0.06$_{7}$	−0.34$_{8}$	−0.27$_{7}$	−0.39$_{7}$	−0.57$_{4}$	−0.32$_{6}$
51 (Hev.) Cephei, S. P. .	−0.38$_{7}$	−0.32$_{8}$	−0.07$_{8}$	−0.17$_{8}$	+0.12$_{8}$	−0.12$_{8}$	−0.35$_{8}$	+0.38$_{7}$	+0.44$_{7}$
α Ursæ Minoris, S. P. .	−0.16$_{9}$	−0.18$_{9}$	+0.04$_{9}$	−0.03$_{9}$	+0.08$_{9}$	+0.40$_{9}$	−0.36$_{9}$	+0.24$_{8}$	0.00$_{9}$
λ Ursæ Minoris, S. P. .	−0.05$_{4}$	+0.39$_{5}$	−0.12$_{6}$	0.00$_{7}$	+0.12$_{7}$	−0.18$_{6}$	−0.49$_{6}$	−0.25$_{6}$	+0.17$_{6}$
λ Ursæ Minoris	−0.04$_{7}$	−0.33$_{7}$	−0.45$_{7}$	−0.27$_{7}$	−0.40$_{8}$	−0.39$_{8}$	−0.88$_{7}$	−0.40$_{7}$	−0.53$_{7}$
α Ursæ Minoris	+0.01$_{8}$	−0.09$_{9}$	−0.08$_{9}$	−0.07$_{9}$	+0.17$_{9}$	+0.22$_{9}$	−0.12$_{9}$	−0.36$_{9}$	−0.31$_{9}$
51 (Hev.) Cephei	−0.38$_{5}$	−0.40$_{7}$	−0.65$_{7}$	−0.37$_{7}$	−0.35$_{8}$	−0.67$_{6}$	−0.59$_{7}$	−0.95$_{7}$	−1.11$_{7}$
δ Ursæ Minoris	−0.16$_{7}$	−0.25$_{8}$	−0.18$_{7}$	+0.03$_{8}$	+0.23$_{8}$	+0.14$_{7}$	−0.40$_{8}$	+0.20$_{6}$	−0.65$_{8}$
Mean.	−0.18$_{52}$	−0.24$_{60}$	−0.22$_{60}$	−0.10$_{62}$	−0.04$_{65}$	−0.08$_{60}$	−0.43$_{61}$	−0.20$_{54}$	−0.29$_{59}$
γ Cephei	−0.47$_{4}$	+0.63$_{3}$	−0.11$_{1}$	−0.20$_{2}$	−0.81$_{1}$	−0.16$_{3}$	+1.73$_{1}$	−0.59$_{2}$	−0.87$_{2}$
β Ursæ Minoris	−0.39$_{4}$	−0.44$_{4}$	−0.15$_{2}$	−0.10$_{2}$	−0.67$_{3}$	−0.17$_{1}$	+0.17$_{3}$	−0.07$_{3}$	−0.37$_{1}$
50 Cassiopeiæ	+1.54$_{1}$	+1.13$_{2}$	+0.82$_{3}$	+0.26$_{2}$	. . .	+1.13$_{1}$	. . .	+0.76$_{1}$	+0.46$_{2}$
κ Draconis	−0.87$_{2}$	+0.04$_{2}$	0.00$_{1}$	. . .	. . .	. . .	−0.03$_{1}$	. . .	+0.46$_{1}$
α Camelopardalis . . .	+1.05$_{1}$	+0.14$_{2}$	. . .	−0.43$_{2}$	+0.11$_{2}$	−0.40$_{1}$	. . .	. . .	. . .
α Ursæ Majoris	+0.80$_{3}$	+0.45$_{3}$	+0.49$_{4}$	+0.07$_{4}$	+0.65$_{4}$	+1.28$_{2}$	−0.10$_{4}$	−0.10$_{3}$	. . .
α Cephei	+0.43$_{3}$	−0.34$_{3}$	+0.52$_{2}$	−0.13$_{2}$	−0.32$_{3}$	+1.03$_{2}$	−0.40$_{2}$	+0.06$_{3}$	+0.06$_{2}$
α Cassiopeiæ	−0.01$_{2}$	−1.24$_{1}$	+0.36$_{3}$	−0.32$_{1}$	. . .	+0.28$_{3}$	+0.16$_{1}$	+0.40$_{3}$	+0.21$_{1}$
γ Draconis	−0.42$_{1}$	−0.42$_{4}$	+0.09$_{2}$	−1.16$_{2}$	−0.12$_{4}$	+1.31$_{1}$	+0.39$_{3}$	+0.12$_{2}$	−0.36$_{1}$
Mean.	+0.03$_{21}$	+0.01$_{24}$	+0.35$_{18}$	−0.21$_{17}$	−0.08$_{17}$	+0.49$_{14}$	+0.16$_{15}$	+0.04$_{17}$	−0.08$_{20}$

TABLE VIII.—*Corrections given by the Greenwich observations to the standard N. P. D.'s of 47 stars, etc.*—**Concluded.**

	1879.	1880.	1881.	1882.	1883.	1884.	1885.	1886.	1887.
	″	″	″	″	″	″	″	″	″
η Ursæ Majoris	-0.41_{2}	-1.18_{2}	-0.30_{2}	. . .	$+1.86_{2}$	-1.08_{1}	$+0.13_{3}$	-0.25_{3}	-0.33_{1}
α Persei	. . .	-0.07_{1}	$+0.28_{4}$	. . .	. . .	-0.53_{2}	$+0.91_{2}$	-1.14_{1}	. . .
α Aurigæ	$+0.48_{4}$	$+0.06_{3}$	-0.32_{2}	$+0.05_{1}$	$+0.32_{1}$	$+0.13_{2}$	. . .	-0.79_{1}	-0.48_{2}
α Cygni	$+0.34_{2}$	$+0.21_{3}$	. . .	$+0.87_{3}$	$+0.24_{2}$	$+0.04_{2}$	. . .	$+0.03_{3}$	$+0.29_{1}$
α Canum Ven.	$+0.06_{3}$	$+0.05_{5}$	. . .	. . .	. . .	. . .	. . .	. . .	. . .
α Lyræ	-0.43_{7}	-0.21_{6}	-0.05_{6}	-0.27_{5}	-0.01_{6}	-0.18_{5}	$+0.31_{5}$	-0.39_{6}	-0.20_{6}
ι Aurigæ	-0.06_{4}	-0.37_{1}	. . .	-0.25_{1}	$+0.49_{1}$	-0.06_{1}	$+0.09_{2}$	-0.69_{1}	$+0.12_{2}$
ζ Cygni	-0.06_{4}	-0.02_{4}	-0.43_{3}	-0.63_{1}	-0.26_{2}	$+0.11_{2}$	-0.14_{2}	-0.04_{4}	-0.60_{2}
β Tauri	-0.22_{4}	-0.70_{3}	-0.47_{2}	-0.17_{1}	-0.57_{1}	-0.27_{2}	-0.33_{3}	-0.41_{3}	-0.79_{1}
β Geminorum	-0.11_{5}	-0.38_{5}	-0.20_{3}	$+0.21_{3}$	$+0.78_{3}$	$+0.52_{4}$	-0.76_{2}	$+0.61_{3}$	-0.17_{3}
α Andromedæ	-0.18_{4}	-0.18_{2}	-0.07_{2}	$+0.51_{1}$	$+0.83_{1}$	-0.22_{3}	$+0.48_{2}$	-0.16_{4}	-0.47_{4}
β Pegasi	$+0.26_{3}$	-0.73_{2}	-1.74_{1}	-1.09_{1}	. . .	. . .	. . .	. . .	. . .
α Coronæ	-0.01_{3}	$+0.15_{4}$	$+0.69_{4}$	$+0.30_{3}$	$+0.45_{4}$	$+0.51_{3}$	-0.06_{3}	$+0.89_{5}$	-0.48_{5}
Mean	-0.07_{45}	-0.20_{41}	-0.08_{29}	$+0.06_{20}$	$+0.38_{23}$	0.00_{27}	$+0.08_{24}$	-0.04_{34}	-0.32_{27}
μ Pegasi	$+0.46_{4}$	-0.32_{2}	-0.95_{1}	. . .	-0.32_{3}	$+0.62_{2}$	$+0.91_{2}$	. . .	. . .
α Arietis	-0.76_{1}	$+0.34_{4}$	$+0.35_{5}$	$+0.36_{3}$	$+0.41_{3}$	$+0.02_{3}$	-0.16_{2}	$+0.40_{3}$	$+0.16_{3}$
μ Geminorum	. . .	$+0.24_{4}$	-0.06_{4}	$+0.23_{3}$	-0.11_{2}	$+0.03_{1}$	-1.47_{1}	-0.36_{2}	$+0.06_{1}$
δ Geminorum	$+0.13_{2}$	$+0.01_{4}$	$+0.02_{3}$	$+0.49_{3}$	-0.32_{1}	$+0.72_{1}$	$+0.12_{2}$	-0.53_{5}	$+0.83_{2}$
β Arietis	$+0.08_{2}$	$+0.53_{4}$	$+0.06_{5}$	$+0.36_{3}$	$+0.09_{3}$	$+0.26_{1}$	-0.31_{2}	$+0.19_{4}$	$+0.09_{4}$
α Bootis	-0.10_{7}	-0.20_{6}	-0.04_{6}	-0.26_{6}	$+0.07_{6}$	$+0.41_{6}$	$+0.07_{6}$	-0.12_{5}	-0.34_{6}
γ Geminorum	$+0.39_{3}$	-0.46_{2}	-0.17_{3}	$+0.58_{4}$	$+0.35_{4}$	$+0.97_{2}$	$+0.99_{2}$	$+0.17_{4}$	-0.07_{4}
α Tauri	$+0.27_{2}$	$+0.72_{3}$	-0.40_{3}	-0.23_{3}	-0.57_{2}	-0.16_{2}	$+0.15_{3}$	-0.01_{5}	-0.05_{3}
β Leonis	$+0.73_{3}$	. . .	$+0.35_{2}$	$+0.19_{4}$	$+0.51_{4}$	-0.43_{3}	-0.03_{2}	$+0.02_{3}$	-0.08_{3}
α¹ Herculis	-0.14_{3}	$+0.09_{3}$	-0.25_{1}	$+0.68_{2}$	-0.10_{2}	$+0.44_{2}$	-0.05_{1}	-0.60_{2}	-1.19_{4}
α Pegasi	$+0.32_{2}$	-0.26_{2}	-0.15_{1}	$+0.64_{2}$	-0.25_{1}	$+0.43_{2}$	$+0.17_{3}$	-0.09_{2}	-0.43_{2}
Mean	$+0.17_{29}$	$+0.11_{34}$	-0.02_{34}	$+0.24_{33}$	$+0.07_{31}$	$+0.27_{25}$	$+0.11_{26}$	-0.05_{33}	-0.18_{32}
γ Pegasi	$+0.40_{4}$	$+0.52_{2}$	. . .	$+0.33_{2}$	$+0.17_{2}$	$+1.10_{2}$	$+0.52_{2}$	$+0.10_{3}$	$+0.32_{2}$
α Ophiuchi	$+0.08_{4}$	$+0.45_{3}$	$+0.26_{4}$	-0.53_{3}	$+0.20_{4}$	$+0.72_{4}$	$+0.64_{4}$	$+0.94_{5}$	-0.25_{6}
α Leonis	$+0.25_{5}$	-0.03_{3}	$+0.03_{3}$	$+1.17_{3}$	$+0.51_{3}$	-0.32_{1}	$+0.73_{4}$	$+0.45_{4}$	$+0.17_{4}$
γ Aquilæ	-0.15_{3}	-1.23_{1}	$+0.54_{1}$	-0.41_{2}	$+0.35_{3}$	$+0.97_{2}$	$+0.37_{2}$	$+1.09_{1}$	. . .
α Aquilæ	$+0.08_{4}$	$+0.34_{4}$	$+0.53_{2}$	$+0.08_{2}$	$+0.55_{4}$	$+0.94_{3}$	-0.47_{4}	$+0.39_{5}$	$+0.30_{4}$
α Orionis	-0.63_{2}	-0.01_{2}	$+0.26_{3}$	-0.53_{2}	$+0.68_{4}$	$+0.97_{1}$	$+0.31_{3}$	$+0.20_{5}$	-0.63_{4}
α Aquarii	-0.98_{2}	-0.36_{2}	-1.05_{1}	-0.36_{3}	-0.47_{3}	$+0.05_{2}$	-0.22_{1}	-0.48_{4}	-0.79_{3}
α Hydræ	-0.40_{2}	-0.25_{2}	-0.93_{1}	. . .	$+1.15_{2}$	-0.05_{3}	$+0.29_{2}$	-0.21_{3}	$+0.46_{2}$
β Orionis	$+0.52_{3}$	-0.13_{3}	-0.52_{3}	-1.18_{2}	-0.61_{3}	$+0.50_{2}$	. . .	$+0.10_{5}$	-0.64_{1}
α Virginis	-0.57_{6}	-0.69_{5}	-0.69_{4}	$+0.56_{5}$	-0.15_{4}	-0.04_{4}	-0.52_{1}	$+0.29_{1}$	-0.68_{1}
Mean	-0.08_{35}	-0.10_{27}	-0.13_{22}	$+0.01_{24}$	$+0.22_{32}$	$+0.47_{24}$	$+0.27_{23}$	$+0.26_{36}$	-0.16_{27}

§ 9. *Latitude of the Royal Observatory and final systematic corrections to the north polar distances as annually reduced.*

In planning the present investigation it was supposed that the observations of the four polar stars, together with those of the nine stars of Groups I and III, would afford all the data necessary for the latitude. This, however, is not the case. The material used in discussing the constant of refraction in § 4 is so much more complete for the period embraced by it that I shall begin with discussing its results.

Adopting the Pulkowa refractions, and neglecting the hypothetical periodic error, the only unknown quantity to be derived from the conclusions of § 4 would be the latitude. The result is

$$2\Delta\varphi = +0''.22$$
$$\Delta\varphi = +0''.11$$

But in § 4 we used the flexure of the Ten-Year Catalogue unchanged. The mean reduction of the latitude to our adopted flexure is $+0''.05$. Thus we have, from all the observations of circumpolar stars, 1877–'86,

	°	′	″
Tabular colatitude, 1877–'86	38	31	21.90
Colatitude, 1877–'86, with Pulkowa refractions	38	31	21.74

But if we had taken only the results from the four polar stars, as given in the Ten-Year Catalogue, we should have had, using Bessel's refraction,

	″
From λ Ursæ Minoris	$2\Delta\varphi = -0.74$
" α Ursæ Minoris	-0.36
" 51 (Hev.) Cephei	-0.78
" δ Ursæ Minoris	-0.69
Mean result for $\Delta\varphi$	-0.32
Reduction for flexure	$+0.05$

The reduction to the Pulkowa refraction is $+0''.12$, giving $\Delta\varphi = -0''.15$ with this refraction. The result is—

The colatitude derivable from observations of the four polar stars during the period 1877–'86 *is greater by* 0″.31 *than that derivable from observations of all circumpolar stars.*

The question now arises whether this discrepancy goes back to the beginning. In investigating this question we remark that the absolute errors of the tabular north polar distances of stars of Group II are so much smaller than the uncertainty affecting the latitude derived from any one of them that, in investigating the latitude, we may safely use these polar distances as known quantities. It will be observed that if we regard the standard north polar distances as quite accurate then the numbers in Table VIII will all be of the nature of corrections to the latitude. In the case of circumpolar stars the effect of errors in the standard north polar distances upon the latitude will be in opposite directions above and below the pole, so that, even granting their existence, they will affect the latitude by

only a small fraction of their amount. I therefore begin by comparing the latitudes derived from Group II with those from Group I–III, when the results for periods of five years each are combined according to their weights. The results are shown in the following table:

TABLE IX.—*Comparison of the latitudes from the four polar stars forming Group II with those from Group I–III.*

Years.	$\Delta\varphi$ Group II.	$\Delta\varphi$ Group I–III.	I–III *minus* II.	Mean reduced $\Delta\varphi$
	$''$	$''$	$''$	$''$
1851–'55	−0.08	+0.15	+0.23	+0.17
1856–'60	+0.06	+0.10	+0.04	+0.22
1861–'65	−0.02	−0.04	−0.02	+0.11
1866–'70	−0.12	+0.04	+0.16	+0.10
1871–'75	−0.07	+0.13	+0.20	+0.17
1876–'80	−0.08	+0.06	+0.14	+0.13
1881–'85	−0.17	+0.16	+0.33	+0.13
1886–'87	−0.25	+0.20	+0.45	+0.11
1877–'86	−0.14	+0.12	+0.26	+0.13

The discordance seems to go back to the beginning, its mean value being $+0''.17$. The next question is, in what way should we derive a definitive latitude from these discordant results? To throw light on this question I have, in the last line of the table, given the mean results for the ten years' observations, which have already been discussed for refraction. This discussion showed, as just stated, that the latitude from the combination of all circumpolar stars was greater by $0''.31$ than that from the polar stars alone, and it is also greater by $0''.05$ than that from Group I–III. I have concluded that, so far as can be decided from the data before us, the best general reduction to the latitude which would probably be given by a discussion of all the observations of circumpolar stars will be found by applying—

To the results of Group II . . . . $\Delta\varphi = +0''.23$

To the results of Group I–III . . . . $+0.05$

Applying these corrections and taking the mean we obtain the values of $\Delta\varphi$ given in the last column of Table IX.

Another determination of the latitude could be obtained by correcting the result for each year, as derived in the annual volumes, so as to reduce it to the elements here adopted. This would require us to find the mean of the corrections of Table VI for each year, when we give to the correction for each zenith distance the weight which all the stars at that zenith distance received in forming the latitude. The rigorous determination of this mean would require a complete recomputation of all the annual determinations of latitude, and a slight examination shows that the result would be vitiated by the periodic error. In fact, in the earlier years the concluded latitude depends almost entirely on the observations of Polaris, and, until recently, the four polar stars received greater weight in the determination than all the others combined. I therefore consider that the mean result of the preceding table, namely,

$$\Delta\varphi = +\ 0''.14$$
$$\text{Colat.} = 38^\circ\ 31'\ 21''.76$$

is that to be adopted as the best general mean if we use a single latitude throughout.

The question still remains whether the observations from year to year show greater changes in the value of this constant than would probably result from errors of observation. Such changes can be determined from observations of equatorial as well as of circumpolar stars. If the error of nadir point changes from year to year, the effect will be the same on the concluded north polar distance of northern and of southern stars. On the other hand, errors of absolute zenith distance, equal on both sides of the zenith, are possible. These will have opposite effects on the deduced north polar distance of northern and southern stars. The following table shows the several mean results of each year's observations. The second and third columns give for each year the quantities whose mean values, for groups of five years, are found in Table IX, that is, the excess of north polar distance of Group II corrected by $+0''.23$ and of Group I–III corrected by $+0''.05$. The quantities used in forming the numbers are those of Table VIII. But in taking the means found in the fourth column regard has been had to the weights, which are very unequally distributed.

The column Δp gives the mean corrections to the north polar distances of the three groups of southern stars, as resulting from the uniform elements of reduction already tabulated, using the constant colatitude $38° 31' 21''.90$. In taking the mean each group has received equal weight. We may, therefore, regard Δp as the correction to the latitude, which results from assuming the tabular north polar distances of the three groups of southern stars to be absolutely correct. Hence, granting the unchangeableness of the angle between the pole and the zenith point given by the mean of the direct and reflex observations, these corrections Δp should be uniform from year to year except for the possible systematic error in the tabular proper motions. They are evidently not uniform. During the 13 years from 1852 to 1864 they are markedly positive, the positive effect reaching its maximum in 1859. From 1865 onward there is no marked change, yet it may be remarked that the results of the last 6 years are decidedly positive.

To judge how far these slight variations are due to errors of zenith point we compare them with the numbers in the column $\Delta\varphi$, which gives the annual apparent corrections to the adopted latitude, obtained as in the formation of Table IX, that is, we compare the variations of latitude derived from northern with those from southern stars.

We may consider the negatives of $\Delta\varphi$ as representing the corrections to the tabular zenith distances of the pole. The system of interpretation is this: Accordance of changes in Δp with those in $\Delta\varphi$ will indicate errors of zenith point, or, if the hypothesis be deemed admissible, changes of latitude. Discordances will indicate the action of causes affecting the measures of absolute zenith distances. The marked maximum about the year 1859 corresponds so well with that of the maximum value of Δp that we may fairly regard constant errors in the zenith point as the source of the discrepancy rather than changes in the instrument or the habits of the observers.

The last column of Table X gives the reductions of the colatitudes employed in the annual reductions to the concluded values found on page 448.

TABLE X.—*Apparent errors of the colatitude 38° 31′ 21″.90, resulting from each year's observations with the Greenwich transit circle and concluded corrections to the colatitudes used in the annual reductions.*

Year.	Values of $\Delta\varphi$			Δp	$\Delta p - \Delta\varphi$	Concluded correction to annually adopted colatitude.
	Group II, +0″.23.	Group I–III, +0″.05.	Concluded.			
	″	″	″	″	″	″
1851	+0. 26$_{39}$	+0. 12$_{26}$	+0. 20 $_{65}$	−0. 18	−0. 38	−0. 05
1852	+0. 27$_{48}$	+0. 29$_{38}$	+0. 28 $_{86}$	+0. 10	−0. 18	−0. 05
1853	+0. 08$_{42}$	+0. 14$_{40}$	+0. 11 $_{82}$	+0. 12	+0. 01	−0. 05
1854	+0. 03$_{53}$	+0. 11$_{39}$	+0. 06 $_{92}$	+0. 23	+0. 17	−0. 25
1855	+0. 11$_{46}$	+0. 34$_{38}$	+0. 21 $_{84}$	+0. 21	0. 00	−0. 25
1856	+0. 22$_{48}$	+0. 09$_{36}$	+0. 17 $_{84}$	+0. 19	+0. 02	−0. 25
1857	+0. 13$_{48}$	−0. 06$_{41}$	+0. 04 $_{89}$	+0. 20	+0. 16	−0. 25
1858	+0. 40$_{58}$	+0. 15$_{37}$	+0. 30 $_{95}$	+0. 32	+0. 02	−0. 49
1859	+0. 51$_{56}$	+0. 41$_{45}$	+0. 47$_{101}$	+0. 70	+0. 23	−0. 49
1860	+0. 12$_{48}$	+0. 01$_{27}$	+0. 08 $_{75}$	+0. 38	+0. 30	−0. 22
1861	+0. 24$_{45}$	+0. 12$_{30}$	+0. 19 $_{75}$	+0. 15	−0. 04	−0. 02
1862	+0. 28$_{39}$	+0. 12 $_{7}$	+0. 26 $_{46}$	+0. 36	+0. 10	−0. 02
1863	+0. 19$_{59}$	−0. 19$_{20}$	+0. 09 $_{79}$	+0. 27	+0. 18	−0. 02
1864	+0. 27$_{49}$	−0. 04$_{25}$	+0. 16 $_{74}$	+0. 17	+0. 01	−0. 02
1865	+0. 07$_{45}$	+0. 06$_{15}$	+0. 07 $_{60}$	+0. 05	−0. 02	−0. 02
1866	+0. 01$_{40}$	+0. 37$_{28}$	+0. 16 $_{68}$	−0. 05	−0. 21	−0. 02
1867	+0. 20$_{28}$	−0. 03$_{34}$	+0. 07 $_{62}$	+0. 04	−0. 03	−0. 02
1868	+0. 16$_{46}$	+0. 06$_{56}$	+0. 10$_{102}$	+0. 05	−0. 05	+0. 18
1869	+0. 13$_{43}$	+0. 10$_{39}$	+0. 12 $_{82}$	+0. 07	−0. 05	+0. 18
1870	+0. 12$_{51}$	+0. 04$_{34}$	+0. 09 $_{85}$	+0. 06	−0. 03	+0. 18
1871	−0. 09$_{50}$	+0. 08$_{24}$	−0. 03 $_{74}$	−0. 15	−0. 12	+0. 18
1872	+0. 08$_{51}$	+0. 12$_{37}$	+0. 09 $_{88}$	−0. 09	−0. 18	+0. 18
1873	+0. 27$_{57}$	+0. 35$_{40}$	+0. 30 $_{97}$	−0. 02	−0. 32	+0. 18
1874	+0. 33$_{54}$	+0. 25$_{24}$	+0. 31 $_{78}$	+0. 07	−0. 24	+0. 18
1875	+0. 17$_{52}$	+0. 07$_{34}$	+0. 13 $_{86}$	−0. 03	−0. 16	+0. 18
1876	+0. 17$_{65}$	+0. 11$_{32}$	+0. 15 $_{97}$	−0. 21	−0. 36	+0. 18
1877	+0. 33$_{60}$	+0. 24$_{30}$	+0. 30 $_{90}$	+0. 13	−0. 17	−0. 12
1878	+0. 18$_{54}$	−0. 10$_{18}$	+0. 11 $_{72}$	+0. 22	+0. 11	−0. 12
1879	+0. 05$_{52}$	+0. 14$_{26}$	+0. 08 $_{78}$	+0. 01	−0. 07	−0. 12
1880	−0. 01$_{60}$	+0. 09$_{37}$	+0. 03 $_{97}$	−0. 06	−0. 09	−0. 12
1881	+0. 02$_{60}$	+0. 31$_{31}$	+0. 12 $_{91}$	−0. 05	−0. 17	−0. 12
1882	+0. 13$_{62}$	−0. 21$_{25}$	+0. 03 $_{87}$	+0. 10	+0. 07	−0. 12
1883	+0. 19$_{65}$	+0. 16$_{27}$	+0. 18 $_{92}$	+0. 22	+0. 04	−0. 12
1884	+0. 15$_{60}$	+0. 58$_{23}$	+0. 27 $_{83}$	+0. 25	−0. 02	−0. 12
1885	−0. 20$_{61}$	+0. 21$_{29}$	−0. 07 $_{90}$	+0. 15	+0. 22	−0. 12
1886	+0. 03$_{54}$	+0. 26$_{27}$	+0. 11 $_{81}$	+0. 06	−0. 05	−0. 12
1887	−0. 06$_{59}$	+0. 03$_{12}$	−0. 04 $_{71}$	−0. 22	−0. 18	−0. 12

NOTE.—For 1877 the places of the planets correspond to an adopted colatitude of 38° 31′ 21″.60, so that the correction to be used is +0″.18, while −0″.12 applies to the concluded north polar distances of the fixed stars.

I regard the hypothesis sometimes made of a secular change in the latitude as too improbable for acceptance except on stronger evidence than any yet adduced. Regarding the absolute value of $\Delta\varphi$ as *prima facie* invariable, we may explain its apparent variations by supposing changes in the instrument or habits of the observers. The first conclusion we reach is that, omitting the abnormal years 1858 and 1859, the changes in the latitude are as good as evanescent, there being no such correspondence between $\Delta\varphi$ and Δp as would result from a variation of the angle between the zenith and polar points. Even including these years the solution by least squares would give a centennial variation of scarcely more than one-third of a second.

To throw yet farther light on the question whether the apparent changes of latitude from year to year are purely accidental, I have taken the mean concluded $\Delta\varphi$ found in Table X for groups of five years with the following results:

	$\Delta\varphi$ ″	Colat. ° ′ ″
1851–'55	$\Delta\varphi = +0.17$	Colat. = 38 31 21.73
1856–'60	+0.21	21.69
1861–'65	+0.15	21.75
1866–'70	+0.11	21.79
1871–'75	+0.16	21.74
1876–'80	+0.14	21.76
1881–'85	+0.11	21.79
1886–'87	+0.04	21.86

The differences would seem to be mainly accidental.

Returning to the quantities Δp we remark that they should vary progressively if the adopted annual variations of the standard declinations are systematically in error. Now, a comparison of Boss's declinations of southern stars with those of AUWERS'S BRADLEY shows a difference in 1755 of about 1″.7 in the sense: Δ N. P. D. (AUWERS—BOSS) = + 1″.7. If the errors indicated by this difference were thrown entirely on the standard the change between 1851 and 1887 would be —0″.60.

The attempt to represent the numbers in the column Δp by an expression varying uniformly with the time would give a centennial variation of about —0″.55. The application of this indicated correction to the centennial motions of the standard places would reduce the discrepancy in 1755 from 1″.7 to 1″.0.

Excepting the years 1858 and 1859 the apparent constancy of the latitude is such that we see no occasion for assuming any change in the zenith point, and shall presumably attain the best results by adopting the following mean values of the colatitude:

	$\Delta\varphi$ ″	Colatitude. ° ′ ″
1851–'57	+0.15	38 31 21.75
1858–'59	+0.39	38 31 21.51
1860–'87	+0.12	38 31 21.78

It will be remembered that in deriving these latitudes three values of the flexure have been used for as many periods. I have taken the mean value of $\Delta\varphi$ for these periods, as actually given in the table, and compared it with the value which would have resulted from using the constant flexure 0″.50. The results are as follows:

	$\Delta\varphi$ With adopted flexure. ″	$\Delta\varphi$ With constant flexure, 0″.50. ″
1851–'77	+0.17	+0.06
1878–'82	+0.08	+0.08
1883–'87	+0.09	+0.19

These results agree with the hypothesis that the change of flexure is real, slightly better than they do with the opposite one, but not enough better to add weight to it. We therefore make another comparison in the following way: The agreement of the exceptionally large values of $\Delta\varphi$ with those of Δp in 1858–'59 justify us in concluding that the nadir point was affected by some abnormal error in these years. We therefore reject them. We also reduce the mean results for certain groups of years to what they would have been had all four values of the flexure given on page 416 been used. The results are then as follows:

	Coefficient of flexure.	$\Delta\varphi$	Coefficient of flexure.	$\Delta\varphi$
	$''$	$''$	$''$	$''$
1851–'57	0.27	+0.17	0.50	+0.04
1860-'67	0.27	+0.15	0.50	+0.02
1868–'77	0.33	+0.14	0.50	+0.05
1878–'82	0.51	+0.08	0.50	+0.08
1883–'87	0.68	+0.09	0.50	+0.19

It will be seen that the derived latitudes would present the smallest variations were the change of flexure assumed to be one-half that adopted.

The column $\Delta p - \Delta\varphi$ now demands our attention. This column gives the mean correction to the standard polar distances of Groups IV–VI when the latitude is determined separately from each year's observations. Since the flexure is applied with opposite signs in the columns $\Delta\varphi$ and Δp, the difference of these two columns should change by about $0''.50$ in consequence of the assumed change of $0''.38$ in the flexure, were this assumption incorrect. As no such change is shown we have another presumption in favor of the hypothesis. In fact the difference between the mean of the first 17 and the last 5 years is only $0''.04$. Still this is not conclusive, owing to the uncertainty of the correction to the standard proper motions.

The difference between the preceding colatitudes and those adopted in the annual reductions, as given in the column adopted $\Delta\varphi$, are of the nature of constant corrections to be applied to the annual results for each year in addition to those found and tabulated in § 7. Applying these corrections we have the following table of definitive corrections to the reduced north polar distances of each year, as given in the annual volumes. In strictness these corrections are applicable only to the results of direct observations, which are not given separately since 1885. But as the reflection observations are reduced to the same uniform system as the direct ones, we may consider the same corrections as applicable to the concluded means as actually printed.

Finally, it should be remarked that these corrections, so far as they depend on the instrument, lead to results substantially identical with those which would have been obtained had all the observations from the beginning been reduced by the method used since 1882. The fact that the corrections for recent years are not evanescent is due to the proposed use of the Pulkowa refractions.

TABLE XI.—*Corrections to the north polar distances, derived annually from the observations with the Greenwich transit circle, to reduce them to the instrumental standard of the present paper and the Pulkowa refractions.*

N. P. D.	Dec.	Z. D.	1851 and 1852.	1853.	1854.	1855.	1856.	1857.	1858.	1859.	1860.	1861.	1862.	1863.
°	°	°	″	″	″	″	″	″	″	″	″	″	″	″
− 46	+44	−84.5	+1.41	+1.13	+0.89	+0.96	+0.85	+0.85	+0.75	+0.47	+1.02	+0.96	+1.48	+1.48
− 45	+45	−83.5	+1.14	+0.86	+0.62	+0.69	+0.58	+0.58	+0.48	+0.20	+0.75	+0.69	+1.21	+1.21
− 44	+46	−82.5	+0.97	+0.69	+0.45	+0.52	+0.41	+0.41	+0.31	+0.03	+0.58	+0.52	+1.04	+1.04
− 43	+47	−81.5	+0.86	+0.57	+0.33	+0.40	+0.29	+0.29	+0.19	+0.09	+0.46	+0.40	+0.92	+0.92
− 42	+48	−80.5	+0.80	+0.51	+0.27	+0.34	+0.23	+0.23	+0.13	−0.15	+0.40	+0.34	+0.86	+0.86
− 40	+50	−78.5	+0.73	+0.45	+0.21	+0.28	+0.17	+0.17	+0.08	−0.20	+0.35	+0.30	+0.81	+0.81
− 35	+55	−73.5	+0.60	+0.34	+0.10	+0.17	+0.06	+0.08	−0.03	−0.30	+0.24	+0.19	+0.66	+0.66
− 30	+60	−68.5	+0.51	+0.24	+0.02	+0.08	−0.02	0.00	−0.11	−0.38	+0.15	+0.11	+0.54	+0.53
− 25	+65	−63.5	+0.43	+0.18	−0.06	0.00	−0.09	−0.08	−0.19	−0.45	+0.08	+0.05	+0.42	+0.41
− 20	+70	−58.5	+0.36	+0.13	−0.11	−0.05	−0.14	−0.13	−0.25	−0.49	+0.02	−0.01	+0.32	+0.30
− 15	+75	−53.5	+0.31	+0.08	−0.15	−0.09	−0.17	−0.16	−0.29	−0.52	−0.02	−0.03	+0.23	+0.21
− 10	+80	−48.5	+0.27	+0.05	−0.17	−0.12	−0.20	−0.19	−0.32	−0.54	−0.05	−0.06	+0.17	+0.15
− 5	+85	−43.5	+0.23	+0.03	−0.19	−0.14	−0.22	−0.21	−0.35	−0.56	−0.08	−0.06	+0.11	+0.09
0	+90	−38.5	+0.19	+0.01	−0.20	−0.16	−0.23	−0.22	−0.37	−0.56	−0.11	−0.07	+0.06	+0.04
+ 5	+85	−33.5	+0.15	0.00	−0.22	−0.18	−0.24	−0.24	−0.40	−0.56	−0.13	−0.08	+0.01	−0.01
+ 10	+80	−28.5	+0.12	−0.01	−0.24	−0.20	−0.26	−0.25	−0.42	−0.56	−0.15	−0.08	−0.02	−0.04
+ 15	+75	−23.5	+0.09	−0.02	−0.24	−0.21	−0.26	−0.25	−0.43	−0.55	−0.16	−0.07	−0.03	−0.05
+ 20	+70	−18.5	+0.06	−0.03	−0.24	−0.22	−0.26	−0.25	−0.45	−0.54	−0.18	−0.06	−0.05	−0.07
+ 25	+65	−13.5	+0.04	−0.04	−0.25	−0.23	−0.26	−0.26	−0.46	−0.53	−0.19	−0.06	−0.05	−0.07
+ 30	+60	− 8.5	+0.01	−0.04	−0.24	−0.23	−0.24	−0.24	−0.46	−0.51	−0.19	−0.03	−0.03	−0.04
+ 35	+55	− 3.5	−0.04	−0.06	−0.26	−0.25	−0.26	−0.26	−0.48	−0.51	−0.21	−0.05	−0.04	−0.04
+ 40	+50	+ 1.5	−0.05	−0.05	−0.25	−0.25	−0.26	−0.25	−0.49	−0.49	−0.22	−0.02	−0.02	−0.02
+ 45	+45	+ 6.5	−0.09	−0.04	−0.26	−0.24	−0.25	−0.24	−0.50	−0.47	−0.23	+0.01	0.00	0.00
+ 50	+40	+11.5	−0.13	−0.07	−0.27	−0.27	−0.27	−0.26	−0.52	−0.47	−0.25	+0.01	−0.01	0.00
+ 55	+35	+16.5	−0.15	−0.06	−0.27	−0.27	−0.24	−0.24	−0.52	−0.45	−0.25	+0.02	+0.01	+0.03
+ 60	+30	+21.5	−0.18	−0.07	−0.27	−0.28	−0.24	−0.25	−0.53	−0.44	−0.26	+0.02	+0.01	+0.03
+ 65	+25	+26.5	−0.21	−0.08	−0.27	−0.29	−0.24	−0.25	−0.55	−0.43	−0.28	+0.03	−0.01	+0.01
+ 70	+20	+31.5	−0.24	−0.09	−0.27	−0.30	−0.24	−0.25	−0.56	−0.42	−0.29	+0.04	−0.02	0.00
+ 75	+15	+36.5	−0.28	−0.10	−0.29	−0.32	−0.26	−0.26	−0.59	−0.42	−0.32	+0.04	−0.07	−0.05
+ 80	+10	+41.5	−0.32	−0.12	−0.30	−0.34	−0.27	−0.28	−0.61	−0.43	−0.34	+0.03	−0.13	−0.11
+ 85	+ 5	+46.5	−0.36	−0.14	−0.32	−0.36	−0.28	−0.29	−0.64	−0.42	−0.37	+0.02	−0.20	−0.18
+ 90	0	+51.5	−0.40	−0.17	−0.34	−0.39	−0.30	−0.31	−0.67	−0.44	−0.40	+0.02	−0.25	−0.23
+ 95	− 5	+56.5	−0.44	−0.21	−0.37	−0.43	−0.34	−0.35	−0.70	−0.47	−0.43	−0.01	−0.33	−0.32
+100	−10	+61.5	−0.50	−0.26	−0.42	−0.48	−0.39	−0.40	−0.76	−0.51	−0.49	−0.06	−0.43	−0.42
+105	−15	+66.5	−0.58	−0.33	0.49	−0.55	−0.45	−0.47	−0.83	−0.57	−0.56	−0.12	−0.55	−0.55
+110	−20	+71.5	−0.67	−0.41	−0.57	−0.64	−0.53	−0.55	−0.92	−0.65	−0.65	−0.20	−0.66	−0.66
+115	−25	+76.5	−0.78	−0.51	−0.67	−0.74	−0.63	−0.65	−1.02	−0.76	−0.75	−0.31	−0.80	−0.80
+120	−30	+81.5	−0.96	−0.67	−0.83	−0.90	−0.79	−0.79	−1.17	−0.89	−0.90	−0.44	−0.96	−0.96
+121	−31	+82.5	−1.07	−0.79	−0.95	−1.02	−0.91	−0.91	−1.29	−1.01	−1.02	−0.56	−1.08	−1.08
+122	−32	+83.5	−1.24	−0.96	−1.12	−1.19	−1.08	−1.08	−1.46	−1.18	−1.19	−0.73	−1.25	−1.25
+123	−33	+84.5	−1.51	−1.23	−1.39	−1.46	−1.35	−1.35	−1.73	−1.45	−1.46	−1.00	−1.52	−1.52

TABLE XI.—*Corrections to the north polar distances, etc.*—Continued.

N. P. D.	Dec.	Z. D.	1864.	1865.	1866.	1867.	1868.	1869.	1870.	1871.	1872.	1873.	1874.	1875.
°	°	°	"	"	"	"	"	"	"	"	"	'	"	"
−46	+44	−84.5	+1.49	+1.68	+0.60	+0.60	−2.06	+2.07	−2.08	−1.84	−1.84	−1.84	−1.84	−1.86
−45	+45	−83.5	+1.22	+1.41	+0.33	+0.33	−1.96	−1.97	−1.98	−1.74	−1.74	−1.74	−1.74	−1.76
−44	+46	−82.5	+1.04	+1.24	+0.16	+0.16	−1.82	−1.83	−1.84	−1.60	−1.60	−1.60	−1.60	−1.62
−43	+47	−81.5	+0.93	+1.12	+0.04	+0.03	−1.69	−1.70	−1.71	−1.47	−1.47	−1.47	−1.47	−1.49
−42	+48	−80.5	+0.87	+1.06	−0.02	−0.03	−1.56	−1.57	−1.58	−1.34	−1.34	−1.34	−1.34	−1.36
−40	+50	−78.5	+0.82	+1.00	−0.04	−0.05	−1.29	−1.30	−1.31	−1.06	−1.06	−1.06	−1.06	−1.08
−35	+55	−73.5	+0.67	+0.83	−0.12	−0.13	−0.95	−0.96	−0.97	−0.72	−0.73	−0.73	−0.73	−0.74
−30	+60	−68.5	+0.55	+0.70	−0.15	−0.16	−0.72	−0.73	−0.74	−0.51	−0.53	−0.52	−0.53	−0.52
−25	+65	−63.5	+0.44	+0.57	−0.17	−0.19	−0.56	−0.58	−0.59	−0.36	−0.39	−0.38	−0.39	−0.44
−20	+70	−58.5	+0.35	+0.45	−0.15	−0.18	−0.42	−0.45	−0.47	−0.25	−0.29	−0.28	−0.29	−0.35
−15	+75	−53.5	+0.27	+0.33	−0.12	−0.15	−0.30	−0.33	−0.35	−0.16	−0.21	−0.20	−0.21	−0.27
−10	+80	−48.5	+0.20	+0.25	−0.09	−0.12	−0.19	−0.22	−0.25	−0.06	−0.11	−0.10	−0.11	−0.19
−5	+85	−43.5	+0.15	+0.17	−0.05	−0.09	−0.10	−0.13	−0.16	+0.02	−0.05	−0.02	−0.03	−0.12
0	+90	−38.5	+0.10	+0.09	−0.02	−0.05	−0.02	−0.05	−0.08	+0.07	0.00	+0.02	0.00	−0.08
+5	+85	−33.5	+0.05	+0.04	+0.01	−0.03	+0.06	+0.03	−0.01	+0.12	+0.05	+0.07	+0.05	−0.03
+10	+80	−28.5	+0.02	0.00	+0.03	−0.01	+0.11	+0.08	+0.04	+0.15	+0.09	+0.11	+0.09	+0.02
+15	+75	−23.5	0.00	−0.03	+0.04	0.00	+0.14	+0.12	+0.08	+0.18	+0.12	+0.14	+0.12	+0.05
+20	+70	−18.5	−0.02	−0.04	+0.04	+0.01	+0.16	+0.16	+0.12	+0.21	+0.15	+0.17	+0.15	+0.08
+25	+65	−13.5	−0.03	−0.05	+0.03	+0.01	+0.17	+0.16	+0.14	+0.20	+0.15	+0.17	+0.15	+0.11
+30	+60	−8.5	−0.02	−0.04	+0.02	+0.01	+0.19	+0.18	+0.16	+0.20	+0.17	+0.18	+0.16	+0.13
+35	+55	−3.5	−0.03	−0.04	0.00	−0.01	+0.18	+0.17	+0.16	+0.18	+0.17	+0.17	+0.16	+0.15
+40	+50	+1.5	−0.02	−0.02	−0.02	−6.02	+0.18	+0.18	+0.18	+0.18	+0.18	+0.18	+0.18	+0.18
+45	+45	+6.5	−0.01	0.00	−0.04	−0.04	+0.18	+0.19	+0.20	+0.18	+0.19	+0.19	+0.20	+0.22
+50	+40	+11.5	−0.02	0.00	−0.07	−0.06	+0.17	+0.19	+0.21	+0.16	+0.19	+0.18	+0.20	+0.24
+55	+35	+16.5	−0.01	+0.01	−0.07	−0.05	+0.19	+0.21	+0.23	+0.16	+0.21	+0.19	+0.21	+0.26
+60	+30	+21.5	−0.02	0.00	−0.08	−0.05	+0.21	+0.22	+0.26	+0.16	+0.22	+0.20	+0.22	+0.29
+65	+25	+26.5	−0.04	−0.01	−0.08	−0.04	+0.24	+0.27	+0.31	+0.19	+0.25	+0.23	+0.25	+0.32
+70	+20	+31.5	−0.07	−0.06	−0.07	−0.03	+0.28	+0.31	+0.35	+0.22	+0.28	+0.26	+0.28	+0.36
+75	+15	+36.5	−0.11	−0.11	−0.04	0.00	+0.34	+0.37	+0.41	+0.27	+0.34	+0.32	+0.34	+0.43
+80	+10	+41.5	−0.17	−0.18	−0.01	+0.03	+0.42	+0.45	+0.48	+0.32	+0.39	+0.37	+0.39	+0.47
+85	+5	+46.5	−0.22	−0.26	+0.03	+0.07	+0.52	+0.54	+0.57	+0.40	+0.47	+0.44	+0.45	+0.53
+90	0	+51.5	−0.28	−0.34	+0.07	+0.10	+0.62	+0.65	+0.68	+0.48	+0.53	+0.52	+0.53	+0.60
+95	−5	+56.5	−0.36	−0.44	+0.10	+0.12	+0.74	+0.76	+0.78	+0.58	+0.63	+0.62	+0.63	+0.69
+100	−10	+61.5	−0.45	−0.57	+0.12	+0.14	+0.86	+0.88	+0.90	+0.68	+0.72	+0.71	+0.72	+0.77
+105	−15	+66.5	−0.56	−0.70	+0.13	+0.14	+1.01	+1.03	+1.04	+0.80	+0.83	+0.82	+0.83	+0.87
+110	−20	+71.5	−0.67	−0.82	+0.10	+0.11	+1.20	+1.21	+1.22	+0.99	+1.00	+1.00	+1.00	+1.01
+115	−25	+76.5	−0.81	−0.98	+0.04	+0.05	+1.50	+1.51	+1.52	+1.28	+1.28	+1.28	+1.28	+1.30
+120	−30	+81.5	−0.97	−1,16	−0.08	−0.09	+2.03	+2.04	+2.05	+1.82	+1.82	+1.82	+1.82	+1.84
+121	−31	+82.5	−1.09	−1.28	−0.20	−0.20	+2.17	+2.18	+2.19	+1.96	+1.96	+1.96	+1.96	+1.98
+122	−32	+83.5	−1.26	−1.45	−0.37	−0.37	+2.31	+2.32	+2.33	+2.10	+2.10	+2.10	+2.10	+2.12
+123	−33	+84.5	−1.53	−1.72	−0.64	−0.64	+2.41	+2.42	+2.43	+2.20	+2.20	+2.20	+2.20	+2.22

TABLE XI.—*Corrections to the north polar distances, etc.*—Continued.

N. P. D.	Dec.	Z. D.	1876.	1877. (*a*)	1877. (*b*)	1878.	1879.	1880.	1881.	1882.	1883 and 1884.	1885.	1886.	1887.
°	°	°	″	″	″	″	″	″	″	″	″	″	″	″
−46	+44	−84.5	−1.86	−1.85	+0.73	+0.53	+0.64	+0.64	+1.19	+1.07	+1.12	+1.15	+1.19	+1.14
−45	+45	−83.5	−1.76	−1.73	+0.46	+0.26	+0.37	+0.37	+0.92	+0.80	+0.85	+0.88	+0.92	+0.87
−44	+46	−82.5	−1.62	−1.60	+0.29	+0.09	+0.20	+0.20	+0.75	+0.63	+0.68	+0.71	+0.75	+0.70
−43	+47	−81.5	−1.49	−1.47	+0.18	−0.02	+0.09	+0.09	+0.64	+0.52	+0.57	+0.60	+0.64	+0.59
−42	+48	−80.5	−1.36	−1.35	+0.12	−0.08	+0.03	+0.04	+0.58	+0.46	+0.51	+0.54	+0.58	+0.53
−40	+50	−78.5	−1.07	−1.07	+0.08	−0.13	0.00	+0.01	+0.52	+0.42	+0.47	+0.49	+0.53	+0.48
−35	+55	−73.5	−0.71	−0.70	−0.03	−0.20	−0.10	−0.09	+0.38	+0.29	+0.34	+0.36	+0.41	+0.36
−30	+60	−68.5	−0.54	−0.56	−0.08	−0.23	−0.16	−0.14	+0.31	+0.20	+0.25	+0.27	+0.31	+0.27
−25	+65	−63.5	−0.43	−0.44	−0.12	−0.26	−0.19	−0.16	+0.23	+0.13	+0.17	+0.20	+0.24	+0.20
−20	+70	−58.5	−0.34	−0.36	−0.15	−0.25	−0.21	−0.17	+0.17	+0.08	+0.12	+0.14	+0.17	+0.14
−15	+75	−53.5	−0.26	−0.28	−0.17	−0.24	−0.21	−0.17	+0.12	+0.03	+0.07	+0.10	+0.14	+0.10
10	+80	−48.5	−0.18	−0.21	−0.17	−0.21	−0.21	−0.16	+0.09	0.00	+0.04	+0.06	+0.09	+0.06
−5	+85	−43.5	−0.10	−0.14	−0.16	−0.17	−0.19	−0.13	+0.06	−0.02	+0.02	+0.03	+0.06	+0.03
0	+90	−38.5	−0.06	−0.09	−0.15	−0.15	−0.17	−0.11	+0.03	−0.04	−0.01	0.00	+0.03	0.00
+5	+85	−33.5	−0.01	−0.04	−0.15	−0.13	−0.16	−0.10	+0.01	−0.06	−0.03	−0.02	+0.01	−0.02
+10	+80	−28.5	+0.04	0.00	−0.14	−0.11	−0.14	−0.09	−0.02	−0.07	−0.05	−0.04	−0.01	−0.04
+15	+75	−23.5	+0.07	+0.03	−0.13	−0.09	−0.12	−0.07	−0.03	−0.08	−0.06	−0.05	−0.03	−0.05
+20	+70	−18.5	+0.10	+0.07	−0.12	−0.09	−0.12	−0.08	−0.05	−0.09	−0.07	−0.06	−0.05	−0.06
+25	+65	−13.5	+0.12	+0.09	−0.12	−0.09	−0.12	−0.08	−0.07	−0.10	−0.09	−0.08	−0.07	−0.08
+30	+60	−8.5	+0.14	+0.12	−0.12	−0.09	−0.11	−0.09	−0.09	−0.10	−0.10	−0.09	−0.08	−0.09
+35	+55	−3.5	+0.15	+0.15	−0.13	−0.10	−0.12	−0.10	−0.10	−0.11	−0.11	−0.11	−0.11	−0.11
+40	+50	+1.5	+0.19	+0.19	−0.12	−0.12	−0.12	−0.12	−0.12	−0.12	−0.12	−0.12	−0.12	−0.12
+45	+45	+6.5	+0.22	+0.22	−0.11	−0.14	−0.12	−0.14	−0.14	−0.13	−0.13	−0.13	−0.13	−0.13
+50	+40	+11.5	+0.23	+0.24	−0.12	−0.15	−0.13	−0.15	−0.15	−0.14	−0.14	−0.15	−0.16	−0.15
+55	+35	+16.5	+0.25	+0.28	−0.12	−0.15	−0.12	−0.16	−0.17	−0.14	−0.15	−0.16	−0.17	−0.16
+60	+30	+21.5	+0.27	+0.31	−0.11	−0.15	−0.12	−0.16	−0.19	−0.15	−0.17	−0.18	−0.19	−0.18
+65	+25	+26.5	+0.30	+0.34	−0.11	−0.15	−0.12	−0.17	−0.21	−0.16	−0.18	−0.19	−0.21	−0.19
+70	+20	+31.5	+0.34	+0.38	−0.10	−0.13	−0.10	−0.15	−0.22	−0.17	−0.19	−0.20	−0.23	−0.20
+75	+15	+36.5	+0.40	+0.44	−0.09	−0.10	−0.08	−0.14	−0.25	−0.18	−0.21	−0.23	−0.26	−0.23
+80	+10	+41.5	+0.44	+0.48	−0.09	−0.08	−0.07	−0.12	−0.28	−0.20	−0.24	−0.26	−0.29	−0.26
+85	+5	+46.5	+0.50	+0.54	−0.08	−0.05	−0.05	−0.10	−0.31	−0.23	−0.26	−0.29	−0.32	−0.29
+90	0	+51.5	+0.59	+0.61	−0.07	−0.01	−0.03	−0.08	−0.34	−0.25	−0.29	−0.32	−0.35	−0.32
+95	−5	+56.5	+0.67	+0.69	−0.07	+0.01	−0.03	−0.07	−0.39	−0.29	−0.33	−0.35	−0.38	−0.35
+100	−10	+61.5	+0.76	+0.77	−0.10	+0.01	−0.03	−0.07	−0.45	−0.35	−0.39	−0.42	−0.45	−0.42
+105	−15	+66.5	+0.86	+0.86	−0.13	0.00	−0.06	−0.09	−0.52	−0.41	−0.46	−0.48	−0.52	−0.48
+110	−20	+71.5	+1.01	+1.01	−0.19	−0.04	−0.11	−0.13	−0.59	−0.49	−0.54	−0.57	−0.61	−0.57
+115	−25	+76.5	+1.30	+1.30	−0.26	−0.09	−0.20	−0.21	−0.69	−0.60	−0.65	−0.68	−0.68	−0.68
+120	−30	+81.5	+1.84	+1.84	−0.42	−0.22	−0.33	−0.33	0.88	−0.76	−0.81	−0.84	−0.88	−0.84
+121	−31	+82.5	+1.98	+1.98	−0.53	−0.33	−0.44	−0.44	−0.99	−0.87	−0.92	−0.95	−0.99	−0.95
+122	−32	+83.5	+2.12	+2.12	−0.70	−0.50	−0.61	−0.61	−1.16	−1.04	−1.09	−1.12	−1.16	−1.12
+123	−33	+84.5	+2.22	+2.22	−0.97	−0.77	−0.88	−0.88	−1.43	−1.31	−1.36	−1.39	−1.43	−1.39

NOTE.—Of the two corrections for 1877, the first, (*a*), is applicable to the north polar distances of the planets; the second, (*b*), to the concluded north polar distances of the fixed stars.

Chapter II.

THE NORTH POLAR DISTANCES OF THE WASHINGTON TRANSIT CIRCLE.

The telescope of the Washington transit circle is approximately of the same aperture and length as that of the Greenwich one, namely, 8 inches clear aperture and 12 feet focal length. The stability of its mounting leaves much to be desired, the level, azimuth, and nadir point being all subject to wide and rapid changes. The accidental errors of the observations, both in right ascension and declination, are consequently large; but there is no evident reason why the systematic errors should not be of the smallest class. The results in declination do, indeed, show apparently wide deviations from year to year; but it is the object of this paper to reduce them to a determinate law, and, if possible, to trace them to their sources and free the results from them.

§ 1. *General description of the instrument, its use and its behavior.*

The instrument is supplied with two finely divided circles, both movable on the axis. That on the clamp end of the axis is called circle A, the other, circle B. Each pier is supplied with four microscopes, arranged diagonally, so that zenith distances can at any time be independently measured with the two circles, or with one circle in any position relative to the telescope. The divisions are so numbered that in either position of the instrument the readings of the east circle increase from the zenith toward the south, and those of the west circle from the zenith toward the north.

These arrangements afford valuable means for investigating the peculiarities of the instrument as regards flexure and analogous sources of error, which, so far as I know, have never been applied in any other instrument. A detailed investigation of certain peculiarities, especially in the flexure, was made in 1865, and was published in the Washington observations for that year. The following is a brief outline of some results of that investigation.

Flexure of circles.—The most remarkable result is that the circumference of neither circle on which the divisions are cut turns uniformly with the axis. In the case of circle B the inequality has a coefficient of $1''.20$, and in that of circle A of $0''.37$. It goes through one period in a revolution, and depends solely upon the position of the circle with respect to the vertical.

It thus appears that the flexure of a meridian instrument as usually determined may be due to the circles as well as to the telescope. In the former case, and perhaps in any case, it may have a term depending upon the cosine of the zenith distance as well as upon the sine.

Flexure of micrometer.—Another result of the investigation is that flexure affects the telescope micrometer as well as the telescope and circle. This is shown by the variability of the micrometer

reading for coincidence of the fixed and movable micrometer threads as the instrument revolves. It is thus shown that instrumental flexure may arise from three different sources—the effect of gravity on the circle, on the tube of the telescope, and on the micrometer itself. I know of no reason why the flexure in other large instruments may not arise in all these ways, instead of being confined to the tube of the telescope, as often assumed.

To state the numerical results of the investigation we premise that the circle reading by which the position of each circle is defined is not that of either of the four microscopes, but that of a horizontal setting microscope midway between the two other microscopes on the south side of the east pier and on the north side of the west pier. This arrangement has the valuable result that all the readings have the same relation to the line of gravity when the instrument is reversed. Let us put

R, the reading of the horizontal microscope.

Then the readings of the four microscopes of circle A require the correction

$$+0''.37 \sin R - 0''.01 \cos R = 0''.37 \sin (R - 1^\circ\ 33')$$

and those of circle B

$$+0''.84 \sin R - 0''.86 \cos R = 1''.20 \sin (R - 45^\circ\ 40')$$

on account of circle flexure. Since these corrections depend on the attraction of gravity they could be included in the same table with the correction for errors of division so long as the position of the reading microscopes is not materially changed. This, however, has not been done.

Micrometer flexure.—The effect of micrometer flexure is that all zenith distances require the correction

$$-0''.31 \sin Z - 0''.29 \cos Z \qquad (a)$$

Z being the zenith distance of that point at which the telescope is directed, measured in the direction of the increasing reading of circle A. Measured in the direction of the increasing reading of circle B the correction will be

$$-0''.31 \sin Z + 0''.29 \cos Z. \qquad (b)$$

If we agree always to measure Z toward the south, and require the corresponding correction, it will be

(*a*) when the clamp is east,
(*b*) when the clamp is west.

Flexure of telescope.—The flexure of the telescope tube is such that all zenith distances measured with the telescope require the correction

$$-0''.59 \sin Z.$$

These various flexures seem to have been determined with all desirable precision. The details of supplementary determinations of the flexure of the telescope and micrometer are given in the Washington observations for 1872. It has throughout been assumed that the telescope tube has no flexure depending upon the cosine of the zenith distance.

The correction of the telescope and micrometer flexure can be combined into the single expression

$$-0''.90 \sin Z \pm 0''.29 \cos Z$$

the upper sign being taken for clamp west and the lower one for clamp east. Moreover, the circle flexure can be included in an expression of the same form, namely,

$$a \sin Z + b \cos Z$$

so long as the position of the circle on the axis remains unchanged. But the position of the circle used in observation is changed from year to year, in order that the declinations of the same star may depend upon different divisions from year to year. It has also been the custom to reverse the telescope at the beginning of every year. This, however, was not done at the beginning of 1872, because the telescope was out of use during the first half of the preceding year.

In 1866 circle A was used in observation; but since that time A has been used only as the setting circle and all the determinations of declination have been made with circle B. This has been done because of evidence that circle B is better graduated than circle A.

§ 2. *Form of the reductions and results as published.*

The results of positions of standard stars resulting from observation are given annually in the form of corrections to the apparent places of the American Ephemeris resulting from the individual observations. In the preliminary reductions and in this table of individual corrections the separate results are given without corrections for flexure, errors of division, or error of latitude. The sum total of these corrections is given once for all in connection with the mean results

In the introduction to the volume for each year is found a comparison of direct and reflected observations, with the determination of the corrections necessary to reduce all the observations to a standard which is the mean of the results obtained directly and by reflection. The resulting correction is included with those for flexure and latitude.

§ 3. *Constants given by the comparison of direct and reflex observations.*

For the sake of clearness I shall state the annual results of the comparison of direct and reflex observations in the following way:

The nadir point being determined by the coincidence of the micrometer thread with its reflected image, and being affected by error of division and by all other accidental causes peculiar to the method of observing and the position of the instrument, I assume that the nadir reading is each year systematically

too great by β.

The fiducial thread on which the star is set being really the mean of two threads some 10 seconds apart, I assume that the observer in setting upon the image of a star places this imaginary mean line

too low by α.

We now readily see that these two causes have the following effects upon the reduced zenith distances south:

Observation.	Circle west.	Circle east.
North direct	$+\ \alpha + \beta$	$+\ \alpha - \beta$
North reflex	$-\ \alpha - \beta$	$-\ \alpha + \beta$
South direct	$-\ \alpha + \beta$	$-\ \alpha - \beta$
South reflex	$+\ \alpha - \beta$	$+\ \alpha + \beta$
North D — R	$= + 2\alpha + 2\beta$	$+ 2\alpha - 2\beta$
South D — R	$= - 2\alpha + 2\beta$	$- 2\alpha - 2\beta$

The comparison and discussion in the introduction to the observations for each year afford data for determining the values of α and β. The value of D — R is called $2\Delta Z$, and is determined separately for north and south stars. We therefore have

$$2\alpha = \Delta Z\ (\text{north}) - \Delta Z\ (\text{south})$$

and

$$2\beta = \quad \Delta Z\ (\text{north}) + \Delta Z\ (\text{south})\ \text{with circle west}$$
$$2\beta = - \Delta Z\ \ \text{north}\ \ - \Delta Z\ (\text{south})\ \text{with circle east}$$

For convenience I also use

$$\beta' = \tfrac{1}{2}\{\Delta Z\ (\text{north}) + \Delta Z\ \text{south}\}$$
$$= \quad \beta\ (\text{circle west})$$
$$= - \beta\ (\text{circle east})$$

It will be seen that the comparison of the direct and reflex observations of each year suffices completely to determine the values of both α and β. Table XII shows the position of the instrument, the circle reading, and the values of α and β derived from the work of each year.

The second column gives the direction of that end of the axis the circle on which was read during the year. As already stated the rule has been to reverse the instrument at the beginning of each year. An exception was made in 1872 because the instrument had been in use only five months of the year preceding. It is also to be remarked that the instrument was in use only during the first half of 1869, it having been dismounted in June of that year and removed to a new foundation some 10 or 12 metres toward the west.

The values of β may first claim our attention. It being always possible that the determination of the nadir point is subject to undiscoverable constant errors, we may regard β as expressing the sum total of those errors If, however, these sources of error remain the same from year to year the values of β ought to remain unchanged on reversal of the instrument, provided the position of the latter on its axis is not altered. This was the case for the years 1867–1868, 1870–1871, 1877–1878, 1879–1880, 1881–1882.

It will be seen that this condition is far from being fulfilled in most of the cases. As there can, I apprehend, be no doubt that we should consider the absolute zenith as that given by the mean of the direct and reflex observations, I have not thought it necessary to investigate the changes in β.

During the earlier years (1866–1870) the adopted coefficient of flexure of telescope and micrometer was $-0.''78$ instead of $-0.''90$, which was used from and after 1871. I have reduced the earlier values of ΔZ to the new flexure; but for 1870 this was done in only a part of the work, so that there is a slight numerical discrepancy in the numbers for that year.

TABLE XII.—*Results of the annual comparisons of the direct and reflex zenith distances south observed with the Washington transit circle.*

Year.	Circle.		Zenith at circle reading.		ΔZ		2α	2β	α	β'
	Name.	Position.			North.	South.				
			°	′	″	″	″	″	″	″
1866	A	East	179	56	+1.04	+0.43	+0.61	−1.47	+0.30	+0.73
1867	B	East	0	0	+0.53	−0.22	+0.75	−0.31	+0.38	+0.16
1868	B	West	0	0	+0.86	+0.18	+0.68	+1.04	+0.34	+0.52
1869	B	East	0	30	−0.19	−0.37	+0.18	+0.56	+0.09	−0.28
1870	B	West	0	0	−0.05	+0.10	−0.15	+0.05	−0.08	+0.02
1871 } 1872 }	B	East	0	0	+1.02	−0.43	+1.45	−0.59	+0.72	+0.30
1873	B	West	42	48	+0.51	+0.09	+0.42	+0.60	+0.21	+0.30
1874	B	East	44	0	+0.02	−0.87	+0.89	+0.85	+0.44	−0.42
1875	B	West	46	0	+0.95	+0.24	+0.71	+1.19	+0.36	+0.60
1876	B	East	48	0	+0.57	−0.24	+0.81	−0.33	+0.40	+0.16
1877	B	West	45	4	+0.78	−0.01	+0.79	+0.77	+0.40	+0.38
1878	B	East	45	4	−0.11	−0.77	+0.66	+0.88	+0.33	−0.44
1879	B	West	47	4	+1.59	+0.73	+0.86	+2.32	+0.43	+1.16
1880	B	East	47	4	−0.88	−2.50	+1.62	+3.38	+0.81	−1.69
1881	B	West	49	4	+0.95	+0.37	+0.58	+1.32	+0.29	+0.66
1882	B	East	49	4	−0.54	−1.11	+0.57	+1.65	+0.28	−0.82
1883	B	West	50	4	+1.05	+0.50	+0.55	+1.55	+0.27	+0.78
1884	B	East	51	4	−0.11	−1.15	+1.04	+1.26	+0.52	−0.63
1885	B	West	52	4	+1.13	+0.38	+0.75	+1.51	+0.38	+0.76
1886	B	East	53	4	−0.26	−1.10	+0.84	+1.36	+0.42	−0.68

§ 4. *Special discussion of the discordance between the results of direct and reflex observations.*

The quantity α demands a fuller investigation. It is fairly constant through the 21 years of observation, with the exception of 1870, 1871, and 1880. I have assumed it to be due to a habit on the part of the observers of setting the mean declination thread too low by α. It might equally arise from a play in the fastenings of the instrument, causing a change of the nadir point as the instrument passed through the zenith; but the most careful examination has failed to show any such play. What we can say with certainty is that there is some cause by virtue of which any arc including the zenith is found to measure more by reflected observations than by direct ones, and this by an amount which appears to be constant, whatever the length of the arc. While the personal habit I have assumed will account for this, we are not required to assume that this is the real cause. Any cause acting in the way described will suffice.

Owing to the symmetry of everything with respect to the line of gravity when the instrument is reversed, we are justified in assuming in the absence of any evidence to the contrary that the error, whatever it is, is equal on the two sides of the zenith; but it is still possible that it acts differently in the case of the direct and reflex observations

To show how the influence of α upon the results of the measures may best be determined, we begin with a comprehensive statement of the system on which the annual results of the observations are derived. The north polar distances, as currently reduced and printed in detail, are derived with

an assumed latitude of 38° 53′ 38″.80 and without any corrections for errors of graduation, flexure, latitude, error of nadir point, or any other systematic cause. At the close of the year's work the corrections for graduation and flexure are then applied once for all to the mean of all the direct and reflex results for each star, and from their discordance are derived in effect, though not ostensibly, the values of α and β. Correcting the zenith distances of stars observed on both sides of the pole for graduation, flexure, and $\alpha + \beta'$, a correction, $\Delta\varphi$, to the latitude is derived separately for each year. The definitive corrections to the north polar distances derived from the current reductions are then, besides those for graduation and flexure,

North direct	$-\alpha - \beta' - \Delta\varphi$
South direct	$+\alpha - \beta' - \Delta\varphi$
North reflex	$+\alpha + \beta' - \Delta\varphi$
South reflex	$-\alpha + \beta' - \Delta\varphi$

The results derived from the application of these corrections are independent of any systematic error in the determination of the nadir point; effectively they are the same as if the pole were a visible point in the heavens and north polar distances were measured by setting the instrument on the pole and on a star and taking the difference of the circle readings. The general result of the comparisons between the direct and reflex measures during the 21 years that the instrument has been in use may then be expressed as follows:

In the case of stars which culminate north of the zenith there is no systematic difference between the polar distances derived from direct and from reflex observations.

South of the zenith the reflex north polar distances exceed the direct ones by the quantity 4α.

To find how far the assumption that α does not vary with the zenith distance is justified by the observations, I have made a comparison of the residuals of R — D for a number of years, scattered through the series. The comparison for the three years, 1866–1868, is found in the volume of observations for 1868; with this I have joined the mean results for 1874, 1875, and 1879–1884.

Mean values of R — D *after subtracting a constant on each side of the zenith.*

N. P. D.	1866 to 1868.	1874 to 1884.	Mean.
°	″	″	″
1	−0.43	−0.12	−0.28
11	−0.05	−0.17	−0.11
21	+0.25	+0.03	+0.04
31	+0.27	+0.08	+0.18
41	+0.08	−0.05	+0.02
51	Zenith.	Zenith.	. . . .
61	−0.02	+0.32	+0.15
71	−0.02	−0.12	−0.07
81	+0.08	−0.34	−0.13
91	+0.20	+0.21	+0.21
98	−0.23	+0.50	+0.13

On the theory on which we have so far proceeded, α arises from some cause which affects both the direct and reflex observations. Although the presumption is in favor of this theory its proof is impossible. We may, however, easily test the question whether the two classes of measures are

equally affected by the cause in question by determining whether the north polar distances of different years are brought into better agreement by the application of α. The following table shows for each year—

1. The mean corrections to the standard north polar distances of 38 stars culminating between 9° and 50° of zenith distance south, as they would result from direct measures of the arc between the pole and the star without any correction for α.

2. The same corrections from measures between the reflected pole and the reflected star.

3. The mean of the two results, which is the common result of both classes of measures when each class is corrected by 2α.

Table XIII.—*Comparison of direct and reflex north polar distances after elimination of zenith point.*

Year.	Δ N. P. D. Direct.	Reflex.	Mean.	p	a	ap
	″	″	″	″	″	″
1866	−0.02	+1.20	+0.59	+0.10	−0.11	−.011
1867	+0.12	+1.62	+0.87	+0.24	+0.03	+.007
1868	+0.11	+1.47	+0.79	+0.23	−0.04	−.009
1869	+0.10	+0.46	+0.28	+0.22	−0.54	−.119
1870	+0.03	−0.59	−0.28	+0.15	−1.03	−.154
1871 } 1872 }	−0.20	+2.70	+1.25	−0.08	+0.73	−.058
1873	+0.50	+1.34	+0.92	+0.62	−0.30	−.186
1874	−0.83	+0.95	+0.06	−0.71	+0.17	−.121
1875	+0.01	+1.43	+0.72	+0.13	−0.01	−.001
1876	−0.18	+1.44	+0.63	−0.06	+0.09	−.005
1877	−0.06	+1.52	+0.73	+0.06	+0.07	+.004
1878	−0.37	+0.95	+0.29	−0.25	−0.06	+.015
1879	−0.04	+1.68	+0.82	+0.08	+0.14	+.011
1880	−0.05	+3.19	+1.57	+0.07	+0.90	+.063
1881	−0.10	+1.06	+0.48	+0.02	−0.14	−.003
1882	−0.38	+0.76	+0.19	−0.26	−0.15	+.039
1883	+0.02	+1.12	+0.57	+0.14	−0.17	−.024
1884	−0.26	+1.82	+0.78	−0.14	+0.32	−.045
1885	−0.50	+1.00	+0.25	−0.38	+0.03	−.011
1886	−0.41	+1.27	+0.43	−0.29	+0.12	−.035
Mean.	−0.12	+1.32	+0.60			

It will be seen, from a comparison of the first three columns of the table, that the discordances between the north polar distances from reflex observations are excessive, and that the direct measures are not made more accordant by reduction to the mean of D and R, that is, by the application of the correction 2α. We may, however, consider the question, *What fraction of the correction* 2α *must we apply to the direct north polar distances in order to secure the best accordance of results?* We reach the answer to this question in the following way:

Put

$p_1, p_2, \ldots . p_{20}$, the excesses of the annual means of the direct measures over the general mean $-0''.12$;

$a_1, a_2, \ldots . a_{20}$, the excesses of the values of 2α for each year over the general mean $0''.72$;

k, the required fraction.

Then, by applying the correction $2k\alpha$ to the value of p for each year, the twenty annual excesses of $p+2k\alpha$ over their mean value will become

$$p_1+a_1k;\ p_2+a_2k;\ \ldots .\ p_{20}+a_{20}k.$$

In order that the sum of the squares of these excesses may be a minimum we must have

$$k=-\frac{a_1p_1+a_2p_2+\ \ldots .\ +a_{20}p_{20}}{a^2_1+a^2_2+\ \ldots .\ +a^2_{20}}$$

From the numbers of the table we find

$$\begin{aligned}\Sigma ap &= -0.643\\ \Sigma a^2 &= \quad 3.05\\ k &= +0.210\end{aligned}$$

It therefore appears that the maximum of accordance among the results of the twenty years' observations is obtained by adopting

$$D+\tfrac{1}{10}(R-D)$$

as the concluded result.

In the case of 1870 the value of α is very uncertain, owing to the paucity of reflex observations, but its exceptionally large value in 1880 is established by an ample series of such observations, both north and south of the zenith, so that the fact of this class of observations giving a mean correction of $+2''.38$ to the north polar distances of equatorial stars in 1880 is also well established. We can not but entertain a strong suspicion that α proceeds from some cause affecting the reflex observations only.

With or without α the direct measures of southern north polar distances show systematic variations from year to year of such a magnitude as to require investigation. We may conceive that they are derived by measuring the zenith distance south of the mean star and the zenith distance north of the pole, and adding the results. We shall, therefore, investigate and compare the separate discordances of these two components of the north polar distance. We may deal with the following classes of determinations of latitude:

1. The measured arc from the zenith point given by nadir observations to the pole as seen directly.
2. The measured arc from the observed nadir point to the reflected pole.

The complements of these arcs are given annually in the published volumes of Washington observations; but each of them is affected by the error β of nadir point, which can not be determined except by the comparison of direct and reflex observations. The value of β, determined in the way already employed, is independent of any hypothesis as to the partition of the discordance 4α between the direct and reflex observations; that is, β retains its value unchanged so long as we assume that α, even if it affects only the reflex observations, proceeds from a cause which acts equally in the north and south. If, then, we correct the separate annual colatitudes by β', we shall get two results, differing by 2α, the one from direct and the other from reflex observations, the discordance of which is to be examined as in the case of the north polar distances.

TABLE XIV.—***Mean results of the measures made each year from the assumed absolute zenith toward the north and the south.***

Year.	$-\Delta\varphi$			ΔZ D. S.		
	Direct.	Reflex.	Mean.	Direct.	Reflex.	Mean.
	"	"	"	"	"	"
1866	+0.39	+1.00	+0.70	—0.41	+0.20	—0.10
1867	+0.10	+0.85	+0.48	+0.02	+0.77	+0.39
1868	+0.33	+1.01	+0.67	—0.22	+0.46	+0.12
1869	+0.03	+0.21	+0.12	+0.07	+0.25	+0.16
1870	—0.18	—0.49	—0.34	+0.21	—0.10	+0.06
1871 } 1872 }	+0.28	+1.73	+1.00	—0.48	+0.97	+0.24
1873	+0.19	+0.61	+0.40	+0.31	+0.73	+0.52
1874	+0.07	+0.96	+0.52	—0.91	—0.02	—0.46
1875	+0.18	+0.89	+0.54	—0.17	+0.54	+0.18
1876	+0.32	+1.13	+0.72	—0.50	+0.31	—0.09
1877	+0.04	+0.83	+0.44	—0.10	+0.69	+0.29
1878	+0.29	+0.95	+0.62	—0.66	0.00	—0.33
1879	—0.45	+0.41	—0.02	+0.41	+1.27	+0.84
1880	+0.75	+2.37	+1.56	—0.80	+0.82	+0.01
1881	—0.04	+0.54	+0.25	—0.06	+0.52	+0.23
1882	+0.29	+0.86	+0.57	—0.67	—0.10	—0.38
1883	—0.25	+0.30	+0.02	+0.27	+0.82	+0.55
1884	+0.29	+1.33	+0.81	—0.55	+0.49	—0.03
1885	—0.61	+0.14	—0.23	+0.11	+0.86	+0.48
1886	+0.08	+0.92	+0.50	—0.49	+0.35	—0.07

This table has a close relation to Table XIII, in that the north polar distances shown in Table XIII are, in effect, the sums of the zenith distances of the pole, or $-\Delta\varphi$, and of the stars, both of which are given in Table XIV.

An examination of this table confirms the view that the reflex observations are subject to very large errors, changing from year to year, and eluding all the explanations of systematic error which I have thus far set forth. Especially noteworthy is the fact that the direct measures from the assumed absolute zenith both toward the north ($-\Delta\varphi$) and south (ΔZ. D. S.) are more discordant than their sum, which forms the observed north polar distance. Instead of the systematic discordances between these two measures agreeing from year to year, as they would do if they arose from a purely personal or instrumental cause, their tendency is to diverge in opposite directions, one being positive when the other is negative. This shows clearly that the assumed absolute zenith point is really subject to systematic errors, varying from year to year by larger amounts than the variations in the measures from any fixed zenith point. The assumption that the mean of the direct and reflex measures on both sides of the zenith gives the absolute zenith is based upon the hypothesis that the causes of errors affecting the reflex observations act equally on both sides of the zenith. We must, therefore, conclude that this hypothesis is not wholly tenable. At the same time the table seems to show that a part of the error

is subject to the assumed law. Thus, the three algebraically smallest values of $-\Delta\varphi$ correspond to values of ΔZ as follows:

$-\Delta\varphi$	ΔZ
$''$	$''$
−0.49	−0.10
+0.21	+0.25
+0.14	+0.86

and the three largest as follows:

$-\Delta\varphi$	ΔZ
$''$	$''$
+2.37	+0.82
+1.73	+0.97
+1.33	+0.49

A certain amount of correspondence is indicated, but no certain estimate of the common element in the two sets of errors can be made.

The most notable feature of the table is the alternation of signs among the direct results during the last ten years. This is undoubtedly associated with the annual reversal of the instrument, and shows that the position of the zenith point given by the mean of the direct and reflex observations may be regarded as constant relative to the instrument, so that it changes its sign relative to the north and south when the instrument is reversed. Now this would result from the use of an erroneous coefficient of the flexure dependent on the cosine of the zenith distance, a quantity which has not, I believe, been determined since 1872. The effect of this error would be—

1. Accordance of direct and reflex measures.

2. Measures of absolute zenith distance, if too small toward the north, would be too large toward the south, and *vice versa.*

3. With reversal of the instrument the error is reversed, and is therefore eliminated from the mean of two consecutive years' work.

The criterion thus supplied shows no well-marked additional cosine-flexure during the years 1866–'76. But for the ten years, 1877–'86, the comparison of the results obtained in the two positions of the instrument is as follows:

	$-\Delta\varphi$		Δ Z. D. S.	
	Direct.	Reflex.	Direct.	Reflex.
	$''$	$''$	$''$	$''$
Clamp east	−0.26	+0.44	+0.13	+0.83
Clamp west	+0.34	+1.29	−0.63	+0.31
East—west	−0.60	−0.85	+0.76	+0.52

The accordance of the four differences is striking, and if we attribute them to the cause suggested we should conclude that the instrument is affected by an omitted flexure term,

$$0''.34 \cos \text{Z. D.}$$

The general result of this comparison and discussion is that the observations made by reflection from quicksilver are subject to large systematic errors, varying from year to year in a way which it is difficult to account for or to reduce to any well-marked law. The general rules seem to be—

1. That all the measures of absolute zenith distance are too great.

2. That measures toward the north are subject to greater changes than those toward the south.

3. That measures made in the direction of decreasing readings of circle B, that is, toward the north when the circle is east and toward the south when it is west, are those most liable to this error.

§ 5. *Comparison of the standard north polar distances with those given by observations with the Washington transit circle.*

From the discussion of the preceding sections the following conclusions seem fairly deducible:

1. The zenith distances measured with the Washington transit circle are, in the general mean, fairly accurate, whether made directly or by reflection, except for a constant error for each year peculiar to all the stars observed on the same side of the zenith during that year.

2. In consequence the absolute north polar distances of all stars which culminate north of the zenith are probably fairly accurate in the general mean.

3. Owing to the break at the zenith all measures of absolute north polar distance of stars culminating south of the zenith must, during any one year, be regarded as subject to an undiscoverable constant error, by which they are too great or too small by the same amount.

4. The much greater discordance of the reflex than of the direct observations shows that the former should be entirely thrown out so far as absolute north polar distances are concerned.

5. Considering only the direct observations, the absolute north polar distances are nearly as accordant as those observed with the Greenwich circle, and may, therefore, be entitled to a certain weight in determining absolute north polar distances.

I shall apply these conclusions in the subsequent investigation.

Table XV contains the corrections to the standard positions given by the direct observations made each year upon 69 standard stars. From the numbers in this table those used in the preceding section have been derived.

The corrections here given are those which result from the definitive reductions published in the annual volumes; they therefore contain the correction 2α, except in the case of 1871–'72, when this correction was omitted. We now have two objects, namely (1), the determination of the corrections which must be applied to the reduced results of each year to reduce them to Boss's standard; (2), the corrections to Boss's standard, given by the observations. By subtracting the correction 2α we may get the mean results given for each year by direct observations alone. I have, however, slightly modified this method in consequence of the following considerations.

Were an equal number of observations made on each star during each year the general mean would be the only one to be adopted. The same would be true were there no systematic differences from year to year.

But an examination of the first column in Table XIV shows that even the direct results are subject to systematic deviations from year to year. Our proper course is, therefore, to regard the general mean of all the years as the standard to which the separate mean of each year shall be reduced, and then to reduce the observations of each separate star in each year to this mean. The mean result will then be that required.

If, then, we determine the mean value for each year of all the corrections given for southern stars in Table XV, this quantity will be a provisional error, which we may, in the first place, subtract from each of the separate results in order to reduce them to the tabular standard. The reductions of this standard to the mean of all observations with the instrument will then be the mean of the quantities given for each year in the first column of Table XIII.

TABLE XV.—*Corrections to* BOSS'S *north polar distances, given by the Washington direct observations of each year, reduced to the mean of the direct and reflex results except in* 1871–'72.

Star.	N. P. D.	1866.	1867.	1868.	1869.	1870.	1871.	1872.
	°	″	″	″	″	″	″	″
ι Cephei, S. P.	− 24.4	$+1.17_1$	. . .	-1.78_1	. . .	-1.63_2	. . .	$+0.19_2$
λ Draconis, S. P.	− 20.0	-0.27_2	$+0.95_3$	$+0.02_4$	. . .	. . .	$+0.67_1$	-0.68_1
β Cephei, S. P.	− 19.9	. . .	$+0.29_2$	-0.48_2	$+0.84_2$	-4.48_2	. . .	$+0.40_1$
κ Draconis, S. P.	− 19.6	. . .	$+0.55_2$	-1.48_2	. . .	. . .	. . .	$+1.24_2$
50 Cassiopeiæ, S. P	− 18.1	-1.20_2	-0.32_3	-0.18_3	-0.59_2	$+0.04_4$	. . .	$+0.79_2$
β Ursæ Minoris, S. P.	− 15.4	-0.89_3	-0.73_2	-0.09_3	. . .	$+1.68_2$	-0.77_3	$+0.09_2$
9 Draconis, S. P.	− 13.7	. . .	$+0.64_3$	$+0.34_4$	. . .	-0.21_1	-0.91_3	-0.54_1
γ Cephei, S. P.	− 13.0	-2.40_1	$+0.36_4$	-0.17_2	. . .	$+0.78_4$	. . .	. . .
48 Cephei, S. P.	− 12.7	$+0.78_4$	$+0.09_3$	-0.60_4	$+1.25_2$	$+0.32_2$	. . .	$+1.09_2$
κ Cephei, S. P.	− 12.6	. . .	-0.25_3	-1.05_2	-0.85_2	$+1.01_2$	. . .	. . .
ζ Ursæ Minoris, S. P.	− 11.8	. . .	. . .	. . .	. . .	. . .	. . .	. . .
4 Draconis, S. P.	− 11.8	$+0.81_3$	$+0.50_2$	-0.46_2	. . .	. . .	$+1.66_1$	-1.29_1
1 Draconis, S. P.	− 8.2	. . .	$+0.17_3$	$+0.45_2$	. . .	-1.18_1	$+0.17_2$	. . .
δ Ursæ Minoris, S. P.	− 3.4	-0.55_8	-0.17_7	-0.11_6	-0.61_6	-0.50_2	-0.05_3	-0.38_6
51 Cephei, S. P.	− 2.8	-0.02_7	$+0.79_3$	-0.28_6	. . .	-0.28_6	$+1.16_3$	$+0.73_5$
α Ursæ Minoris, S. P.	− 1.3	$+0.05_9$	$+0.07_9$	-0.07_9	$+0.24_7$	$+0.19_8$	-0.07_6	$+0.15_8$
λ Ursæ Minoris, S. P.	− 1.0	$+0.09_4$	-0.02_3	-0.33_7	-0.19_7	-0.14_5	$+1.44_1$	$+0.25_6$
λ Ursæ Minoris	+ 1.0	$+0.17_5$	$+0.18_7$	$+0.13_7$	. . .	-0.47_4	$+0.22_5$	-0.25_5
α Ursæ Minoris	+ 1.3	-0.44_9	$+0.15_9$	$+0.08_9$	$+0.17_7$	$+0.44_8$	-0.87_6	-1.12_8
51 Cephei	+ 2.8	-0.31_6	$+0.00_6$	-0.58_4	-0.11_4	-1.41_1	-0.83_3	-0.74_4
δ Ursæ Minoris	+ 3.4	-0.14_8	-0.24_7	$+0.33_8$	-0.31_3	$+0.11_7$	-0.34_4	-0.29_6
ε Ursæ Minoris	+ 7.8	-0.51_5	-0.09_4	-0.11_5	-0.24_2	-1.24_2	-0.18_3	-0.91_2
1 Draconis	+ 8.2	. . .	$+0.15_3$	-0.80_2	. . .	-0.25_3	-0.54_1	-0.65_2
4 Draconis	+ 11.8	-0.26_2	-0.09_4	-0.47_4	-0.39_2	$+0.36_5$	. . .	$+0.01_2$
ζ Ursæ Minoris	+ 11.8	-1.08_3	$+0.00_2$	$+0.57_3$	$+0.25_2$	$+0.48_2$	. . .	$+1.61_1$
κ Cephei	+ 12.6	. . .	$+0.07_2$	$+0.56_4$	. . .	-0.25_3	-0.02_3	. . .
48 Cephei	+ 12.7	$+0.10_3$	-0.01_2	-0.42_4	. . .	. . .	-0.32_3	$+0.28_2$
γ Cephei	+ 13.0	-1.14_4	-0.52_2	$+0.17_3$	. . .	-0.70_1	. . .	. . .
9 Draconis	+ 13.7	-0.19_2	-0.45_2	-0.83_2	-0.41_3	$+0.18_4$	. . .	. . .
β Ursæ Minoris	+ 15.4	-0.78_3	$+0.09_5$	-0.02_4	$+0.22_4$	$+0.08_4$	. . .	. . .
50 Cassiopeiæ	+ 18.1	-0.87_3	$+0.90_1$	-1.24_1	$+0.11_1$	. . .	-1.70_2	-0.20_1
κ Draconis	+ 19.6	-0.70_1	-0.04_2	-0.74_2	-0.45_2	-0.54_1	. . .	$+1.05_1$
β Cephei	+ 19.9	-0.25_4	$+0.10_2$	$+0.05_2$	-0.53_2	-0.21_2	$+0.11_1$	$+1.95_3$
λ Draconis	+ 20.0	$+0.56_3$	-0.13_4	-0.69_4	$+0.45_2$	-0.12_2	. . .	. . .
ι Cassiopeiæ	+ 23.1	$+0.07_2$	$+1.60_1$	$+1.00_4$	$+0.40_1$	. . .	$+0.03_3$	0.00_1
ι Cephei	+ 24.4	-0.27_3	-0.45_1	-1.20_3	. . .	. . .	$+0.30_2$	$+0.10_1$
α Ursæ Majoris	+ 27.6	$+1.32_3$	$+0.49_4$	-0.62_3	-0.53_3	-0.37_6	-0.45_1	-0.40_5
α Cephei	+ 27.9	$+0.17_5$	-0.09_4	$+1.84_4$	-0.55_3	$+0.66_2$	-0.88_1	-0.59_4
η Draconis	+ 28.2	. . .	-0.21_3	$+0.09_3$	. . .	. . .	. . .	. . .
α Cassiopeiæ	+ 34.1	$+0.31_2$	$+0.47_3$	$+0.68_4$	. . .	. . .	$+1.33_1$	$+1.25_2$
β Draconis	+ 37.6	. . .	. . .	$+0.80_2$	. . .	. . .	. . .	-0.01_3
γ Draconis	+ 38.5	$+0.21_3$	$+0.11_4$	-0.33_5	. . .	-0.54_5	$+1.01_1$	-0.22_4
α Persei	+ 40.6	$+0.42_4$	$+0.51_2$	$+1.43_3$	$+1.18_1$	-0.41_4	$+0.29_2$	-0.05_4

TABLE XV.—*Corrections to* **BOSS'S** *north polar distances, etc.*—**Continued.**

Star.	N. P. D.	1873.	1874.	1875.	1876.	1877.	1878.	1879.
	°	"	"	"	"	"	"	"
ι Cephei, S. P.	− 24.4	$+1.14_{1}$	. . .	$+0.74_{2}$	-0.87_{3}	-0.18_{3}	. . .	. . .
λ Draconis, S. P.	− 20.0	. . .	-0.66_{2}	-0.30_{4}	$+1.08_{3}$	-1.07_{2}	. . .	. . .
β Cephei, S. P.	− 19.9	$+0.35_{1}$	$+0.59_{1}$	. . .	-1.01_{1}	$+1.39_{1}$	. . .	-0.20_{1}
κ Draconis, S. P.	− 19.6	. . .	-2.09_{2}	-0.99_{3}	$+0.86_{3}$	$+0.18_{2}$	. . .	. . .
50 Cassiopeiæ, S. P.	− 18.1	-0.08_{2}	$+0.28_{2}$	-0.92_{2}	$+1.01_{3}$	-0.06_{3}	$+0.74_{1}$	. . .
β Ursæ Minoris, S. P.	− 15.4	0.00_{1}	$+1.02_{4}$	-0.74_{3}	-0.32_{3}	$+1.25_{2}$	. . .	-1.00_{3}
9 Draconis, S. P.	− 13.7	. . .	$+0.10_{2}$	-0.93_{2}	-0.90_{2}	-0.06_{2}	-0.18_{2}	. . .
γ Cephei, S. P.	− 13.0	$+1.16_{1}$	$+1.58_{3}$	$+0.20_{3}$	$+1.43_{3}$	-0.14_{2}	-0.78_{1}	$+0.90_{2}$
48 Cephei, S. P.	− 12.7	. . .	-1.10_{2}	. . .	$+0.16_{2}$	$+0.26_{2}$	$+1.04_{2}$	$+1.78_{2}$
κ Cephei, S. P.	− 12.6	. . .	-0.65_{2}	. . .	-0.69_{2}	$+0.40_{2}$	. . .	-0.20_{1}
ζ Ursæ Minoris, S. P.	− 11.8	. . .	$+0.34_{1}$	-1.55_{1}	$+0.52_{2}$	-0.17_{2}	. . .	. . .
4 Draconis, S. P.	− 11.8	. . .	$+0.01_{1}$	-0.32_{1}	$+0.89_{2}$	-0.32_{1}	. . .	. . .
ι Draconis, S. P.	− 8.2	$+2.47_{1}$	-1.43_{3}	$+0.42_{4}$	-0.04_{2}	-0.95_{2}	$+5.65_{1}$	. . .
δ Ursæ Minoris, S. P.	− 3.4	$+0.20_{6}$	$+0.31_{6}$	$+0.03_{7}$	$+0.04_{8}$	$+1.02_{6}$	$+0.10_{6}$	$+0.02_{3}$
51 Cephei, S. P.	− 2.8	$+1.33_{6}$	$+0.42_{6}$	$+0.58_{5}$	-0.06_{6}	$+0.24_{6}$	0.00_{5}	$+1.05_{6}$
α Ursæ Minoris, S. P.	− 1.3	$+0.43_{8}$	-0.40_{8}	-0.62_{8}	-0.46_{9}	-0.35_{8}	-0.75_{8}	-0.08_{8}
λ Ursæ Minoris, S. P.	− 1.0	$+1.80_{1}$	-0.23_{6}	-0.28_{4}	-0.52_{5}	$+0.26_{5}$	$+0.59_{5}$	$+0.34_{3}$
λ Ursæ Minoris	+ 1.0	$+0.54_{5}$	$+0.17_{6}$	$+0.86_{6}$	$+0.39_{5}$	$+0.51_{5}$	$+0.92_{5}$	$+0.76_{5}$
α Ursæ Minoris	+ 1.3	$+0.38_{8}$	$+0.22_{8}$	-0.01_{9}	-0.73_{9}	-0.13_{9}	-1.03_{8}	-0.17_{8}
51 Cephei	+ 2.8	$+0.21_{4}$	-0.14_{6}	$+0.28_{6}$	-0.32_{7}	-0.49_{5}	-0.13_{5}	-0.09_{3}
δ Ursæ Minoris	+ 3.4	-0.37_{6}	-1.02_{7}	-0.22_{6}	-1.07_{7}	-0.28_{7}	$+0.06_{6}$	$+0.15_{6}$
ε Ursæ Minoris	+ 7.8	-0.62_{3}	-0.61_{4}	$+0.36_{5}$	-0.04_{4}	-0.54_{5}	-0.03_{4}	$+0.51_{3}$
ι Draconis	+ 8.2	-0.08_{1}	-0.84_{4}	-0.08_{1}	-0.23_{2}	$+0.33_{2}$	-0.75_{3}	$+1.09_{2}$
4 Draconis	+ 11.8	$+0.20_{2}$	$+0.74_{2}$	$+0.01_{2}$	-0.38_{3}	$+0.61_{3}$	$+0.09_{1}$	. . .
ζ Ursæ Minoris	+ 11.8	. . .	-0.51_{3}	-1.41_{2}	0.00_{3}	$+0.62_{4}$	-0.15_{1}	. . .
κ Cephei	+ 12.6	-1.85_{1}	-0.34_{2}	-0.12_{2}	$+0.26_{3}$	$+0.34_{1}$	$+1.00_{1}$	$+1.08_{2}$
48 Cephei	+ 12.7	$+1.13_{2}$	$+0.66_{3}$	$+0.29_{2}$	-0.15_{3}	-0.20_{3}	. . .	. . .
γ Cephei	+ 13.0	. . .	$+0.33_{2}$	-0.19_{4}	-0.86_{3}	-0.28_{1}	-1.35_{1}	-0.11_{1}
9 Draconis	+ 13.7	. . .	-0.82_{2}	$+1.67_{2}$	$+0.89_{4}$	$+0.58_{3}$	-0.16_{2}	. . .
β Ursæ Minoris	+ 15.4	$+0.89_{2}$	-0.39_{6}	$+0.75_{5}$	$+0.49_{4}$	$+0.12_{4}$	-0.23_{3}	$+1.12_{5}$
50 Cassiopeiæ	+ 18.1	. . .	-1.91_{2}	$+0.28_{4}$	-0.43_{4}	$+0.56_{4}$	$+0.24_{1}$	$+0.98_{1}$
κ Draconis	+ 19.6	-0.24_{1}	-0.51_{1}	$+0.58_{3}$	-0.15_{3}	-0.05_{2}	-0.01_{1}	$+0.31_{2}$
β Cephei	+ 19.9	-0.36_{2}	-0.72_{2}	-1.07_{4}	$+0.19_{4}$	-0.38_{3}	-0.37_{1}	-2.04_{2}
λ Draconis	+ 20.0	-2.27_{1}	-0.28_{2}	-0.26_{4}	-0.27_{4}	$+0.15_{3}$	. . .	-0.88_{2}
ι Cassiopeæ	+ 23.1	$+0.48_{2}$	$+0.14_{4}$	$+0.90_{4}$	$+0.23_{4}$	$+0.01_{3}$	-0.94_{2}	. . .
ι Cephei	+ 24.4	$+1.07_{1}$	-0.24_{2}	$+0.19_{3}$	-0.13_{5}	-0.96_{2}	-0.13_{1}	-0.20_{3}
α Ursæ Majoris	+ 27.6	-1.91_{2}	-0.92_{5}	$+0.47_{4}$	$+0.55_{7}$	$+0.18_{4}$	$+0.18_{5}$	-0.06_{6}
α Cephei	+ 27.9	. . .	-0.67_{5}	$+0.48_{5}$	$+0.11_{5}$	-0.12_{5}	$+0.09_{3}$	-0.47_{4}
η Draconis	+ 28.2	-0.53_{1}	$+0.66_{1}$	-0.13_{2}	-0.58_{3}	$+1.26_{1}$	. . .	$+0.10_{1}$
α Cassiopeiæ	+ 34.1	-1.92_{3}	-0.66_{2}	$+0.78_{6}$	-0.32_{4}	$+0.48_{3}$	-0.81_{3}	-1.44_{2}
β Draconis	+ 37.6	. . .	-1.58_{2}	-1.17_{3}	$+0.40_{4}$	$+0.52_{1}$	. . .	-0.09_{2}
γ Draconis	+ 38.5	$+0.55_{2}$	$+0.46_{3}$	-0.21_{4}	$+0.18_{5}$	-0.10_{5}	-0.08_{2}	-0.10_{4}
α Persei	+ 40.6	$+0.69_{5}$	-0.11_{4}	-0.28_{4}	-0.12_{6}	$+0.38_{4}$	-0.04_{5}	$+0.12_{6}$

TABLE XV.—*Corrections to* BOSS'S *north polar distances, etc.*—Continued.

Star.	N. P. D.	1880.	1881.	1882.	1883.	1884.	1885.	1886.
	°	″	″	″	″	″	″	″
ι Cephei, S. P.	− 24.4	. . .	-1.91_1	-0.64_2	. . .	. . .	-1.49_1	. . .
λ Draconis, S. P.	− 20.0	$+1.08_1$	-0.64_4	. . .	. . .	-0.10_3	. . .	. . .
β Cephei, S. P.	− 19.9	-0.27_1	. . .	$+0.80_3$	$+0.30_2$	-0.48_1	-0.55_1	$+1.25_2$
κ Draconis, S. P.	− 19.6	. . .	$+0.61_2$	-0.83_1	-0.93_2	. . .	. . .	$+2.01_1$
50 Cassiopeiæ, S. P.	− 18.1	. . .	$+1.26_2$	-0.41_1	. . .	-0.14_1	. . .	-0.45_1
β Ursæ Minoris, S. P.	− 15.4	$+0.69_1$	-0.87_1	$+0.04_4$	$+0.02_4$	$+1.65_2$	-0.56_2	-0.09_2
9 Draconis, S. P.	− 13.7	$+0.34_3$	-0.76_2	$+0.29_4$	-0.25_2	$+0.75_2$	$+0.77_2$	$+0.85_2$
γ Cephei, S. P.	− 13.0	$+0.22_4$	-0.81_3	$+0.32_3$	-1.05_2	$+1.39_2$	-1.14_1	$+0.25_2$
48 Cephei, S. P.	− 12.7	-0.88_2	$+0.80_3$	$+0.24_3$	-0.58_3	. . .	$+0.70_2$	$+0.05_2$
κ Cephei, S. P.	− 12.6	. . .	. . .	$+1.03_2$	$+0.51_2$	. . .	. . .	. . .
ζ Ursæ Minoris, S. P.	− 11.8	. . .	. . .	$+0.51_1$	-1.08_1	-0.80_1	-0.47_1	. . .
4 Draconis, S. P.	− 11.8	-0.68_1	$+1.36_2$	. . .	-0.73_4	. . .	. . .	-1.05_1
1 Draconis, S. P.	− 8.2	-0.11_2	$+0.71_2$	. . .	-0.41_2	. . .	-1.12_2	. . .
δ Ursæ Minoris, S. P.	− 3.4	$+0.25_5$	-0.18_6	$+0.95_6$	-0.02_5	-0.13_4	-0.14_6	$+0.01_4$
51 Cephei, S. P	− 2.8	-0.30_6	$+1.03_7$	$+0.62_6$	$+0.39_5$	$+0.85_7$	$+0.94_7$	$+0.70_6$
α Ursæ Minoris, S. P.	− 1.3	$+0.15_8$	-0.14_8	-0.63_8	-0.53_8	-0.87_8	-0.41_8	-0.93_8
λ Ursæ Minoris, S. P.	− 1.0	$+0.39_4$	$+0.65_3$	$+0.40_6$	$+0.41_3$	$+0.29_4$	$+0.03_3$	-0.09_5
λ Ursæ Minoris	+ 1.0	-0.25_6	$+1.06_7$	$+0.13_5$	$+0.50_5$	$+0.06_7$	$+0.56_4$	-0.02_5
α Ursæ Minoris	+ 1.3	-0.37_8	-0.05_8	-0.68_8	-0.16_8	-0.99_8	-0.39_8	-0.08_8
51 Cephei	+ 2.8	-0.71_6	-0.12_6	-0.14_6	-0.13_5	-0.73_4	-0.73_6	-1.02_6
δ Ursæ Minoris	+ 3.4	-0.17_7	$+0.15_7$	$+0.12_5$	-0.37_6	$+0.48_7$	$+0.05_7$	-0.10_7
ε Ursæ Minoris	+ 7.8	$+0.06_3$	-0.43_3	-0.17_1	-0.28_5	$+0.12_4$	-0.23_3	-0.28_3
1 Draconis	+ 8.2	. . .	$+0.38_2$	-0.72_3	$+1.32_3$	-1.12_1	$+0.38_1$	-0.67_2
4 Draconis	+ 11.8	$+0.18_1$	$+0.12_4$	$+0.38_2$	$+0.08_2$	$+0.24_2$	-1.38_1	-0.15_1
ζ Ursæ Minoris	+ 11.8	-1.63_2	. . .	-0.14_4	$+0.84_2$	$+0.39_2$	$+0.77_3$	-0.28_2
κ Cephei	+ 12.6	$+0.42_4$	$+1.02_2$	$+0.24_2$	$+0.84_2$	. . .	$+0.97_1$	$+0.12_3$
48 Cephei	+ 12.7	$+0.60_3$	$+0.87_3$	$+0.14_2$	$+0.35_2$	$+0.10_2$	-1.14_1	. . .
γ Cephei	+ 13.0	-0.43_3	-0.06_3	-0.28_5	-0.34_1	-2.28_2	. . .	-0.38_2
9 Draconis	+ 13.7	$+0.04_2$	-0.59_3	$+0.11_5$	. . .	. . .	-0.91_2	-0.72_1
β Ursæ Minoris	+ 15.4	-0.08_6	-0.16_6	-0.21_5	-0.53_5	-0.55_5	-0.48_5	-0.20_4
50 Cassiopeiæ	+ 18.1	$+1.23_3$	$+0.45_3$	. . .	$+0.20_2$	$+1.71_3$	$+1.46_1$	-1.14_1
κ Draconis	+ 19.6	. . .	-0.93_4	-0.87_1	$+0.20_2$	$+0.41_2$	-0.78_2	-0.45_2
β Cephei	+ 19.9	$+0.05_3$	-1.33_2	-0.05_3	$+0.29_3$	-0.63_2	. . .	-0.04_2
λ Draconis	+ 20.0	$+1.59_4$	-0.40_3	-0.37_3	-0.81_3	$+0.45_2$	-0.18_2	-0.26_2
ι Cassiopeiæ	+ 23.1	$+0.28_4$	. . .	. . .	. . .	-0.33_3	$+0.55_2$	$+0.08_1$
ι Cephei	+ 24.4	-0.22_4	-0.36_6	$+0.38_5$	$+0.27_3$	-0.13_4	$+0.33_2$	$+0.06_4$
α Ursæ Majoris	+ 27.6	$+0.20_6$	$+0.35_7$	-0.13_5	$+0.29_5$	$+0.55_6$	-0.59_4	$+0.31_5$
α Cephei	+ 27.9	-0.23_4	$+0.06_6$	$+0.34_3$	$+0.96_3$	-0.89_2	-0.31_6	$+0.12_4$
η Draconis	+ 28.2	$+0.86_2$	$+0.38_1$	. . .	. . .	-0.31_3	-0.31_2	-0.67_2
α Cassiopeiæ	+ 34.1	-0.34_2	$+0.05_5$	$+0.06_5$	$+0.19_5$	$+0.16_5$	-0.28_1	-0.16_3
β Draconis	+ 37.6	-0.07_1	$+0.28_5$	-0.35_5	$+0.24_5$	$+0.11_5$	-0.09_1	-0.20_3
γ Draconis	+ 38.5	$+0.55_4$	$+0.06_4$	-0.60_4	-0.17_4	$+0.17_6$	-0.18_6	-0.64_4
α Persei	+ 40.6	$+0.87_4$	-0.97_5	$+0.10_6$	-0.21_6	-0.02_5	-0.40_5	-0.61_4

TABLE XV.—***Corrections to* BOSS'S *north polar distances, etc.***—**Continued.**

Star.	N. P. D.	1866.	1867.	1868.	1869.	1870.	1871.	1872.
	°	"	"	"	"	"	"	"
α Aurigæ	+ 44.2	-0.13_2	$+0.59_2$	$+1.31_3$	-2.22_1	$+0.07_4$	-1.14_2	-1.42_4
α Cygni	+ 45.2	+0.056	$+0.14_4$	+1.086	$+0.37_2$	-1.05_4	$+0.44_3$	-0.30_4
α Lyræ	+ 51.3	−0.108	+0.568	-1.07_7	-0.11_5	−0.466	-0.03_3	-0.73_7
ι Aurigæ	+ 57.0	$+0.76_4$	$+0.97_5$	$+0.63_5$	$+1.06_4$	-0.36_3	$+0.06_3$	−0.816
ζ Cygni	+ 60.2	+0.466	+0.346	+1.166	$+0.38_3$	$+0.29_4$	-1.76_4	−0.086
β Tauri	+ 61.5	$+0.12_7$	$+0.70_7$	+0.686	$+0.23_4$	−0.086	-0.64_2	+0.026
α Andromedæ	+ 61.6	$+0.39_7$	+0.536	+0.466	$+0.56_4$	-0.71_5	-0.92_4	-1.09_5
β Geminorum	+ 61.7	+0.748	$+0.73_7$	$+0.69_7$	$+0.11_5$	−0.536	$+0.06_4$	-1.28_7
ε Bootis	+ 62.4	+0.356	$+0.70_7$	+0.686	$+0.57_4$	$+0.09_5$	-0.09_1	$+0.26_7$
α Coronæ	+ 62.9	$+0.85_7$	$+1.64_7$	$+0.50_5$	$+0.63_4$	-1.00_5	$+0.27_2$	-0.36_4
α Arietis	+ 67.1	$+0.32_5$	$+0.83_7$	$+0.85_5$	-0.31_5	+0.136	-0.47_4	-0.19_7
μ Geminorum	+ 67.4	$+0.47_5$	$+0.89_3$	$+0.75_2$	-0.23_2	$+0.03_2$	. . .	-0.42_3
δ Geminorum	+ 67.8	$+0.72_5$	+1.196	$+1.59_5$	-0.20_5	$+0.48_4$	-0.56_3	−0.316
δ Leonis	+ 68.8	+0.616	$+1.22_7$	$+1.05_3$	$+0.58_4$	−1.376	$+0.42_2$	-0.32_7
γ Leonis	+ 69.6	$+0.25_5$	+1.356	$+1.04_5$	-0.03_5	−0.486	-0.31_3	$+0.12_7$
β Arietis	+ 69.7	$+0.33_7$	$+1.49_7$	$+0.63_5$	-0.87_2	-0.18_5	-0.33_3	−0.356
α Bootis	+ 70.3	+0.408	$+0.95_7$	$+1.15_7$	$+0.49_5$	−0.116	$+1.45_2$	$+0.20_7$
η Bootis	+ 71.1	$+0.90_7$	+1.046	+0.336	-0.04_4	+0.046	$+1.54_1$	-0.10_7
γ Geminorum	+ 73.5	+0.408	+0.906	$+0.37_5$	−0.176	$+0.40_5$	-0.96_3	$+0.07_7$
α Tauri	+ 73.7	$+0.63_7$	$+0.54_7$	+0.386	$+0.39_4$	−0.646	$+0.42_3$	+0.358
β Leonis	+ 74.8	+0.906	$+1.02_7$	$+1.10_5$	$+0.22_5$	−0.216	$+1.33_2$	−0.236
α Pegasi	+ 75.4	+0.516	+0.836	$+1.25_5$	-0.49_5	-0.08_4	$+0.06_5$	+0.046
γ Pegasi	+ 75.4	$+0.18_5$	$+0.79_7$	+1.896	-0.07_5	-0.30_2	-0.24_3	$+0.08_5$
α Herculis	+ 75.5	$+0.81_7$	$+0.95_5$	$+0.39_5$	-0.74_2	-0.33_5	-0.29_2	$+0.11_5$
α Ophiuchi	+ 77.3	$+1.37_7$	$+0.68_7$	-0.14_5	$+0.09_2$	-0.91_5	-0.11_2	$+0.20_5$
α Leonis	+ 77.5	+0.686	$+0.98_7$	$+1.40_5$	$+0.12_5$	-0.56_7	-1.24_2	-0.62_7
γ Aquilæ	+ 79.6	$+0.91_5$	+0.926	+0.246	$+1.74_3$	−0.146	-0.14_5	-0.42_7
ζ Pegasi	+ 79.7	$+1.18_5$	$+1.14_4$	-0.02_4	$+0.42_1$	-0.42_2	-0.21_5	-0.43_4
ε Pegasi	+ 80.6	$+0.41_5$	$+0.48_7$	$+1.25_5$	$+0.13_5$	-1.02_5	-0.09_3	−0.576
α Aquilæ	+ 81.6	$+0.35_7$	$+0.77_7$	$+0.09_7$	$+1.45_2$	−1.086	$+0.07_4$	-0.77_7
α Hydræ	+ 81.8	$+0.83_5$	$+0.38_7$	+1.296	$+0.25_4$	-0.71_4	-0.32_2	-0.67_5
α Orionis	+ 82.6	$+0.60_7$	+1.136	$+0.66_4$	$+0.24_4$	+0.076	-1.13_2	+0.166
ε Piscium	+ 82.7	$+0.60_5$	+1.276	$+1.34_5$	. . .	$+0.16_2$	-0.61_4	-0.02_5
ε Hydræ	+ 83.2	$+1.10_3$	+0.726	$+0.85_5$	+0.456	−1.116	-0.93_2	-0.34_4
α Serpentis	+ 83.7	+0.816	$+0.90_7$	+1.186	$+0.60_4$	+0.286	-0.76_2	-0.26_5
β Aquilæ	+ 83.9	$+0.38_4$	$+0.68_5$	$+1.10_5$	$+1.62_1$	-0.09_5	-0.25_4	-0.36_5
α Ceti	+ 86.3	−0.266	$+0.31_7$	$+0.37_5$	$+0.38_2$	$+0.24_3$	-1.03_4	-0.98_5
ζ Virginis	+ 90.0	$+0.84_7$	+0.776	+1.096	$+0.78_4$	+0.466	-0.19_2	−0.476
α Aquarii	+ 90.8	$+0.97_5$	$+0.12_5$	$+1.30_5$	$+0.33_2$	-0.80_2	$+0.08_4$	-0.62_4
η Serpentis	+ 92.9	. . .	. . .	. . .	. . .	. . .	. . .	. . .
β Orionis	+ 98.3	$+0.28_7$	+0.966	-0.19_4	$+0.61_2$	-0.37_7	$+1.65_2$	-0.34_7
ζ Ophiuchi	+100.4	$+0.82_5$	$+1.03_5$	$+0.84_4$	$+0.59_2$	-0.02_2	$+0.26_2$	$+0.56_4$
α Virginis	+100.6	+0.408	+0.618	−0.256	-0.56_5	−0.056	-0.34_2	-0.24_7

TABLE XV.—*Corrections to* **BOSS'S** *north polar distances, etc.*—Continued.

Star.	N. P. D.	1873.	1874.	1875.	1876.	1877.	1878.	1879.
	°	″	″	″	″	″	″	″
α Aurigæ	+ 44.2	−0.244	−0.483	−1.183	+0.015	+0.535	−1.262	−0.545
α Cygni	+ 45.2	−0.843	−0.746	−0.056	−0.667	+0.037	−0.104	−0.156
α Lyræ	+ 51.3	+1.196	−0.437	−0.247	+0.477	−0.768	−0.366	+0.507
ι Aurigæ	+ 57.0	+0.474	+0.615	+0.597	+1.316	+1.615	+0.145	+0.974
ζ Cygni	+ 60.2	+0.486	−0.087	+0.226	+0.177	+0.097	+0.294	−0.046
β Tauri	+ 61.5	+0.625	+0.196	+0.796	+0.917	+1.196	−0.094	+1.356
α Andromedæ	+ 61.6	+1.544	+0.176	+0.937	+0.588	+0.648	+0.616	+0.807
β Geminorum	+ 61.7	+1.126	−0.037	+0.517	+1.057	+1.945	+0.944	+1.086
ε Bootis	+ 62.4	+0.886	−0.457	+1.307	+0.717	+0.376	+0.896	+0.757
α Coronæ	+ 62.9	+0.926	+0.467	+0.437	+0.806	+0.615	+0.895	+1.667
α Arietis	+ 67.1	+1.246	−0.028	+0.938	+0.258	+1.017	+0.527	+0.626
μ Geminorum	+ 67.4	+0.425	−0.965	+0.655	+0.387	+0.504	−0.135	+0.063
δ Geminorum	+ 67.8	+0.483	+0.276	+0.585	+0.945	+0.515	+0.125	+0.663
δ Leonis	+ 68.8	+0.936	−0.587	+0.706	+0.607	+0.685	+0.817	+0.296
γ Leonis	+ 69.6	+0.455	+0.107	+0.646	+1.058	+1.587	−0.426	+0.336
β Arietis	+ 69.7	+1.497	+0.257	+0.828	+0.538	+0.626	+0.757	+0.457•
α Bootis	+ 70.3	+1.046	+0.177	+0.597	+0.607	+0.697	−0.256	+0.527
η Bootis	+ 71.1	+1.055	+0.177	+1.377	+0.538	+0.627	−0.247	+0.887
γ Geminorum	+ 73.5	+1.366	+0.576	+0.967	+0.708	+1.064	+0.166	+1.386
α Tauri	+ 73.7	+0.616	+0.49	+0.72	+0.51	+1.456	+0.277	+2.035
β Leonis	+ 74.8	+0.675	−0.107	+0.127	+0.887	+1.135	+0.026	+0.426
α Pegasi	+ 75.4	+0.695	+0.726	+0.467	+0.917	+0.876	+0.406	+0.637
γ Pegasi	+ 75.4	+0.954	+0.457	+0.517	+1.337	+0.948	+0.446	+0.877
α Herculis	+ 75.5	+0.525	+0.406	+1.176	+0.395	+0.77	+0.314	+1.134
α Ophiuchi	+ 77.3	+0.745	−0.325	+0.595	+1.036	+0.675	+0.284	+0.344
α Leonis	+ 77.5	+0.905	−0.508	+1.146	+0.468	+0.707	−0.016	+0.877
γ Aquilæ	+ 79.6	+1.096	+0.367	+0.687	+0.057	+0.477	+0.346	+0.673
ζ Pegasi	+ 79.7	+1.572	+0.055	+0.336	+0.046	+0.576	+0.545	+0.774
ε Pegasi	+ 80.6	+1.024	+0.066	+0.206	+0.266	+0.937	+0.674	+0.395
α Aquilæ	+ 81.6	+0.736	+0.057	+1.147	+0.107	−0.247	+0.025	−0.214
α Hydræ	+ 81.8	+0.814	−0.097	+1.176	+0.367	+0.236	+0.567	+0.785
α Orionis	+ 82.6	+1.245	+0.196	+0.674	+0.807	+1.574	+0.134	+0.846
ε Piscium	+ 82.7	+1.766	−0.337	+0.607	+0.605	+0.176	+0.516	+1.096
ε Hydræ	+ 83.2	+0.643	+0.595	+0.925	+0.027	+0.526	+0.825	+1.344
α Serpentis	+ 83.7	+1.285	−0.207	+0.697	+0.157	+0.586	+0.576	+0.986
β Aquilæ	+ 83.9	+3.202	+0.684	+0.012	. . .	+0.213	−0.773	+1.373
α Ceti	+ 86.3	+1.154	−0.706	+0.637	+0.726	+0.625	+0.196	+0.485
ζ Virginis	+ 90.0	+1.495	−0.015	+1.207	+0.797	+1.565	+0.536	+0.785
α Aquarii	+ 90.8	+0.804	+0.016	+0.266	+0.215	+0.537	−0.505	+1.555
η Serpentis	+ 92.9	. . .	. . .	+2.135	+0.674	+1.355	+2.015	+2.404
β Orionis	+ 98.3	+0.396	−0.457	+0.967	+0.697	+1.616	−0.214	+0.395
ζ Ophiuchi	+100.4	+0.703	+0.925	+1.315	+0.615	+0.665	+0.415	+1.354
α Virginis	+100.6	+0.436	−0.107	+0.488	+0.507	+0.156	−0.665	+0.537

TABLE XV.—*Corrections to* BOSS'S *north polar distances, etc.*—Concluded.

Star.	N. P. D.	1880.	1881.	1882.	1883.	1884.	1885.	1886.
	°	″	″	″	″	″	″	″
α Aurigæ	+ 44.2	-0.55_6	-0.54_6	-0.97_6	$+0.01_5$	-0.72_6	-0.66_6	-1.59_5
α Cygni	+ 45.2	-0.95_5	$+0.37_6$	-0.17_6	-0.09_6	-0.29_5	-0.23_6	-0.55_5
α Lyræ	+ 51.3	-0.03_7	$+0.05_7$	-0.16_7	$+0.03_6$	-0.10_7	$+0.18_8$	-0.36_7
ι Aurigæ	+ 57.0	$+1.65_6$	$+0.65_4$	-0.31_5	$+0.49_6$	$+1.09_6$	$+0.22_6$	-0.03_5
ζ Cygni	+ 60.2	$+1.64_5$	$+0.03_6$	$+0.07_4$	$+0.21_6$	$+0.54_7$	-0.01_5	-0.26_5
β Tauri	+ 61.5	$+1.14_6$	$+0.46_6$	-0.12_6	$+1.10_6$	$+0.68_7$	$+0.22_7$	$+0.27_5$
α Andromedæ	+ 61.6	$+1.70_7$	$+0.31_7$	$+0.19_8$	$+0.79_7$	$+1.10_6$	$+0.05_7$	$+0.61_7$
β Geminorum	+ 61.7	$+1.09_5$	$+1.12_6$	$+0.22_7$	$+0.82_4$	$+1.16_7$	$+0.96_6$	$+0.45_4$
ε Bootis	+ 62.4	$+1.73_7$	-0.08_7	$+0.06_6$	$+0.22_6$	$+0.89_6$	-0.24_7	$+0.96_6$
α Coronæ	+ 62.9	$+2.38_6$	$+0.53_6$	$+0.32_6$	$+0.71_7$	$+1.19_7$	$+0.84_7$	$+0.37_6$
α Arietis	+ 67.1	$+1.05_7$	$+0.37_7$	$+0.37_5$	$+0.86_6$	$+0.73_7$	$+0.14_5$	$+0.74_6$
μ Geminorum	+ 67.4	$+1.33_5$	$+0.40_4$	$+0.64_6$	$+0.66_5$	$+0.38_5$	-0.28_6	$+0.24_4$
δ Geminorum	+ 67.8	$+1.43_5$	-0.15_4	$+0.61_6$	$+0.24_5$	$+0.47_6$	$+0.11_6$	$+0.39_6$
δ Leonis	+ 68.8	$+1.54_7$	-0.48_7	-0.91_4	$+.064_4$	$+0.70_7$	$+0.57_6$	$+0.27_6$
γ Leonis	+ 69.6	$+1.29_6$	$+0.79_5$	$+0.25_7$	$+1.11_4$	$+0.88_6$	$+0.16_7$	$+0.08_7$
β Arietis	+ 69.7	$+2.05_6$	$+0.23_7$	$+0.24_6$	$+0.10_6$	$+0.83_6$	-0.23_5	$+0.33_7$
α Bootis	+ 70.3	$+2.31_6$	$+0.43_7$	$+0.61_7$	$+0.16_6$	$+0.35_8$	$+0.46_7$	$+1.14_7$
η Bootis	+ 71.1	$+2.20_6$	$+0.79_6$	$+0.62_6$	-0.09_6	$+0.61_7$	$+0.12_7$	$+0.53_6$
γ Geminorum	+ 73.5	$+1.32_6$	$+0.59_7$	$+0.53_7$	$+0.64_6$	$+0.61_6$	$+0.76_6$	$+0.17_5$
α Tauri	+ 73.7	$+2.00_6$	$+0.98_6$	$+0.68_6$	$+1.27_6$	$+1.38_6$	$+1.14_7$	$+0.55_5$
β Leonis	+ 74.8	$+0.67_5$	$+0.18_7$	$+0.82_5$	$+0.81_6$	$+1.40_7$	$+0.15_6$	$+0.83_7$
α Pegasi	+ 75.4	$+1.47_6$	$+0.11_6$	$+0.80_7$	$+0.50_5$	$+0.99_7$	-0.64_5	$+0.41_5$
γ Pegasi	+ 75.4	$+2.25_6$	$+0.38_6$	$+0.03_6$	$+0.59_7$	$+0.98_7$	$+0.03_7$	$+0.42_6$
α Herculis	+ 75.5	$+1.95_5$	$+0.72_5$	-0.16_3	$+0.21_4$	$+0.64_7$	-0.23_5	$+0.21_6$
α Ophiuchi	+ 77.3	$+2.07_6$	$+1.32_5$	-0.01_4	$+0.02_6$	$+0.70_7$	-0.28_7	-0.09_5
α Leonis	+ 77.5	$+2.03_6$	$+0.42_6$	$+0.23_7$	$+1.22_6$	$+1.08_7$	$+0.81_7$	$+0.75_7$
γ Aquilæ	+ 79.6	$+1.01_7$	$+0.30_6$	$+0.73_5$	$+0.43_5$	$+0.71_5$	$+0.96_5$	$+0.83_5$
ζ Pegasi	+ 79.7	$+0.88_5$	$+0.95_5$	-0.16_7	$+0.38_5$	$+0.71_{10}$	$+0.18_3$	$+0.51_5$
ε Pegasi	+ 80.6	$+1.75_7$	-0.08_6	$+0.26_4$	$+0.75_5$	-0.09_6	-0.19_6	$+0.46_7$
α Aquilæ	+ 81.6	$+1.32_6$	-0.52_6	$+0.86_5$	-0.43_5	$+0.39_6$	$+0.27_6$	$+0.39_7$
α Hydræ	+ 81.8	$+1.89_5$	$+0.53_6$	$+0.45_7$	$+1.03_5$	$+1.17_5$	$+1.19_5$	$+0.34_6$
α Orionis	+ 82.6	$+1.70_7$	$+0.70_7$	$+0.51_6$	$+0.16_4$	$+0.77_7$	-0.18_5	$+0.52_6$
ε Piscium	+ 82.7	$+0.76_6$	$+0.16_5$	-0.64_4	-0.48_5	-0.11_5	$+0.09_7$	$+1.11_5$
ε Hydræ	+ 83.2	$+1.79_5$	$+0.11_4$	$+0.09_6$	$+1.25_5$	$+0.52_5$	$+0.04_4$	$+0.03_5$
α Serpentis	+ 83.7	$+1.60_6$	$+0.36_5$	-0.89_5	$+0.60_7$	$+0.82_7$	$+0.24_7$	$+0.11_6$
β Aquilæ	+ 83.9	$+1.05_5$	$+1.13_3$	$+0.10_4$	$+0.08_4$	$+1.18_6$	-0.05_4	$+0.24_6$
α Ceti	+ 86.3	$+1.58_6$	$+0.08_6$	$+0.03_6$	-0.15_5	-0.20_7	-0.25_6	-0.18_5
ζ Virginis	+ 90.0	$+2.38_5$	$+0.16_6$	$+0.19_5$	$+0.76_5$	$+0.43_4$	$+1.48_3$	-0.13_6
α Aquarii	+ 90.8	$+1.43_6$	$+0.94_5$	-0.01_5	$+0.78_5$	$+0.15_6$	-0.27_5	$+0.58_5$
η Serpentis	+ 92.9	$+1.56_3$	$+2.02_5$	$+0.79_5$	$+1.55_4$	$+1.82_5$	$+1.41_4$	$+1.47_5$
β Orionis	+ 98.3	$+1.47_6$	$+0.78_5$	-0.92_5	$+1.07_5$	$+0.32_6$	$+0.48_7$	-0.35_5
ζ Ophiuchi	+100.4	$+2.62_5$	$+0.62_5$	$+0.09_2$	$+0.69_4$	$+1.19_4$	$+0.35_5$	$+0.12_5$
α Virginis	+100.6	$+0.75_6$	$+0.63_5$	-0.24_6	-0.76_4	-0.26_7	-0.93_4	$+0.73_4$

§ 6 *Corrections to reduce the annually concluded north polar distances of the Washington transit circle to* Boss's *standard.*

We have now to derive the corrections necessary to reduce the places of southern stars in Table XV to Boss's standard. One of the first questions to be considered is, whether this reduction can be taken as constant for each year through the entire arc from the zenith to the southern limit of the zodiac. To test this I have divided the southern stars of Table XV into three groups, as follows:

Group I.	ι Aurigæ to δ Leonis, north polar distance	. . . .	57°.0 to	68°.8
II.	γ Leonis to ζ Pegasi, " " "	. . . .	69.6 to	79.7
III.	ϵ Pegasi to α Virginis, " " "	. . . .	80.6 to	100.6

In order to extend the comparison to the southern zodiacal limits I have added a fourth group, comprising those 15 stars of the American Ephemeris between the limits of 12° and 27° of south declination, which were most frequently observed. But, as Boss's places were not used in the Ephemeris until 1881, I have deemed it sufficient to make the comparison for the four years, 1881–'84, when it can be effected directly.

In taking the means equal weight was given to each star, but from 1866 to 1874 η Serpentis was excluded, owing to the small number of observations upon it.

The resulting mean corrections to Boss's north polar distances are shown in the following table:

TABLE XVI.—*Mean corrections to* BOSS'S *north polar distances by zones.*

Year.	Zone I, 57° to 69°.	Zone II, 69° to 80°.	Zone III, 80° to 101°.	Zone IV. 102° to 117°.	Mean, I to III.
	"	"	"	"	"
1866	+0.53	+0.65	+0.58	. . .	+0.59
1867	+0.89	+0.97	+0.72	. . .	+0.86
1868	+0.82	+0.76	+0.78	. . .	+0.79
1869	+0.31	+0.11	+0.49	. . .	+0.30
1870	−0.28	−0.28	−0.29	. . .	−0.28
1871	−0.36	+0.01	−0.26	. . .	−0.19
1872	−0.18	−0.07	−0.35	. . .	−0.22
1873	+0.83	+0.94	+1.12	. . .	+0.97
1874	−0.04	+0.20	+0.05	. . .	+0.08
1875	+0.63	+0.72	+0.82	. . .	+0.74
1876	+0.70	+0.64	+0.46	. . .	+0.58
1877	+0.83	+0.87	+0.75	. . .	+0.79
1878	+0.45	+0.18	+0.29	. . .	+0.30
1879	+0.75	+0.81	+0.94	. . .	+0.84
1880	+0.52	+1.68	+1.58	. . .	+1.60
1881	+0.29	+0.59	+0.51	+0.42	+0.47
1882	+0.10	+0.37	+0.04	+0.07	+0.18
1883	+0.61	+0.52	+0.46	+0.53	+0.52
1884	+0.81	+0.85	+0.54	+0.75	+0.72
1885	+0.23	+0.24	+0.25	. . .	+0.24
1886	+0.36	+0.48	+0.36	. . .	+0.40
Mean.	+0.47	+0.53	+0.47	. . .	+0.49

We have the following mean deviations of the several groups from the mean of Groups I to III:

		$''$
Group	I	-0.02
	II	$+0.04$
	III	-0.02
	IV	$+0.03$

The evidence of no marked variation with the declination is so strong that I deem it unnecessary to investigate the subject farther, and regard the results in the last column as definitive corrections for reducing all the declinations south of $+30^\circ$ north, as observed with the transit circle, and finally derived in the annual volumes of Washington Observations, to Boss's system.

§ 7. *Corrections to* Boss's *north polar distances, given by observations with the Washington and Greenwich transit circles.*

In Table XIII we have given, for each year, the mean corrections to the standard north polar distances of southern stars, as inferred from direct observations only. If we express these means in the form of a quantity varying uniformly with the time, the result will be

$$\Delta \text{N. P. D.} = -0''.09 - 2''.13T$$

T being the fraction of a century after 1875.

The evidence, whatever it may be worth, is that in former times the north polar distances required a systematic positive correction, and the declinations a negative correction. In 1755 the correction would be $2''.5$. The weight of this result is, of course, very small, yet its agreement with that derived from the Greenwich observations is worthy of remark. We have, in fact, the following three results for the systematic correction to the north polar distances of Boss's standard in 1755:

	$''$
Washington transit circle, 1866–'86	$+2.5$
Greenwich transit circle, 1851–'87	$+1.0$
AUWER'S BRADLEY	$+1.7$

In deriving a correction to individual stars, if we ignore the systematic differences of the results for different years, and confine ourselves to the results of direct observations, our course will be to correct each year in Table XV, except 1871–'72, by -2α, and then take the mean result for each star. Were the same number of observations made on each star in each year the result thus derived would be the definitive one. But, as undoubted systematic differences in different years are shown by Table XIII, we proceed as follows:

Firstly, we apply a certain correction k to all the numbers for each year in Table XV to reduce the results approximately to one standard. Let

$$k_1, k_2, k_3 \quad . \quad . \quad . \quad k_n$$

be the values of these corrections for n consecutive years. The result will be that the mean of all

the north polar distances for these n years, as thus corrected, will be greater than the mean instrumental standard by the quantity

$$K=\frac{k_1+k_2+k_3\ \ .\ \ .\ \ .\ \ +k_n}{n}$$

Then by subtracting this constant from the mean of the corrected north polar distances of each star we obtain its north polar distance, as measured with the instrument itself, reduced to the mean of all the instrumental results as the standard. It will be seen that were an equal number of observations made on any star in each year we should thus have the mean result of all the observations made upon it without correction.

It will be seen that each value of k merges with the value of 2α for the year. The adopted values of $k-2\alpha$ are as follows:

	$k-2\alpha$	k		$k-2\alpha$	k
	″	″		″	″
1866	−0.59	+0.02	1877	−0.73	+0.06
1867	−0.87	−0.12	1878	−0.29	+0.37
1868	−0.79	−0.11	1879	−0.82	+0.04
1869	−0.28	−0.10	1880	−1.57	+0.05
1870	+0.28	−0.03	1881	−0.48	+0.10
1871	+0.16	+0.16	1882	−0.19	+0.38
1872	+0.23	+0.23	1883	−0.57	−0.02
1873	−0.92	−0.50	1884	−0.78	+0.26
1874	−0.05	+0.84	1885	−0.25	+0.50
1875	−0.72	−0.01	1886	−0.43	+0.41
1876	−0.63	+0.18			

The values of k are substantially the negatives of the ΔN. P. D. from direct observations found in Table XIII. For 1871–'72 2α is not necessary, as it was not applied in obtaining the printed results.

I have thought it advisable to divide the results into two periods, the one from 1866 to 1876, the other from 1877 to 1886. The mean values of k are

Period I, 1866–'76	K = +0.05″
Period II, 1877–'86	K = +0.21

We thus have the results given under the heading "Washington" in the first three columns of the following table.

For the sake of comparison I have added, in the last three columns, the mean results of the Greenwich observations as they follow from the corrections derived in the first chapter of the present paper. I have, however, limited the comparison to the stars made use of in that chapter.

TABLE XVII.—***Mean corrections to the north polar distances of 69 standard stars as concluded from observations with the Washington and Greenwich transit circles.***

A.—NORTHERN STARS.

Stars.	Washington.			Greenwich.		
	1866 to 1876.	1877 to 1886.	Mean.	1851 to 1869.	1870 to 1887.	Mean.
	"	"	"	"	"	"
λ Ursæ Minoris	—0.02	—0.17	—0.10	+0.08	—0.13	—0.02
α Ursæ Minoris	—0.10	—0.19	—0.15	+0.01	—0.01	0.00
Cephei 51	—0.39	—0.71	—0.55	—0.13	—0.21	—0.17
δ Ursæ Minoris	—0.13	—0.30	—0.21	+0.13	+0.16	+0.15
ε Ursæ Minoris	—0.35	—0.38	—0.36	...	...	...
1 Draconis	—0.22	—0.42	—0.32	...	...	...
4 Draconis	0.00	—0.14	—0.07	...	...	...
ζ Ursæ Minoris	—0.04	—0.21	—0.12	...	...	...
κ Cephei	+0.09	+0.13	+0.11	...	...	...
48 Cephei	—0.10	—0.36	—0.23	...	...	...
γ Cephei	—0.42	—0.44	—0.43	+0.13	—0.24	—0.05
9 Draconis	+0.02	—0.40	—0.19	...	...	...
β Ursæ Minoris	+0.07	—0.20	—0.07	—0.15	—0.45	—0.30
50 Cassiopeæ	—0.26	+0.27	0.00	+0.06	+0.79	+0.42
κ Draconis	—0.09	—0.39	—0.24	+0.08	—0.31	—0.12
β Cephei	—0.07	—0.63	—0.35	...	...	...
λ Draconis	—0.25	—0.38	—0.32	...	...	...
ι Cassiopeæ	+0.45	—0.25	+0.10	...	...	...
ι Cephei	+0.07	—0.08	0.00	...	...	...
α Ursæ Majoris	—0.15	—0.06	—0.10	+0.09	+0.21	+0.15
α Cephei	+0.06	—0.26	—0.10	+0.21	+0.15	+0.18
η Draconis	—0.20	—0.16	—0.18	...	...	...
α Cassiopeæ	+0.08	—0.30	—0.11	—0.03	+0.03	0.00
β Draconis	—0.29	—0.18	—0.23	...	...	...
γ Draconis	—0.10	—0.30	—0.20	+0.17	+0.10	+0.13
α Persei	+0.16	—0.30	—0.07	+0.08	—0.23	—0.07
α Aurigæ	—0.35	—0.80	—0.57	+0.26	+0.23	+0.25
α Cygni	—0.26	—0.40	—0.33	+0.08	—0.02	+0.03

TABLE XVII.—*Mean corrections to the north polar distances of 69 standard stars, etc.*—**Concluded.**

B.—SOUTHERN STARS.

Star.	Washington.				Greenwich.		
	Direct observations.			½(D+R) with Pulkowa refractions.	1851 to 1869.	1870 to 1887.	Mean.
	1866 to 1876.	1877 to 1886.	Mean.				
	"	"	"	"	"	"	"
α Lyræ	−0.14	−0.31	−0.23	. . .	+0.21	−0.21	0.00
ι Aurigæ	+0.03	−0.18	−0.08	+0.40	+0.07	−0.21	−0.07
ζ Cygni	−0.24	−0.59	−0.42	+0.05	+0.14	−0.04	+0.05
β Tauri	−0.08	−0.19	−0.13	+0.33	+0.20	−0.23	−0.01
α Andromedæ	−0.18	−0.15	−0.17	+0.29	+0.14	−0.05	+0.04
β Geminorum	−0.16	+0.15	0.00	+0.46	−0.05	−0.07	−0.06
ε Bootis	0.00	−0.27	−0.13	+0.33	. . .	.	. . .
α Coronæ	+0.04	+0.14	+0.09	+0.55	+0.18	+0.19	+0.18
α Arietis	−0.09	−0.19	−0.14	+0.31	+0.33	+0.40	+0.36
μ Geminorum	−0.34	−0.41	−0.37	+0.08	+0.24	−0.13	+0.06
δ Geminorum	+0.09	−0.36	−0.14	+0.31	+0.26	−0.06	+0.10
δ Leonis	−0.16	−0.39	−0.28	+0.17	. . .	. . .	. . .
γ Leonis	0.00	−0.23	−0.11	+0.33	. . .	. . .	. . .
β Arietis	+0.02	−0.28	−0.13	+0.31	+0.32	+0.13	+0.22
α Bootis	+0.08	−0.18	−0.05	+0.39	+0.23	0.00	+0.12
η Bootis	+0.06	−0.23	−0.09	+0.35	. . .	. . .	. . .
γ Geminorum	+0.05	−0.10	−0.03	+0.40	+0.11	+0.12	+0.12
α Tauri	−0.05	+0.35	+0.15	+0.58	+0.26	−0.07	+0.09
β Leonis	−0.02	−0.17	−0.09	+0.34	+0.37	+0.41	+0.39
α Pegasi	+0.06	−0.24	−0.09	+0.33	+0.41	+0.08	+0.25
γ Pegasi	+0.13	−0.13	0.00	+0.42	+0.40	+0.23	+0.31
α Herculis	−0.04	−0.28	−0.16	+0.26	+0.52	−0.04	+0.24
α Ophiuchi	−0.10	−0.32	−0.21	+0.21	+0.24	+0.18	+0.21
α Leonis	−0.14	−0.01	−0.08	+0.34	+0.13	+0.21	+0.17
γ Aquilæ	−0.03	−0.20	−0.12	+0.29	+0.34	+0.34	+0.34
ζ Pegasi	−0.17	−0.30	−0.23	+0.18	. . .	. . .	. . .
ε Pegasi	−0.27	−0.35	−0.31	+0.10	. . .	. . .	. . .
α Aquilæ	−0.28	−0.63	−0.45	−0.04	+0.25	+0.22	+0.23
α Orionis	+0.08	−0.21	−0.06	+0.34	+0.09	−0.03	+0.03
ε Piscium	+0.12	−0.61	−0.25	+0.15	. . .	. . .	. . .
ε Hydræ	−0.18	−0.18	−0.18	+0.22	. . .	. . .	. . .
α Serpentis	+0.02	−0.30	−0.14	+0.26	. . .	. . .	. . .
β Aquilæ	+0.14	−0.35	−0.10	+0.30	. . .	. . .	. . .
α Ceti	−0.40	−0.61	−0.50	−0.11	. . .	. . .	. . .
ζ Virginis	+0.20	−0.06	+0.07	+0.44	. .	. . .	. . .
α Aquarii	−0.20	−0.32	−0.26	+0.11	−0.02	−0.54	−0.28
η Serpentis	+0.55	+0.84	+0.70	+1.05	. . .	. . .	. . .
α Hydræ	−0.12	−0.02	−0.07	+0.34	−0.06	−0.18	−0.12
β Orionis	−0.20	−0.32	−0.26	+0.07	0.00	−0.19	−0.10
ζ Ophiuchi	+0.27	0.00	+0.13	+0.44	. . .	. . .	. . .
α Virginis	−0.33	−0.82	−0.57	−0.26	−0.27	0.18	−0.22

§ 8. *Effect of reduction to the Pulkowa refractions and application of* $\frac{1}{2}(R-D)$.

Notwithstanding the much better accordance of the direct than of the reflex observations, the systematic difference between the two classes, as represented by 4α, is, except in the three cases, 1870, 1871–'72, and 1880, fairly accordant.

The mean values for the two periods are:

	Including discordant years.	Excluding discordant years.
	$''$	$''$
1866–'76	$2\alpha=+0.62$	$2\alpha=+0.63$
1877–'86	$2\alpha=+0.83$	$2\alpha=+0.75$

It is, therefore, quite possible that, in obtaining the general mean correction to equatorial stars, the systematic error from which α arises may affect the two classes equally, so that $\frac{1}{2}(R+D)$ may be the correct standard.

Again, all the reductions have been made with the refractions of the *Tabulæ Regiomontanæ*, and I have made no change on this account. So few observations have been made within 10° of the north horizon that the observations afford no data whatever for correcting the constant of refraction. The preponderance of evidence seeming to be in favor of the Pulkowa refractions, I have also found the mean result of adopting the Pulkowa refractions.

The constant terms of the reductions of the north polar distances on these two accounts are:

	$''$
For reduction of latitude to Pulkowa refractions	-0.23
For mean value of 2α	$+0.72$
Sum of constant terms	$+0.49$

This sum is diminished by the difference between the Pulkowa refraction and that of the *Tabulæ Regiomontanæ* in the case of each star, and the difference added to the third column of Table XVII. Thus we have the results given in the fourth column.

A partial test of the general accuracy of these mean results may be applied by comparing them with the mean corrections to the sun's tabular north polar distance. The reductions of the annual volumes give—

	$''$
Mean correction to sun's north polar distance, 1866–1884	$+0.60$
Mean correction to Boss's standard	$+0.62$

This agreement is evidence in favor of the correctness of the standard, but when the comparison is made by years a negative correction to the proper motion in north polar distance is shown, and the conclusions on page 471 receive additional support.

CHAPTER III.

DETERMINATION OF THE CONSTANT OF NUTATION FROM THE RESULTS OF THE PRECEDING DISCUSSIONS AND FROM THE RIGHT ASCENSIONS OF THE FOUR POLAR STARS OBSERVED AT GREENWICH.

§ 1. *General considerations.*

The number of observations on which the preceding corrections to the north polar distances rest is so great that, systematic errors aside, an extremely accurate value of the constant of nutation should be derivable from them. But as systematic errors undeniably exist, we have to inquire in what way they may affect the value of this constant. It has been shown that the systematic errors which remain after correcting the Washington observations of the circumpolar stars for the apparently varying latitude are probably small. They are rendered yet smaller by the fact that separate determinations from upper and lower culminations of stars near the pole are available. Probably the same thing is true of the Greenwich results, though I have not investigated the subject so fully. It is, therefore, in the observations of stars south of the zenith that we are most in danger of error from the cause in question.

The examination of the question will be facilitated if we begin by constructing the formula for correcting the nutation and show in what manner the results derived from it will be affected by various systematic errors. Let us put—

N; the constant of nutation,

☊; the longitude of the Moon's ascending node,

then the apparent north polar distance of a star whose right ascension is α will be affected by the inequality—

$$-\mathrm{N} \cos ☊ \sin \alpha + 0.745\, \mathrm{N} \sin ☊ \cos \alpha.$$

In this formula the solar nutation is omitted, not only on account of the minuteness of any admissible correction which it may require, but because its only appreciable term goes through two periods in the course of any year.

If we determine the two quantities M and N from the equations—

$$m \sin \mathrm{M} = 0.745 \sin ☊$$
$$m \cos \mathrm{M} = \cos ☊$$

we have

$$\text{Nutation in north polar distance} = \mathrm{N}m \sin (\mathrm{M} - \alpha)$$

If we put ν for the correction of the adopted constant of nutation (that of PETERS), the quantity $m \sin (\mathrm{M} - \alpha)$ will be the coefficient of ν in the apparent north polar distance of the star.

As the probable correction to PETERS's constant of nutation is very small, probably as small a quantity as the totality of the observations with any one instrument during a period of the Moon's node suffice to certainly indicate, I have assumed that one value of this coefficient, namely, that corresponding to the middle of the year, will answer for an entire year. This assumes that the mean epoch of all the observations of the star made in any one year coincides with the middle of the year. The greatest error of this hypothesis in the case of any one star will ordinarily not exceed three months, and the small errors thus introduced into the coefficients, if systematic at all, will nearly neutralize each other in the course of one period of the Moon's node.

The following table shows the values of the coefficients for the years in question.

TABLE XVIII.—*For computing coefficients of the constant of nutation.*

Year.	Ω	M	Log *m*.	Year.	Ω	M	Log *m*.
	°	°			°	°	
1851	117.1	124.6	9.905	1870	109.6	115.6	9.891
1852	97.8	100.4	9.875	1871	90.3	90.4	9.872
1353	78.4	74.6	9.881	1872	71.0	65.2	9.890
1854	59.1	51.2	9.913	1873	51.6	43.2	9.930
1855	39.8	31.8	9.957	1874	32.2	25.2	9.970
1856	20.4	15.5	9.988	1875	12.9	9.7	9.995
1857	1.0	0.7	0.000	1876	353.5	355.1	9.999
1858	341.7	346.1	9.990	1877	334.2	339.2	9.981
1859	322.3	330.1	9.960	1878	314.8	323.1	9.945
1860	303.0	311.0	9.918	1879	295.5	302.7	9.902
1861	283.7	288.1	9.881	1880	276.1	278.1	9.874
1862	264.3	262.4	9.874	1881	256.8	252.5	9.881
1863	245.0	237.9	9.901	1882	237.4	229.4	9.918
1864	225.7	217.4	9.944	1883	218.1	210.3	9.960
1865	206.3	200.2	9.981	1884	198.8	194.2	9.990
1866	187.0	185.2	9.999	1885	179.4	179.6	0.000
1867	167.7	170.8	9.996	1886	160.1	164.9	9.988
1868	148.3	155.3	9.972	1887	140.8	148.7	9.957
1869	129.0	137.4	9.932				

Now, any error peculiar to one star and affecting all the measures of that star by the same amount will, of course, be eliminated in the nutation, and will be seen only in the concluded north polar distance of the star after the nutation is determined. Such errors are those arising from all the observations being made on a single graduation the error of which is imperfectly determined; those arising from the color, and therefore the refraction of the star being different from those of other stars. If the instrument is subject to any error by virtue of which the observations in one hour of right ascension are systematically in error, this result will also, for the same reason, be eliminated from the nutation, because each star maintains practically its own right ascension We may, therefore, say in general, that all errors which remain the same from year to year will be eliminated.

But the preceding investgation shows that the Washington observations and, in a less degree, those at Greenwich are also subject to systematic errors, by which all the north polar distances observed in one year are systematically different from those observed in other years. The question arises how far this error will affect the nutation. The reply is that, during any one year, or, in fact,

during any one night, the coefficients of nutation have opposite signs in opposite hours of right ascension. Hence supposing, for the moment, that these errors extend through the entire year, they may be completely eliminated, either by introducing a correction peculiar to each year into the equations of condition, or by so adjusting the weights of the observations in different hours of right ascension that the errors shall be completely eliminated from the result. The first is, of course, the correct way of proceeding.

There is, however, in the case of the Greenwich observations no reason for assigning the beginning of the calendar year as the moment at which systematic errors of the kind in question change. The question therefore arises how far the nutation will be affected if the systematic error in question varies in the course of a year.

The answer is that if the variation in question is the same from year to year it will be, in its systematic effect, eliminated from the constant of nutation and appear only in the concluded polar distances of the individual stars.

So far, however, as the years are unlike in this particular the constant of nutation will be systematically affected. That is to say, the constant of nutation will be affected only by systematic changes in the annual law of error. Just in so far as these changes happen to follow a 19-year period so far will the constant of nutation be affected. Since there is no conceivable reason why such changes in the changes of error should follow the period of the Moon's node we have no systematic errors to fear that extend through more than one revolution of that node and none common to any one instrument or any one observatory. Whatever minute errors may thus be introduced into the work of one 19-year period will not affect the work of any other 19-year period with the same instrument.

It appears therefore that in order to determine the constant in question with the utmost freedom from systematic error we should seek to determine as unknown quantities not only the constant of nutation, but the following quantities:

(1) Apparent correction to the north polar distance of each individual star, which apparent correction will include all errors peculiar to observations of that star.

(2) A common correction to all the north polar distances observed during any one year.

Let us put,

δq, the common correction to all the north polar distances of any one year.

δp_1, δp_2, δp_3, etc., the additional corrections to the north polar distances of the separate stars.

ν, the correction to PETERS's constant of nutation.

N, the coefficient of this correction in the apparent N. P. D. of any one star.

w, the weight of all the observations on one star in one year.

ΔP, the mean apparent correction to the north polar distance of a star from all the observations of any one year.

The observations upon all the stars during any one year will give rise to equations of condition as follows:

$$\begin{array}{c} \delta q + \delta p_1 + N_1 \nu = \Delta P_1 \\ \delta q + \delta p_2 + N_2 \nu = \Delta P_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \end{array}$$

and the normal equation for determining δq may be written in the form

$$W\delta q = \Sigma w \Delta P - (\Sigma w N)\nu - \Sigma w \delta p$$

W being the sum of all the weights for the year.

In this equation the last term represents as many unknown quantities as there are stars.

The equations of condition arising from the observations of any one star through a period of 19 years will be of the form

$$\begin{array}{c} \delta p + \delta q_1 + N_1\nu = \varDelta P_1 \\ \delta p + \delta q_2 + N_2\nu = \varDelta P_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \end{array}$$

and the normal equation for the nutation will be

$$(\Sigma w N^2)\nu = \Sigma w N \varDelta P - (\Sigma w N)\delta p - w_1 N_1 \delta q_1 - w_2 N_2 \delta q_2 - \ldots\ldots.$$

The final normal equation for ν from all the observations of all the stars will be the sum of all the normal equations of this form. In this sum the coefficient of any one δq will be of the form

$$w'N' + w''N'' + w'''N''' + \ldots\ldots$$

in which the values of w and N are those corresponding to the different stars for the years in question. But each of these values of N goes through a complete period; in 18.6 years, and if the stars and their weights are evenly distributed in right ascension this coefficient will vanish. Since its value depends only upon the unequal distribution it will probably be very small.

Hence we may use an approximate expression for the value of δq in each year without systematically affecting the nutation in any appreciable degree. If we determine it by the simple rule that, after its application, the mean residual without respect to a change in the constant of nutation shall be the same for each year of the period, we shall have no systematic error to fear in the present case, when the southern stars are scattered so evenly round the circle of right ascension.

§ 2. *Nutation from the Greenwich north polar distances.*

The residuals which I have used for determining what nutation is indicated by the Greenwich north polar distances are fundamentally those of Table VIII. As just pointed out, we may use the residuals unchanged for the northern stars; but for southern stars we should apply for each year such common corrections to all the residuals as will approximately reduce their mean value for the year to some quantity which shall be the same for all the years. There are, however, two deviations from this method.

In the first place I too hastily reached the conclusion that the systematic differences from year to year were so small that they might be left out of consideration, in view of the fact that their effects will be nearly equal and opposite in opposite hours of right ascension.

In the next place, the original numbers of Table VIII were in error for three years. In 1851–'52 the flexure coefficient was originally supposed to be 0''.50, instead of 0''.74, so that an ulterior correction of $-0''.24 \sin Z$ was necessary. And in reducing 1877 it was not noticed that, in the concluded tabular results, the reduction from STONE's to BESSEL's refractions had been applied.

Instead of repeating the entire computation an approximate correction for these errors was made. The results will be fully shown subsequently.

Since the interval of observations embraces almost two revolutions of the Moon's node it was divided into two periods, 1851–'69 and 1870–'87. For reasons already made apparent, stars culminating north and south of the zenith have been treated separately.

To show exactly the method of computation I subjoin a copy of the complete computation of

the nutation from the observations of α Andromedæ during the period 1851–'69, using the uncorrected values of the residuals ΔP.

TABLE XIX.—*Computation of the correction to* PETERS'S *constant of nutation from the north polar distances of α Andromedæ observed with the Greenwich transit circle from* 1851 *to* 1869.

Year.	M − α	*log* sin.	*log* N.	*log* N²	N	N²	ΔP	a w	b wN	c wN²	P′ wΔP	P″ wNΔP	
	°						″				″	″	
1851	124.2	9.918	9.823	9.646	+0.665	.443	−0.35	7	+4.65	3.10	− 2.5	−1.6	$\frac{b}{a} = + 0.09$
1852	100.0	9.993	9.868	9.736	+0.738	.544	−0.03	6	+4.43	3.26	− 0.2	−0.1	
1853	74.2	9.983	9.864	9.728	+0.731	.534	+0.08	5	+3.65	2.67	+ 0.4	+0.3	$-\frac{b}{a} b = - 0.63$
1854	50.8	9.889	9.802	9.604	+0.634	.404	+0.02	4	+2.54	1.62	+ 0.1	+0.1	$c = +22.95$
1855	31.4	9.717	9.674	9.348	+0.471	.222	−0.20	4	+1.88	0.89	− 0.8	−0.4	$-\frac{b}{a} b + c = +22.32$
1856	15.1	9.416	9.404	8.808	+0.254	.064	−0.50	2	+0.51	0.13	− 1.0	−0.3	
1857	0.3	7.719	7.719	5.438	+0.005	.000	+0.49	4	+0.02	0.00	+ 2.0	−0.0	$-\frac{b}{a} P' = - 1.04$
1858	345.7	9.393	9.383	8.766	−0.242	.058	−0.20	2	−0.48	0.11	− 0.4	+0.1	$P'' = - 7.10$
1859	329.7	9.703	9.663	9.326	−0.461	.212	+0.96	6	−2.76	1.27	+ 5.8	−2.7	$-\frac{b}{a} P' + P'' = - 8.14$
1860	310.6	9.880	9.798	9.596	−0.629	.395	+0.70	4	−2.51	1.58	+ 2.8	−1.8	
1861	287.7	9.979	9.860	9.720	−0.725	.526	+0.41	2	−1.45	1.05	+ 0.8	−0.6	
1862	262.0	9.996	9.870	9.740	−0.743	.550	+0.45	3	−2.23	1.65	+ 1.4	−1.0	$v = -0''.36$
1863	237.5	9.926	9.827	9.654	−0.672	.451	−0.32	3	−2.01	1.35	− 1.0	+0.6	
1864	217.0	9.779	9.723	9.446	−0.529	.280	+0.31	5	−2.64	1.40	+ 1.6	−0.8	
1865	199.8	9.530	9.511	9.022	−0.325	.105	+0.05	4	−1.30	0.42	+ 0.2	−0.1	
1866	184.8	8.922	8.921	7.842	−0.083	.007	−0.16	4	−0.33	0.03	− 0.6	0.0	
1867	170.4	9.213	9.209	8.418	+0.162	.026	+0.18	3	+0.48	0.08	+ 0.5	+0.1	
1868	154.9	9.628	9.600	9.200	+0.399	.159	+0.40	4	+1.59	0.64	+ 1.6	+0.6	
1869	137.0	9.834	9.766	9.532	+0.583	.340	+0.18	5	+2.92	1.70	+ 0.9	+0.5	
								77	+6.96	22.95	+11.6	−7.1	

The resulting corrections to the nutation from all the southern stars are given in column v (a) of Table XX.

The evident periodicity of the results in this column arises from the systematic excess of the measured north polar distances in 1858–'60, and their deficiency in 1851 and 1864–'69. The effect of this will, as already shown, be nearly eliminated from the mean of all the v's, but to ascertain its influence in this special case I reduced the residuals ΔP for the southern stars approximately to one standard by the application of the following corrections:

	″
1851	+0.20
1858	−0.20
1859	−0.60
1860	−0.20
1865 to 1869	+0.20

The effect of applying these corrections was then computed, and the results are given in the column (b). Round numbers, and in most cases equal numbers, were taken for the corrections to ΔP in order to simplify the computation of the corrections to v.

The column w gives the weight to the nearest unit.

In the columns wa, etc, the weights are multiplied by the values of v, but, as will be seen from Table XIX, the products are formed in the course of the computation, and, in fact, each value of v is found by dividing this product by w.

To test the hypothesis that the effect of the large systematic differences during the period 1851–'69 on the general mean is purely periodic, I have computed the mean value of ν on the supposition that it is, accidental errors aside, of the form

$$\nu = \nu_0 + y \sin \alpha + z \cos \alpha$$

The numbers from which I have started to form the equations of condition are found in the columns sin α and cos α. The resulting normal equations are

$$\begin{array}{llll} 988\,\nu_0 + 81 & y + 39 & z = - & 128''.8 \\ 81 & + 623 & + 19 & = - 158 \\ 39 & + 19 & + 355 & = - 24 \end{array}$$

The coefficients of ν, y, and z may be in error by three or four units, owing to omitted decimals, but it is easily seen that this error is unimportant.

The values of the unknown quantities from these equations are

$$\begin{aligned} y &= -0''.24 \\ z &= -0.04 \\ \nu_0 &= -0.109 \end{aligned}$$

The mean of column (b) gives

$$\nu = -0''.108$$

We have, therefore, completely eliminated the error introduced by the systematic differences of the north polar distances by treating it as periodic.

The numbers in the columns 1870–1887 are formed like those of column (a), no systematic correction being applied. The results of applying any such correction may be safely assumed to be evanescent.

Table XX.—*Corrections to* **Peters's** *constant of nutation given by the north polar distances of southern stars observed with the Greenwich transit circle.*

Stars.	1851 to 1869.					sin α	cos α	*ws*	*wc*	1870 to 1887.		
	v (*a*)	*v* (*b*)	*w*	*wa*	*wb*					*v*	*w*	*wv*
	″	″								″		″
α Andromedæ	−0.36	−0.18	23	− 8.1	− 4.2	+0.01	+1.00	+ 0	+23	−0.14	14	− 1.8
γ Pegasi	+0.21	+0.37	20	+ 4.1	+ 7.6	+0.03	+1.00	+ 1	+20	−0.26	10	− 2.7
β Arietis	−0.35	−0.13	18	− 6.2	− 2.3	+0.45	+0.89	+ 8	+16	+0.07	16	+ 1.2
α Arietis	−0.52	−0.29	30	− 15.5	− 8.6	+0.50	+0.87	+15	+26	−0.20	18	− 3.6
α Persei	−0.12	+0.14	17	− 1.9	+ 2.3	+0.75	+0.66	+13	+11	−0.42	11	− 4.5
α Tauri	−0.36	−0.15	46	− 16.6	− 6.9	+0.92	+0.39	+42	+18	−0.19	31	− 5.7
ι Aurigæ	−0.29	−0.03	25	− 7.1	− 0.7	+0.95	+0.30	+24	+ 8	+0.01	12	+ 0.1
α Aurigæ	−0.20	0.00	36	− 7.2	+ 0.1	+0.97	+0.23	+35	+ 8	−0.39	15	− 5.7
β Orionis	−0.60	−0.28	39	− 23.3	− 10.9	+0.97	+0.22	+38	+ 9	−0.03	21	− 0.6
β Tauri	−0.17	+0.05	39	− 6.4	+ 1.8	+0.98	+0.18	+38	+ 7	−0.17	25	− 4.4
α Orionis	−0.48	−0.28	44	− 20.7	− 12.2	+1.00	+0.05	+44	+ 2	+0.31	30	+ 9.3
η Geminorum	−0.27	−0.11	32	− 8.6	− 3.4	+1.00	−0.07	+32	− 2	−0.25	9	− 2.3
γ Geminorum	−0.37	−0.19	20	− 7.4	− 3.8	+0.99	−0.13	+20	− 3	+0.27	23	+ 6.0
δ Geminorum	−0.26	−0.12	30	− 8.2	− 3.6	+0.95	−0.31	+28	− 9	+0.27	16	+ 4.3
β Geminorum	−0.19	−0.05	44	− 8.4	− 2.3	+0.91	−0.41	+40	−18	+0.15	31	+ 4.8
α Hydræ	−0.25	−0.18	18	− 4.6	− 3.2	+0.64	−0.77	+12	−14	+0.32	14	+ 4.4
α Leonis	−0.15	−0.10	32	− 4.7	− 3.1	+0.49	−0.87	+16	−28	+0.34	23	+ 7.6
β Leonis	−0.26	−0.34	20	− 5.1	− 6.7	+0.07	−1.00	+ 1	−20	−0.03	14	− 0.5
α Canum Venat.	−0.26	−0.45	12	− 3.3	− 5.6	−0.22	−0.98	− 3	−12	+0.32	11	+ 3.6
α Virginis	+0.12	−0.10	39	+ 4.5	− 3.9	−0.34	−0.94	−13	−37	−0.41	24	− 9.7
η Ursæ Majoris	−0.27	−0.48	15	− 3.9	− 7.0	−0.43	−0.90	− 6	−14	−0.45	14	− 6.2
α Bootis	+0.04	−0.21	41	+ 1.6	− 8.4	−0.54	−0.84	−22	−34	−0.01	36	− 0.5
α Coronæ	+0.11	−0.11	39	+ 4.4	− 4.4	−0.79	−0.61	−31	−24	−0.01	25	− 0.2
α Herculis	+0.09	−0.13	31	+ 2.7	− 4.0	−0.98	−0.22	−30	− 7	+0.13	19	+ 2.3
α Ophiuchi	+0.07	−0.13	50	+ 3.6	− 6.5	−0.99	−0.14	−50	− 7	−0.02	37	− 0.9
α Lyræ	+0.29	+0.13	53	+ 15.0	+ 6.8	−0.99	−0.14	−52	+ 7	−0.10	54	− 5.4
γ Aquilæ	+0.14	−0.03	32	+ 4.4	− 1.1	−0.91	−0.42	−29	+13	+0.04	18	+ 0.8
α Aquilæ	+0.12	−0.02	42	+ 5.2	− 0.8	−0.90	+0.43	−38	+19	−0.11	33	− 3.7
α Cygni	+0.09	−0.06	21	+ 1.9	− 1.2	−0.77	+0.63	−16	+13	−0.46	18	− 8.2
ζ Cygni	−0.28	−0.38	28	− 7.7	− 10.8	−0.68	+0.73	−19	+20	+0.20	19	+ 3.6
α Aquarii	−0.25	−0.25	14	− 3.7	− 3.6	−0.50	+0.86	− 7	+12	−0.11	14	− 1.5
α Pegasi	−0.13	−0.07	17	− 2.2	− 1.2	−0.26	+0.97	− 4	+16	−0.19	11	− 2.0
β Pegasi	−0.44	−0.37	5	− 2.4	− 2.0	−0.27	+0.96	− 1	+ 5	+0.45	9	+ 3.9
μ Pegasi	+0.43	+0.44	16	+ 6.9	+ 7.0	−0.33	+0.95	− 5	+15	+0.24	10	+ 2.3
Sums	....	....	988	−128.9	−106.8	....	....	+81	+39	....	685	−15.9

Mean results: 1851–'69, $\nu = -0''.108 \pm 0''.037$
1870–'87, $\nu = -0''.023 \pm 0''.043$

TABLE XXI.—*Corrections to* PETERS'S *constant of nutation given by the north polar distances of circumpolar stars observed with the Greenwich transit circle.*

Stars.	1851 to 1869.			1870 to 1887.			
	v	w	wv	v	w	wv	
	″		″	″		″	
α Cassiopeæ	+0.10	11	+1.1	—0.07	7	—0.5	
Polaris	—0.06	51	—2.8	—0.03	48	—1.3	Mean results:
50 Cassiopeæ	—0.60	4	—2.4	+0.09	7	+0.6	1851–'69. $v = -0''.020 \pm 0''.029$.
51 Cephei	+0.08	49	+3.7	—0.13	62	—8.3	1870–'87. $v = -0''.001 \pm 0.''026$.
α Ursæ Majoris	—0.10	15	—1.6	+0.23	16	+3.7	
κ Draconis	—0.29	9	—2.6	—0.37	7	—2.6	
β Ursæ Minoris	—0.09	24	—2.2	+0.21	18	+3.7	
α Camelopardalis	+0.25	14	+3.6	+0.75	3	+1.9	
γ Draconis	+0.10	26	+2.6	—0.10	20	—2.0	
δ Ursæ Minoris	+0.01	54	+0.6	+0.03	64	+2.0	
λ Ursæ Minoris	—0.15	28	—4.2	+0.05	51	+2.5	
α Cephei	—0.04	18	—0.8	00	18	0.0	
γ Cephei	—0.12	9	—1.1	—0.01	10	—0.1	
Sums	....	312	—6.1	....	331	—0.4	

Table XXI, giving the results from stars culminating north of the zenith, needs no explanation.

§ 3. *Nutation from the Washington north polar distances.*

To obtain the nutation from the Washington observations, the corrections $k - 2\alpha$, found on p. 472 were applied to all the results of each year for southern stars, and the corrections to the nutation were derived from the residuals. This course is justified by the minuteness of the effect of any one value of $k - 2\alpha$ upon the final result for the constant. The results are given in detail in Table XXII. The method of computation is so nearly the same as in the case of Greenwich that no additional statement is required.

In the case of northern stars, for reasons already set forth, no systematic correction is required.

TABLE XXII.—*Corrections to* PETERS'S *constant of nutation given by the north polar distances observed with the Washington transit circle,* 1866–1886.

I.—SOUTHERN STARS.

Star.	v	w	wv
	"		"
α Andromedæ	−0.20	29	− 5.8
γ Pegasi	+0.02	25	+ 0.4
ε Piscium	+0.41	26	+10.5
α Arietis	+0.02	42	+ 0.7
β Arietis	−0.09	37	− 3.5
α Ceti	−0.30	43	−13.1
α Tauri	−0.08	67	− 5.1
ι Aurigæ	−0.13	57	− 7.2
β Orionis	−0.12	65	− 8.1
β Tauri	−0.15	69	−10.4
α Orionis	−0.12	61	− 7.1
η Geminorum	+0.15	53	+ 7.9
γ Geminorum	−0.17	70	−11.8
δ Geminorum	+0.02	56	+ 1.3
β Geminorum	+0.07	67	+ 4.6
ε Hydræ	+0.02	37	+ 0.8
α Hydræ	+0.33	40	+13.2
α Leonis	+0.37	40	+14.7
γ Leonis	−0.02	35	− 0.8
δ Leonis	−0.06	30	− 1.8
β Leonis	−0.04	28	− 1.1
α Virginis	−0.08	33	− 2.6
ζ Virginis	+0.05	32	+ 1.6
η Bootis	+0.07	39	+ 2.7
α Bootis	−0.21	45	− 9.6
ε Bootis	0.00	47	0.0
α Coronæ	+0.07	52	+ 3.6
α Serpentis	−0.12	58	− 6.8
ζ Ophiuchi	+0.14	48	+ 6.6
α Herculis	+0.21	52	+10.8
α Ophiuchi	+0.14	61	+ 8.9
η Serpentis	+0.08	38	+ 3.0
α Lyræ	0.00	79	− 0.1
γ Aquilæ	−0.13	60	− 7.9
α Aquilæ	0.00	64	0.0
β Aquilæ	+0.11	33	+ 3.7
ζ Cygni	−0.03	46	− 1.2
ε Pegasi	+0.02	37	+ 0.8
α Aquarii	−0.13	34	− 4.2
ζ Pegasi	−0.06	26	− 1.6
α Pegasi	+0.11	30	+ 3.2
		1,891	−10.8

II.—NORTHERN STARS.

Star.	v	w	wv
	"		"
α Cassiopeæ	+0.27	12	+ 3.3
50 Cassiopeæ	−0.24	15	− 3.7
Polaris	0.00	47	+ 0.1
ι Cassiopeæ	+0.32	14	+ 4.5
48 Cephei	0.00	21	− 0.1
α Persei	+0.02	35	+ 0.8
α Aurigæ	−0.07	44	− 3.0
51 Cephei	−0.11	64	− 6.7
1 Draconis	+0.05	18	+ 0.8
9 Draconis	−0.49	13	− 6.2
α Ursæ Majoris	+0.55	24	+13.1
λ Draconis	0.00	11	0.0
4 Draconis	−0.27	10	− 2.6
κ Draconis	+0.03	7	+ 0.2
β Ursæ Minoris	+0.12	28	+ 3.1
ζ Ursæ Minoris	−0.19	18	− 3.6
η Draconis	+0.06	16	+ 0.9
ε Ursæ Minoris	+0.05	46	+ 2.4
β Draconis	−0.25	23	− 5.6
γ Draconis	+0.12	46	+ 5.6
δ Ursæ Minoris	−0.23	75	−17.5
λ Ursæ Minoris	+0.02	53	+ 1.1
κ Cephei	−0.27	15	− 4.2
α Cygni	−0.25	47	−11.5
α Cephei	−0.15	29	− 4.3
β Cephei	+0.10	16	+ 1.5
ι Cephei	−0.05	14	− 0.7
γ Cephei	−0.44	9	− 3.8
		770	−36.1

RESULTS.

	"	"
From southern stars	$v = -0.006$	± 0.023
From northern stars	$v = -0.047$	± 0.037

§4. *Constant of nutation from the right ascensions of the four polar stars observed with the Greenwich transit circle.*

Although not included in the main object of the present paper, I have deemed it well to supplement the preceding determination of the constant of nutation by one derived from the right ascensions observed with the same instrument during the same period. Of course the only observations of this class at all adapted to this purpose are those of the four polar stars. Were the utmost refinement attempted we should have to determine the relative personal equations of the individual observers for each of the stars, so as to reduce the observations of each to a common standard. Such an investigation would, however, not add enough to the precision of the result to be worth entering upon, and I have allowed the error thus arising to add itself to the accidental errors. It is true that during any one nodal period the error thus arising will be systematic, since a change of observers may well correspond to changes in the coefficient of nutation. But there is no reason why such a correspondence should be of the same nature in any two nodal periods; it is therefore certain that we have no error to fear from this cause which will affect two determinations in the same way. Indeed in the same period it will tend to act differently in the case of different stars; its effect will be nearly equal and opposite in the cases of δ Ursæ Minoris and Cephei 51, and will be markedly different for Polaris and λ Ursæ Minoris.

Nor have I seen that any advantage would arise from an attempt to correct the annual means found in the published volumes. A result is there printed only when it is real; that is, when the azimuth of the instrument depends on double transits of the same star. We have, then, two cases, (1) when the right ascension of a star, say Polaris, depends on a double transit of the star itself; (2) when it depends on a double transit of some other star. In the former case the result of each transit appears, though logically we have but a single determination But it is easily seen that the right ascension thus resulting would be entitled to double weight, because the effect of any error of azimuth is completely eliminated from the mean of the two transits. It is true that the azimuth may have changed in the interval; but the error thus produced is probably no larger than the error of azimuth derived from a double transit of any other star.

I conclude, therefore, that nothing better is to be done than to take the annual means as given in the several published volumes of the observations.

In forming the equations of condition I have taken as unknown quantities the mean correction to the right ascension of each star, the factor by which the adopted nutation of PETERS should be corrected, and the correction of the annual variation. It is true that the latter quantity can not be determined from the observations with an accuracy to compare with that of its standard value; but it is desirable to know the effect of any change upon the adopted constant.

If, then, we put

x, the correction to the tabular right ascension of the star, for which I take that given in the *Catalogue of* 1,098 *standard stars,* which is identical with that of the American Ephemeris since 1870,

y, the correction to the annual variation in right ascension,

N, the tabular amount of the nutation in right ascension,

μ, the factor by which N must be multiplied to obtain the actual nutation,

the mean of each year will give an equation of condition of the form

$$x + ty + N\mu = \delta\alpha$$

In strictness the factor N should be diminished by that portion of the nutation which is common to all the stars; but this was not done. In the case of the polar stars the effect upon the correction of the nutation will be but a small fraction of the probable error of the result.

The equations of condition have been formed on the same general plan as those given by the polar distances. For each coefficient has been taken its value for the middle of the year. The nearest entire number has been taken for each coefficient, as the product of any fraction of the coefficient by the correction must be evanescent. The weights have been assigned by the rule adopted in the north polar distances. On this system the weights of Polaris would be so nearly equal for each year that no distinction has been made between the weights for different years. The equations of condition and the results of solution are as follows:

Equations for correcting the nutation from right ascensions of the four polar stars observed at Greenwich.

POLARIS.

First Period, 1851–'69.

1851,	$1x$	$-9y$	$+5\mu$	$=-0.16$
1852,	1	-8	-3	$=-0.19$
1853,	1	-7	-10	$=-0.18$
1854,	1	-6	-17	$=+0.16$
1855,	1	-5	-22	$=-0.08$
1856,	1	-4	-24	$=+0.18$
1857,	1	-3	-23	$=-0.08$
1858,	1	-2	-20	$=-0.01$
1859,	1	-1	-15	$=-0.27$
1860,	1	0	-8	$=-0.05$
1861,	1	$+1$	$+1$	$=-0.57$
1862,	1	$+2$	$+9$	$=-0\ 52$
1863,	1	$+3$	$+16$	$=-0\ 51$
1864,	1	$+4$	$+21$	$=-0\ 30$
1865,	1	$+5$	$+24$	$=-0.68$
1866,	1	$+6$	$+25$	$=-0.39$
1867,	1	$+7$	$+22$	$=-0.55$
1868,	1	$+8$	$+17$	$=-0.65$
1869,	1	$+9$	$+10$	$=-0.42$

Result: $5551\mu = -69\ 4 - 1282y$;
Putting $y = -0^s.021$ we have
Correction of nutation $= -0''.071$.

POLARIS.

Second Period, 1870–'88.

1870,	$1x$	$-9y$	$+2\mu$	$=-1.33$
1871,	1	-8	-7	$=-1.01$
1872,	1	-7	-15	$=-0.82$
1873,	1	-6	-21	$=-1.13$
1874,	1	-5	-25	$=-0.81$
1875,	1	-4	-26	$=-0.79$
1876,	1	-3	-24	$=-0.83$
1877,	1	-2	-19	$=-1.11$
1878,	1	-1	-12	$=-1.06$
1879,	1	0	-4	$=-0.90$
1880,	1	$+1$	$+4$	$=-1.14$
1881,	1	$+2$	$+13$	$=-1.09$
1882,	1	$+3$	$+20$	$=-1.08$
1883,	1	$+4$	$+25$	$=-1.39$
1884,	1	$+5$	$+27$	$=-1.11$
1885,	1	$+6$	$+26$	$=-0.97$
1886,	1	$+7$	$+22$	$=-0.82$
1887,	1	$+8$	$+15$	$=-0.63$
1888,	1	$+9$	$+7$	$=-0.84$

Result: $6487\mu = -17.2 - 1438y$;
Putting $y = -0^s.021$ we have
Correction of nutation $= +0''.018$.

Equations for correcting the nutation from right ascensions of the four polar stars, etc.—Continued.

CEPHEI 51.

First Period, 1851–'69.

1851,	$1x$	$-9y$	-10μ	$=+0.47$	Wt. $=6$
1852,	1	-8	-10	$=-0.43$	3
1853,	1	-7	-10	$=+1.15$	2
1854,	1	-6	-8	$=+0.87$	7
1855,	1	-5	-5	$=+0.68$	6
1856,	1	-4	-2	$=-0.11$	7
1857,	1	-3	$+2$	$=+1.09$	5
1858,	1	-2	$+5$	$=+0.32$	8
1859,	1	-1	$+8$	$=+0.38$	8
1860,	1	0	$+10$	$=+0.56$	6
1861,	1	$+1$	$+10$	$=-0.46$	5
1862,	1	$+2$	$+10$	$=+0.25$	3
1863,	1	$+3$	$+8$	$=+0.52$	6
1864,	1	$+4$	$+6$	$=+0.91$	6
1865,	1	$+5$	$+3$	$=-0.38$	4
1866,	1	$+6$	-1	$=+0.32$	4
1867,	1	$+7$	-4	......	0
1868,	1	$+8$	-7	$=+0.40$	6
1869,	1	$+9$	-9	$=+0.78$	1

Result: $4796\mu = -46.7 - 1375y$;

Putting $y = 0$ we have

Correction of nutation $= -0''.089$.

CEPHEI 51.

Second Period, 1870–'88.

1870,	$1x$	$-9y$	-10μ	$=+0.53$	Wt. $=6$
1871,	1	-8	-10	$=+0.32$	7
1872,	1	-7	-9	$=+0.35$	6
1873,	1	-6	-6	$=+0.29$	6
1874,	1	-5	-4	$=+0.49$	6
1875,	1	-4	0	$=+0.43$	7
1876,	1	-3	$+3$	$=+0.17$	7
1877,	1	-2	$+7$	$=+0.26$	6
1878,	1	-1	$+9$	$=+0.05$	7
1879,	1	0	$+11$	$=+0.79$	5
1880,	1	$+1$	$+10$	$=-0.01$	6
1881,	1	$+2$	$+9$	$=+0.65$	7
1882,	1	$+3$	$+7$	$=+0.63$	7
1883,	1	$+4$	$+4$	$=+0.51$	7
1884,	1	$+5$	$+1$	$=+0.65$	7
1885,	1	$+6$	-3	$=+0.40$	7
1886,	1	$+7$	-6	$=+0.59$	6
1887,	1	$+8$	-8	$=+1.47$	6
1888,	1	$+9$	-10	$=+0.15$	6

Result: $6515\mu = -44.0 - 783y$;

Putting $y = 0$ we have

Correction of nutation $= -0''.062$.

Equations for correcting the nutation from right ascensions of the four polar stars, etc.—**Continued.**

δ URSÆ MINORIS.

First Period, 1851–'69.

Year	x	y	μ		Wt.
1851,	$1x$	$-9y$	$+6\mu$	$=+0.06$	Wt. $=7$
1852,	1	-8	$+6$	$=+0.36$	6
1853,	1	-7	$+6$	$=+0.23$	4
1854,	1	-6	$+5$	$=-0.33$	7
1855,	1	-5	$+4$	$=-0.20$	6
1856,	1	-4	$+2$	$=+0.44$	7
1857,	1	-3	-1	$=-0.03$	5
1858,	1	-2	-3	$=+0.17$	8
1859,	1	-1	-5	$=+0.07$	8
1860,	1	0	-6	$=-0.17$	6
1861,	1	$+1$	-6	$=+0.26$	5
1862,	1	$+2$	-6	$=+0.24$	3
1863,	1	$+3$	-6	$=+0.05$	6
1864,	1	$+4$	-4	$=-0.15$	7
1865,	1	$+5$	-2	$=-0.03$	3
1866,	1	$+6$	0	$=-0.72$	4
1867,	1	$+7$	$+2$	$=\ ...$	0
1868,	1	$+8$	$+4$	$=-0.13$	7
1869,	1	$+9$	$+6$	$=-0.07$	2

Result: $2215\mu = -3.8 + 1120y$;

Putting $y = -0^{s}.015$ we have

Correction of nutation $= -0''.077$.

δ URSÆ MINORIS.

Second Period, 1870–'88.

Year	x	y	μ		Wt.
1870,	$1x$	$-9y$	$+6\mu$	$=-0.25$	Wt. $=6$
1871,	1	-8	$+6$	$=-0.62$	7
1872,	1	-7	$+6$	$=-0.34$	6
1873,	1	-6	$+5$	$=-0.29$	6
1874,	1	-5	$+3$	$=-0.83$	6
1875,	1	-4	$+1$	$=-0.16$	7
1876,	1	-3	-1	$=-0.53$	7
1877,	1	-2	-3	$=-0.35$	7
1878,	1	-1	-5	$=+0.02$	7
1879,	1	0	-6	$=-0.15$	6
1880,	1	$+1$	-6	$=-0.35$	6
1881,	1	$+2$	-6	$=-0.56$	6
1882,	1	$+3$	-5	$=-0.75$	6
1883,	1	$+4$	-4	$=-0.66$	7
1884,	1	$+5$	-2	$=-0.39$	7
1885,	1	$+6$	0	$=-0.14$	8
1886,	1	$+7$	$+3$	$=+0.06$	6
1887,	1	$+8$	$+4$	$=-0.11$	7
1888,	1	$+9$	$+6$	$=-0.27$	7

Result: $2494\mu = +14.4 + 766y$;

Putting $y = -0^{s}.015$ we have

Correction of nutation $= +0''.011$.

***Equations for correcting the nutation from right ascensions of the four polar stars, etc.*—Concluded.**

λ URSÆ MINORIS.

First Period, 1851–'69.

1851,	$1x$	$-9y$	$+24\mu$	$=+0.58$	Wt. $=6$
1852,	1	-8	$+20$	$=-0.08$	4
1853,	1	-7	$+14$	$=+0.33$	4
1854,	1	-6	$+7$	$=-0.62$	5
1855,	1	-5	-1	$=+0.68$	5
1856,	1	-4	-9	$=-0.03$	6
1857,	1	-3	-16	$=+0.53$	4
1858,	1	-2	-22	$=-0.63$	7
1859,	1	-1	-23	$=+0.68$	5
1860,	1	0	-25	$=-0.22$	3
1861,	1	$+1$	-22	$=-1.03$	4
1862,	1	$+2$	-17	$=-0.26$	2
1863,	1	$+3$	-11	$=-1.43$	5
1864,	1	$+4$	-3	$=+1.33$	2
1865,	1	$+5$	$+6$	$=-4.75$	2
1866,	1	$+6$	$+14$	$=-1.88$	2
1867,	1	$+7$	$+20$	$=-2.37$	1
1868,	1	$+8$	$+24$	$=$ . . .	0
1869,	1	$+9$	$+25$	$=-2.79$	2

Result: $20026\mu = -136.4 + 1684y$;

Putting $y = 0$ we have

Correction of nutation $= -0''.063$.

λ URSÆ MINORIS.

Second Period, 1870–'88.

1870,	$1x$	$-9y$	$+24\mu$	$=-0.43$	Wt. $=7$
1871,	1	-8	$+20$	$=-1.83$	6
1872,	1	-7	$+14$	$=-1.44$	4
1873,	1	-6	$+6$	$=-2.32$	7
1874,	1	-5	-2	$=-2.16$	6
1875,	1	-4	-10	$=-1.92$	6
1876,	1	-3	-17	$=-2.25$	7
1877,	1	-2	-22	$=-0.97$	6
1878,	1	-1	-25	$=-1.13$	6
1879,	1	0	-26	$=-1.44$	5
1880,	1	$+1$	-22	$=-0.86$	6
1881,	1	$+2$	-17	$=-1.12$	7
1882,	1	$+3$	-10	$=-2.58$	6
1883,	1	$+4$	-2	$=-1.65$	6
1884,	1	$+5$	$+7$	$=-1.24$	7
1885,	1	$+6$	$+14$	$=-0.83$	7
1886,	1	$+7$	$+20$	$=-0.20$	6
1887,	1	$+8$	$+24$	$=-1.93$	6
1888,	1	$+9$	$+26$	$=+0.18$	6

Result: $37321\mu = +466.2 - 1584y$;

Putting $y = 0$ we have

Correction of nutation $= +0''.115$.

It will be seen that in the case of Polaris the resulting nutation is largely dependent upon the adopted correction of the proper motion in right ascension. There is, however, a remarkable difference between the proper motion which would be given by all the extant observations and that given by the series of 38 years in question. The tabular places represent the observations from 1755 to 1869 so closely that we can scarcely suppose an error of more than one second per century possible. But if we consider only the series in question the 37 years' observations give the result

$$y = -0^{s}.031$$

This large value of y from the observed places of the star is perhaps to be attributed to personal differences among the observers. Assuming that difference to be in the mean progressive it would undoubtedly give the value of y which ought to be adopted in the equations. But, as a matter of fact, we should rather consider it as the result of certain small changes taking place *per saltum* as one observer retired and another took his place. Differences thus arising would be so much in the nature of progressive ones that I consider the best result will be obtained in the case of Polaris by giving double weight to the proper motions derived from the observations themselves. In the case of 51 Cephei the tabular proper motion is so uncertain that I shall use the tabular motions given by the observations. We thus have the following values of the corrections to the constant of nutation given by the observations in question:

First period, 1851–'69:

Polaris	$\nu = -0''.071$
Cephei 51	-0.089
δ Ursæ Minoris	-0.077
λ Ursæ Minoris	-0.063
Mean	-0.075

Second period, 1870–'88:

Polaris	$\nu = +0''.018$
Cephei 51	-0.062
δ Ursæ Minoris	$+0.011$
λ Ursæ Minoris	$+0.115$
Mean	$+0.020$